3D FINITE ELEMENT MODELLING OF BOND-SLIP BETWEEN REBAR AND CONCRETE IN PULL-OUT TEST

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ABSTRACT
A reinforced concrete material is a composite material made up of two components with unequal mechanical behaviour and physical features. In general, the external load is already applied to concrete and the reinforcing bars receive its part of the load only from the surrounding concrete by bond. In composite structures, the bond between different components of reinforced concrete member has a primordial role and its negligence conducted to poor structural response. Therefore, for modeling of reinforced concrete structures one needs a simple and realistic bond-slip model. There are various finite element models for bond-slip relationship between reinforcement and concrete. In this paper, modeling of the transition region between steel and concrete as a cohesion layer in the finite element program (Ansys) is discussed. A 3D finite element model to represent this layer has been introduced. The layer involves modeling the ribs and effects of slip and bond stress of the bar. The accuracy of the models is assessed by comparison of the finite element numerical response with experimental data from pullout test.

Keywords: pull-out test, finite element, bond-slip relationship

1. INTRODUCTION
A reinforced concrete (RC) structure is a composite structure made up of two materials with different characteristics, namely, concrete and steel. In general, the external load is already applied to concrete and the reinforcing bars receive its part of the load only from the surrounding concrete by bond. “Bond stress” is the name assigned to the shear stress at the bar-concrete interface which, by transferring load between the bar and the surrounding concrete, modifies the steel stresses. This bond, when efficiently developed, enables the two materials to form a composite structure. In composite structures, the bond between different components of reinforced concrete member has a primordial role and its negligence is conducted to poor structural response. These complex phenomena have led engineers in the past to rely heavily on empirical formulas for the design of concrete structures, which were derived from numerous experiments. For these reasons, the incorporation of bond is carried out considerably in recent works. The properties of this interaction depend on several factors, such as friction, mechanical interaction and chemical adhesion [1, 2].
In the past, a number of experimental investigations have been carried out in order to clarify and understand the behaviour of deformed bars pulled out from a concrete block under monotonic and cyclic loading conditions. These experimental results are well documented in the specific literature [9]. Based only on the experimental results it is difficult to filter out the influences of material and geometrical parameters on the bond behaviour. Therefore, to better understand the bond behaviour, a reliable bond model (simulation of the transmission of forces in the bond zone, see Figure 1a) that can be employed in a three-dimensional finite element, an analysis is needed. The numerical modelling of the bond behaviour is principally possible at two different levels: (1) detailed modelling (see Figure 1b) in which the geometry of the bar and the concrete is modelled by three-dimensional elements and (2) phenomenological modelling (see Figure 1c) based on a smeared or discrete formulation of the bar-concrete interface [3].

In the phenomenological modelling of bond the concrete and the reinforcement are discretised by two- or three-dimensional finite elements. The link between the bar and the concrete can be realized by a discontinuous approach where bond is defined by discrete, zero-thickness elements (springs) whose behaviour is controlled by the bond stress-slip relationship. This approach is able to realistically predict the bond behaviour for different geometries and for different boundary conditions only if a realistic constitutive model for the surrounding concrete is used. However, the model is not able to automatically predict the bond behaviour of a given bar geometry. Consequently, the influence of these parameters must be stored in advance in the basic parameters of the bond model. Thus one has the possibility to realistically simulate the behavior of reinforced concrete structures with relatively low effort in modelling and computing time. By the use of detailed modelling, such as both modelling of the ribs of the reinforcement and the concrete lugs (see Figure 1b) between the ribs of the reinforcement a quite fine finite element mesh has to be generated. This leads again to a high effort in modelling work and also to a really long computing time in particular while carrying out a finite element analysis on complex reinforced concrete structures [3, 4].

2. REINFORCEMENT FINITE ELEMENT MODELS
In finite element modelling of reinforced concrete structures, there are three
different alternative representations of reinforcement: smeared, embedded and discrete reinforcement models. The first one is rarely used and therefore it depends on the nature of used structure. The discrete and embedded representations are formulated and introduced in the developed program [4, 5].

2.1. Discrete Reinforcement Representation
The discrete modelling of steel reinforcement is the first approach used in finite element analysis of reinforced concrete structures [10]. The discrete representation of reinforcement uses one dimensional truss elements and it is the only way for accounting for bond slip and dowel action effects, Figure 2.

![Figure 2. Discrete representation of steel bars [4]](image)

A significant advantage of discrete representation is that it can account for possible displacement of the reinforcement with respect to the surrounding concrete. The bond effects are usually related with this representation and the bond-link or cohesive models can be used to connect the steel and concrete nodes in order to consider this effect. The main disadvantage is that the finite element mesh patterns are only restricted by the location of reinforcement and consequently the increase of the number of concrete elements and the degrees of freedom. In this way, Lagrange or Serindipity isoparametric concrete elements are used and a line three node truss elements is used to represent the steel and the compatibility between concrete and steel must be guaranteed [4].

2.2. Embedded Reinforcement Representation
In this representation, the reinforcement bar is considered as an axial member incorporated in the concrete element such that its displacements are consistent with membranous concrete elements and bond loss can be considered, Figure 3.

In this scope, many works have presented different formulations for this model. Embedded models allow for an independent choice of concrete mesh. So, the same number of nodes and degrees of freedom are used for both concrete and steel. The disadvantage of this procedure is that additional degrees of freedom increase the computational and numerical treatment [4].
3. FINITE ELEMENT MODELS FOR BOND
Two different elements have been typically proposed to include the bond-slip effect in the finite element analysis of RC structures. One is bond link element and the other is bond zone element as also known as contact element. These elements are associated with the discrete reinforcement model, which has the advantage of representing different material properties more precisely. Afterwards other bond conditions at different nodes can be easily represented [5].

To describe the bond behaviour between concrete and steel, the vertical and horizontal relative displacement between concrete and steel in the local coordinates can be considered. The same type of isoparametric elements and it has, at the unloaded stage, no physical dimension in the transverse direction. It uses linear, quadratic or cubic interpolation functions corresponding to the number of nodes per element. In linear analysis, the vertical relative displacements are too small compared to the horizontal displacement [4].

3.1. Analysis With Bond Link Element
Bond-link element consists of two orthogonal springs which connect and transmit shear and normal forces between a reinforcing bar node and an adjacent concrete node (Figure 4). Since the link has no physical dimensions, the two connected nodes originally occupy the same location in the finite element of undeformed structure [5].

The bond element is a two-node finite element. The element displacement field is a slip which is defined as a relative movement between the reinforcing bar and concrete in the direction parallel to the axis of the reinforcing bar.

The bond effect is assumed as an interaction between reinforcing bars and surrounding concrete. When the change of stresses in concrete and steel occurs, the effect of bond begins and becomes more pronounced at the end anchorages of reinforcing bars and in the vicinity of cracks.
3.2. Analysis With Contact Element
The behaviour of the concrete-steel interface must be described from stress-strain laws. Many constitutive relationships are presented in the literature. In this element, the contact surface between the steel bar and the concrete in the immediate vicinity of the steel bar is modeled by a bond stress-slip law which considers the special properties of the bond zone. The most important differences are that contact element has the dimension along the steel-concrete interface (it does not have physical dimension in other two directions) and it provides a continuous contact surface between steel bar and concrete [5].

3.3. Analysis Without Bond
In this case, the stiffness matrices of the steel elements are computed in local axis at the nodes of non bond. The concrete element stiffness matrices are calculated in the global axis and they are transformed steel local axes at common nodes. In y-direction, concrete and steel have the same degree of freedom but have different degree of freedom in x-direction at common nodes [4].

4. LOCAL BOND SLIP RELATIONSHIP
4.1. Differential Equation Governing The Slip
In Figure 5 a steel reinforcement embedded in a concrete mass is shown. Over a small piece of the bar, dx, the change in the relative displacement of the steel to concrete, dΔ, is equal to change in steel deformation, δs, minus the change in concrete deformation, δc. That is [7]:

\[ d\Delta = \delta_s - \delta_c \] (6)

The magnitudes of differential deformation for the reinforcement and concrete, if we assume an elastic state, are given in equation 7 and 8 respectively as follows:

\[ \delta_s = \left( \frac{\sigma_s}{E_s} \right) dx \] (7)
\[ \delta_c = \left( \frac{\sigma_s}{E_s} \right) dx \]  

Figure 5. Bond consideration for steel reinforcement in concrete.

where the sub-scripts “s” and “c” refer to steel and concrete respectively. The terms used in equation 1 are general (independent of the type of reinforcement) and apply to local level (vary with location and stage of the test) [7].

In practice, the value of \( \delta_c \) is negligible relative to \( \delta_s \) because the concrete section is usually much larger than the steel section and the normal stress in concrete is much lower. Therefore, the second term in equation 6 is neglected and whole differential slip at local level is attributed to the steel deformation. It follows that equation 6 reduce to [7]:

\[ d \Delta \sim \delta_s \]  

Substituting from equation 7 into equation 9 and re-arranging, we can write:

\[ \frac{d \Delta}{dx} = \frac{\sigma_s}{E_s} \]  

If we differentiate both sides of the above equation with respect to \( dx \), the following equation will be obtained:

\[ \frac{d^2 \Delta}{d^2 x} = \left( \frac{1}{E_s} \right) \frac{d \sigma_s}{dx} \]  

On the other hand, the bond stress and steel stress (over segment \( dx \)) are inter-related from the condition of equilibrium that states (Figure 5):

\[ (\sigma_s + d\sigma_s)A_s = \sigma_s A_s + \tau \times dx \times \pi \times d_b \]

Simplifying:
If we substitute from equation 12 into equation 11, the following equation will be attained:

\[
\frac{d^2 \Delta}{dx^2} = \tau(s(x)) \times \left( \frac{\pi d_b}{A_s} \right)
\]  

(13)

Where \(d_b\) is the diameter, \(A_s\) is the cross-sectional area, \(E_s\) is the Young’s modulus of the reinforcing bars and \(s(x)\) is the slip between concrete and steel abscissa \(x\) [7]. Equation 13 is known as the fundamental differential equation for the bond between a steel reinforcement and concrete. This equation has been drawn in the same form as shown above or in other forms (but with the same concept) by various authors.

It is assumed that the bond characteristics of reinforcing bar are analytically described by a local relationship of bond \(\tau=\tau(s)\), in which \(\tau\) is the shear stress acting on the contact surface between bars and concrete and \(s\) is the slip; that is the relative displacement between those of the steel bar and concrete.

4.2. Analytical Expressions for Bond-Slip Relationship

The experimental evidence indicates that the load transfer between reinforcement and concrete is mainly accomplished through bearing of the reinforcing bar lugs on the surrounding concrete and through friction at large slip values (Figure 1a). The adhesion is negligible. This behavior can be described using so-called bond stress-slip relationships.

The simple bi-linear bond stress-slip model is selected and the parameters of the model are derived from the experiment data corresponding to material features of each specimen. The bond stress-slip relationship which is used in the model and the corresponding components are shown in Figure 6.

\[
\tau_1(s) = E_b s \quad s \leq s_1
\]  

(14)
\[ \tau_2(s) = \tau_1 + E_b(s - s_1) \quad s_1 \leq s \leq s_2 \quad (15) \]

\[ s_1 = 2 \text{ mm}, \quad s_2 = 10.5 \text{ mm}, \quad \tau_f = 10.55 \text{ MPa} \quad \text{and} \quad \tau_1 = 13.50 \text{ MPa} \]

5. NUMERICAL EXAMPLES
5.1. Finite Element Modeling of Pullout Test
To study the bond behavior of steel reinforcement in a concrete matrix, we use pull-out tests of a steel bar (Ø12mm) with ribs (see Figure 1) which was performed by Eligehausen (2003) [9].
To investigate the performance of the cohesion layer, numerical investigations on pullout specimens have been carried out. The specimen is an anchor of a reinforcing bar d_b=12 mm in a well confined cylinder of concrete of 150mm height and 60mm diameter which corresponds to anchorage length of 5 bar diameters (embedding length l_E=5 d_b =60 mm).
For the numerical investigations the finite element software (Ansys) has been used and a detailed FE model in 3D mode with and without bond-slip effect as cohesion layer to simulate bond have been employed. Since rib of reinforcement are being simulated, the mesh size close to the rib in steel bar, concrete and cohesion layer should be small enough to accurately describe the deformation and stress gradients. However, for the remaining regions coarse mesh can be used in order to reduce the computational costs. The results of these numerical investigations are compared with the results of the experimental investigations [8]. The test specimen used in the finite element model is shown in Figure 7.

![Finite element model](image)

**Figure 7. Finite element model**

Table 1 shows the summary of the basic material variable used in the experimental and numerical investigations.
Table 1: Summary of the material parameters

<table>
<thead>
<tr>
<th>Material properties</th>
<th>Values (kg/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete compressive strength</td>
<td>300</td>
</tr>
<tr>
<td>Concrete tensile strength</td>
<td>30</td>
</tr>
<tr>
<td>Concrete E modulus</td>
<td>273664</td>
</tr>
<tr>
<td>Concrete Poisson’s coefficient</td>
<td>0.2</td>
</tr>
<tr>
<td>Steel E modulus</td>
<td>2100000</td>
</tr>
<tr>
<td>Steel yield stress</td>
<td>3000</td>
</tr>
<tr>
<td>Steel Poisson’s coefficient</td>
<td>0.3</td>
</tr>
</tbody>
</table>

A displacement control load being applied to the end of the reinforcement in pull-out test Figure 8.

In this paper, the concrete and the reinforcement bar was modeled by eight-node Serendipity axisymmetric elements (Plane 82, Axisymmetric) with 2×2 Gauss integration points.

To display the bond slip effect between concrete and steel, two distinct models are selected, such as: (1) full perfect, (2) bi-linear model. These models are introduced in finite element program (Ansys) and the collected results are analysed and discussed in the next section.

Figure 8. Displacement control load applied to the end of the reinforcement

Figure 9. Stress distribution in reinforced concrete with bi-linear law of bond, (a) $\sigma_{xx}$, (b) $\sigma_{yy}$, and (c) $\tau_{xy}$. 
The bond stress-slip relation obtained in the finite element calculation when the three-dimensional modelling of the reinforcement is used are substantially corresponding to the curves of the experimental investigations.

According to the normal and shear stress curves (Figures 9-10), it is possible to appreciate how the connection influences the transmission of the efforts from steel bar towards the concrete and vice versa.

6. CONCLUSION
In this paper the methods of modeling of reinforcing bars and bond-slip models between steel rebar and concrete in the finite element program is described. Then one analytical expression of bond-slip relationship is selected and the pull-out test with slip and without slip modeled by finite element software (Ansys) in 3d mode and then the obtained results are presented and compared with experimental data from pullout test. It was found that stress distribution in the steel bar and concrete of pull-out tests may principally be influenced by the properties of the interface.
1. In the improvement of finite element models of composite material, it is necessary to use not only the constitutive laws of concrete and steel but also one of the interface.
2. The stress distribution in the steel bar of pull-out tests may principally be
influenced by the properties of the interface.
3. The finite element studies of pullout tests with a short embedment length (local bond conditions) show relatively good agreement between experimental and numerical results.
4. The cohesion layer is able to predict transfer of bond stresses from reinforcement into concrete realistically.
5. The proposed approach predicts the stress field in the concrete and along the steel bars (local behaviour)

REFERENCES