A NONLINEAR DYNAMIC BASED REDUNDANCY INDEX FOR REINFORCED CONCRETE FRAMES

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ABSTRACT
In almost all codes of practice for seismic resistant design of buildings, a behavior factor is used to reduce design base shear. The behavior factor is affected by several parameters such as ductility, overstrength and redundancy reduction factors. There are two common approaches to assess the effects of redundancy on the strength of a structural system, which are as follows: Static Pushover Analysis and Incremental Dynamic Analysis. The two indices: redundancy strength coefficient and redundancy variation coefficient have been introduced to measure these effects. Simplified methods are developed and presented to calculate these parameters. In this paper the redundancy strength and the redundancy variation parameters are evaluated for the reinforced concrete plane frames with different number of stories, bays and ductility capacities. The investigations indicate that these two parameters are mainly the results of redundancy reduction factors.

Keywords: redundancy, behavior factor, ductility, concrete frames, redundancy strength index, redundancy variation index

1. INTRODUCTION
Although behavior factor (R) has been an important subject in structural engineering studies in recent years but most important studies in this field get back to the last two decade. Among the researchers in this field, Freeman is the one who calculated behavior factor with using the capacity spectrum method. According to this method, to compute the quantity of R by an analytical method, it can be formulated as follows:

\[
R = R_A \times R_B \times R_C \times \ldots \times R_N
\]  \hspace{1cm} (1)

Where Rx are parameters such as arrangement of frames, type of structural system, composition of loads, degree of uncertainty, damping, characteristics of nonlinear behavior in structure, characteristics of materials, ratio of building dimensions, failure mechanism and other effective parameters. The range of effective factors in determining R is such that it would almost be impossible to find two buildings with identical behavior factors. In other words, each building has its own unique
features. Therefore, instead of adding all effective factors, as mentioned in the behavior factor relation, usually only the factors having more determinant role in the behavior factor are studied. In this paper, two main coefficients namely the structural capacity and the force resulting from earthquake are primarily considered, and the factors that help increase the capacity and reduce the seismic forces are determined in the following steps.

In 1991, separate researches on behavior factor, also known as Uang plasticity coefficient method were accomplished by Uang [1]. In 1997, Pandey and Barai [2] studied the structural sensitivity response to uncertainty. They assumed that for every structure subjected to a given loading, the general uncertainty is proportional to the reverse structural sensitivity response; thus, the structural response sensitivity reduces with increasing uncertainty.

In 1999, Bertero and Bertero studied uncertainty in the seismic resistant design. In this study, they explained the main concepts of seismic uncertainty and defined the probabilistic effect of uncertainty on structural failure.

In 2003, Wen, and Song [3], studied the reliability of structural behavior under earthquakes. They believed that when more elements are involved in resistance against lateral load, the probability for collapse of all elements, at the same time, is lower than the case when less elements with equal resistance are involved.

In 2004, Hosain and Tsopelas tried to determine structural uncertainty in reinforced concrete buildings. In that, they studied $r_u$ (uncertainty resistance coefficient) and $r_v$ (uncertainty variation coefficient) and their relation with the component’s plastic rotation ductility factor ($\mu_\theta$). The effects of number of stories and bays, the length of bays and story height were studied as well. They then studied the effect of uncertainty on behavior factor ($R_R$). Here, the effect of number of stories and bays, bays’ length, story height and also the effect of gravity loads on uncertainty coefficient are studied. Even the effect of number of frames present at each lateral load direction has been considered, and finally, the procedure to compute uncertainty coefficient using uncertainty resistance and uncertainty change coefficients were studied.

2. REDUNDANCY

The redundancy concept has been considered by engineers, especially after Kobe, Northridge and Turkey earthquakes, during which many buildings with low redundancy degree were damaged. Therefore, the redundancy topic was introduced seriously, and the degree of redundancy in structural systems was considered for seismic design.

There is some information about the useful effects of redundancy in structural resistance, but the efficient methods measurement methods are not available as yet. The effects of three parameters are usually considered to measure redundancy degree, which include:

1. Static redundancy degree of system
2. The ratio of probability in system failure to parts failure
3. Involvement of additional capacity which was not necessary for design
Some researchers studied the effects of redundancy degree with deterministic method; that is, use of nonlinear static analyses. There are few studies in which probabilistic method is applied to determine the effects of redundancy degree using structural reliability.

Seismic redundancy degree (n) for a structural system is actually the number of critical areas (plastic hinges) in a structural system which continue to yield until the structure exceeds the allowable limit leading to emergency disasters like plastic displacement or complete collapse. In engineering problems of earthquake, it is assumed that if all critical points (plastic hinges) yield simultaneously, the structure would fail under earthquake shaking. The redundancy degree is defined using the parallel and serial structural system reliability theory, determining the probability of failure in serial systems by weakest connection model and setting the probability of failure in parallel systems through secure decay model [4].

In 1999, Bertero et al studied the effect of redundancy and redistribution of internal forces in seismic design and stated that a part of behavior factor is originated from redundancy degree and can not be determined independent from overstrength and ductility. They also assumed that when the structure can not withstand gravity loads under the effect of earthquake forces, it would collapse. About structural resistance against displacement due to increasing lateral load, the resistance in the first yielding point is considered, and the maximum resistance is predicted using the reliability of displacement capacity.

A structure takes advantage of the positive effects of redundancy degree when:

a) Change coefficient in "structural demand" reduces in comparison to change coefficient in "structural capacities"
b) Addition resistance increase
c) Curvature capacity increases in plastic hinges
d) A minimum rotation capacity is ensured in all elements of structural system

According to much uncertainty in structural capacity and demand, one of the methods in studying the redundancy of structural systems under seismic loads, is to use the reliability concept. In one kind of structural system without change in materials and configuration, the redundancy degree factor can only influence the reliability on structural stability against earthquake induced lateral loads and the structure behavior factor, seriously. It should be considered that the redundancy degree is different in similar frames. If the size of an element, its reinforcement and implementation details change, the failure mechanism may naturally change, but even for two completely similar frames, redundancy degree will be different for various lateral load models [5].

Behavior factor used in codes, which reduces the level of elastic forces in the design process and in its primary formulation, is defined in terms of ductility coefficient ($R_{\mu}$) and the additional resistance coefficient ($R_s$)[6]. Ductility coefficient is computed considering nonlinear response of structural system. The relationships for computing functional ductility coefficient is formulated by some researchers by involving the natural period of structure and its ductility capacity, which are commonly based on nonlinear response change of a multi-story building relative to nonlinear response of a system with single degree of freedom [7].
The overstrength capacity show the actual lateral resistance in comparison to modeling resistance overstrength may be divided into two general parts. The first part is related to the overstrength resistance modeling until the first hinge yields in a structure and the second part is related to formation of the first hinge until a mechanism for total failure of a structure is developed [8].

In ATC-19, the formulation of behavior factor (R) is introduced [8]. This coefficient includes an additional factor (R_R) used to account for the effect of redundancy degree in a structure. These effects include probability effects and others related to structural systems geometry either in a plan or at a point in height. Therefore, behavior factor (R) is equal to:

\[ R = R_\mu \cdot R_S \cdot R_R \]  

Some effective parameters in redundancy and structural systems reliability are the ratio of demand to the capacity of structural systems, the kind of failure mechanism formed, building high, the number of stories, the length and the number of bays. This study computes the probabilistic and deterministic effects of redundancy through obtaining two redundancy resistance index (r_s) and redundancy variation index (r_v). These two indexes are used in measuring the resistance reduction coefficient from R_R redundancy for structural frames with two-dimensional reinforced concrete.

3. REDUNDANCY INDEXES

Redundancy resistance index r_s represent the ability of a structural system in redistributing forces while failure and the capability of a structure in transferring the forces of elements yielded to the elements with higher resistance. This index is a function of static redundancy, ductility, strain hardening and the average resistance of elements in a structural system. Second index having probability nature is an r_v redundancy variations index. This index measures the probability effect of elements resistance on structural system resistance. It is also a function of static redundancy in a structural system, and on the other hand is a function of statistical nature in ductility and structural elements resistance. Following variables are used in computing above indexes:
- Base shear in the beginning of yielding system.
- Ultimate base shear.
- The number of local failure or the number of plastic hinges caused during ultimate failure of structure.
- The access of elements curvature to ultimate curvature.

4. REDUNDANCY RESISTANCE INDEX

Redundancy resistance index r_s are defined as the ratio of average ultimate resistance \( \bar{S}_u \) to yielding resistance \( \bar{S}_y \). In which \( \bar{S}_y \) is the average system resistance non redundant system.
In a method suggested for this paper in studying the effects of redundancy using nonlinear dynamic analysis with increased acceleration, the base shear during failure and yielding is considered. In previous studies, this method is applied for studying the effects of overstrength [10]. In this study, the system failure standards that will be considered in nonlinear static and dynamic analyses with increased acceleration are as follows:

- Limitations related to storey drift which according to code [11] for buildings which period lower than 0.7 second are limited to 2.5% and for structures with period more than 0.7 second are limited to 2%.
- The index of structure stability which in a structure with high ductility is limited to 0.125 and in a structure with low ductility is limited to 0.25.
- The formation of failure mechanism in a structure and collapsing structure.
- The access of structure failure index to a number one according to park-Ang criterion [12].

In pushover static analyses performed in this study, it is assumed that lateral loads with reverse triangular distribution are inserted into a structure which is proportional to Iran 2800 standard earthquake force. In nonlinear dynamic analysis with increased acceleration, the maximum acceleration of any record is coordinated to a primary number (here it is considered to be 0.02g) and in one stage in increased to 0.02g and the structure is analyzed in every step until when one of the four above-mentioned criteria’s is occurred. In this stage, the analysis is stopped.
and base shear is used during yielding and maximum base shear is used for measuring redundancy resistance index.

5. REDUNDANCY VARIATION INDEX
The relation between resistance of a structural system and the resistance of its composing elements is obtained using plastic analysis of structure. In this relation, the selection of failure mechanism is important because it can result in non-actual estimates from redundancy variation index. For simplify computations, one sway mechanism according to Figure (2) is considered. This mechanism is based on the "strong column" and "weak beam" assumption which column resistance is at least 20% more than the resistance of beams.

![Figure 2. Sway type failure mode of a generic plane frame](image)

The frame strength (base shear strength) for any failure mode could be represented by the following expression:

$$S = \sum_{i=1}^{n} C_i M_i$$  \hspace{1cm} (4)

Where $S=$ frame strength (base shear); $n=$ number of plastic hinges in the frame resulting from the particular failure mode or collapse mechanism considered; $M_i=$yield moment of the structural element where plastic hinge "i" is formed; and $C_i=$coefficient with units radians length that is a function of the plastic rotation and geometry of the structure. Eq. (4) is of the form of the strength equation of a parallel system type.
The mean value of the frame strength can be derived from the following expression:
\[
\bar{S} = \sum_{i=1}^{n} C_i \bar{M}_i
\]  

(5)

Where \( \bar{S} \) = mean value of the strength of the structural element where plastic hinge "i" is formed.

Accordingly the standard deviation of the frame strength \( \sigma_f \) can be obtained from:

\[
\sigma_f = \sqrt{\sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} C_i C_j \rho_{ij} \sigma_{Mi} \sigma_{Mj}}}
\]  

(6)

Where \( \rho_{ij} \) = correlation coefficient between the strengths \( Mi \) and \( Mj \) and \( \sigma_{Mi} \) = standard deviation of the yield moment \( Mi \). also \( \rho_{ii} = 1 \) for \( i=j \). To further simplify the deviation, a regular multistory multi-bay frame with the following properties is considered.

1. The frame is composed of elements with identical normally distributed strengths:

\[
e_{Mi} = \bar{M}_i = \bar{M}_j = \bar{M}_e
\]  

(7)

\[
\sigma_{Mi} = \sigma_{Mj} = \sigma_e
\]  

(8)

2. The correlation coefficient between the strength of any two pairs of elements is the same.

\[
\rho_{ij} = \rho_e
\]  

(9)

3. The bays of the frame have identical spans and the stories identical high which result in:

\[
C_i = C_j = C
\]  

(10)

Eq.(5) and (6) now become:

\[
\bar{S} = n.C \bar{M}_e
\]  

(11)

\[
\sigma_f = C \sigma_e \sqrt{n + n(n-1) \rho_e}
\]  

(12)

The following relationship between the coefficient of variation (COV) of the frame strength \( \upsilon_f \) and the COV of the element strength \( \upsilon_e \) is calculated by dividing Eq. (12) to Eq. (11):
The redundancy variation index $r_v$ is defined as the ratio between $v_f$ and $v_e$:

$$r_v = \frac{v_f}{v_e} = \sqrt{\frac{1+(n-1)\rho_e}{n}}$$

(14)

For a parallel system with unequally correlated elements, $\rho_e$ could be substituted with the average correlation coefficient $\bar{\rho}$ defined as:

$$\bar{\rho} = \frac{1}{n(n-1)} \sum_{i \neq j} \rho_{ij}$$

(15)

Therefore, Eq. (14) can be modified using the average correlation coefficient of the strengths of the plastic hinges as follows:

$$r_v = \sqrt{\frac{1+(n-1)\bar{\rho}}{n}}$$

(16)

Hence the redundancy variation index $r_v$ is a function of the number of plastic hinges "n" and their average correlation coefficient between their strengths, and represents a measure of the probabilistic effects of redundancy on the system strength, its values range between 0 and 1.

For a building structure where a single plastic hinge causes collapse (n=1), $r_v = 1$ and the structure under consideration is non-redundant. The other extreme value $r_v = 0$ indicates an infinitely redundant structural system and is reached either when an infinite number of plastic hinges are required to cause collapse (practically "n" attains large values) or when element strengths in a structure are uncorrelated (the average correlation coefficient in Eq. (16) is zero).

Using Eq. (16) $r_v$ can be estimated from a pushover or dynamic analysis and for a particular value of the average correlation coefficient of the structural member strength.

6. REDUNDANCY FACTOR "$R_R$"

The overall effects of redundancy on the structural strength may be completely described by the ratio of the ultimate strength of a structural system to the ultimate strength of non-redundant structure. Thus:

$$v_f = \frac{\sigma_f}{S} = \frac{\sigma_e}{M_e} \sqrt{\frac{1+(n-1)\rho_e}{n}} = v_e \sqrt{\frac{1+(n-1)\rho_e}{n}}$$

(13)
Where $S_u = \text{structural system strength which includes all the effects of redundancy;}$ and $S_{nr} = \text{the same strength but for non-redundant structural system.}$ Assuming that the strength of a structure is distributed normally, the characteristic or design strength of a structural system, its standard deviation, the coefficient $k$ is formed. Therefore, both $S_u$ and $S_{nr}$ may be written as follows:

$$f_{uS} k S = \sigma$$

$$f_{nrS} k S = \sigma$$

Where $\sigma_f = \text{standard deviation of the frame strength; } \sigma = \text{standard deviation of the non-redundant frame strength; } \overline{S}_u = \text{average of the ultimate frame strength and } \overline{S}_{nr} = \text{average of the non-redundant frame strength.}$

An expression for $\sigma_f$ could be obtained as follows:

$$r_v = \frac{\sigma_f}{\sigma_{nrS}} \Rightarrow \sigma_f = r_v r_s v_e \overline{S}_{nr}$$

By virtue of $\overline{S}_u = r_v \overline{S}_{nr}$; Eq. (19) results into:

$$S_u = r_v \overline{S}_{nr} - kr_v r_s v_e \overline{S}_{nr} = r_s (1 - kr_v v_e) \overline{S}_{nr}$$

Where $r_v = \text{redundancy variation index; } r_s = \text{redundancy strength index and } v_e = \text{COV of the strength of the structural system elements.}$

Using Eq. (19) and (21), Eq.(17) becomes:

$$R_R = \frac{r_s (1 - kr_v v_e) \overline{S}_{nr}}{\overline{S}_{nr} - k \sigma_{nr}} = r_s \left( \frac{1 - kv_e r_v}{1 - k v_e} \right)$$

Where $v_{nr}$ is the COV (coefficient of variation) for non-redundant frame strength. A non-redundant frame structure could be modeled as a parallel system consisting of ideal elastic-brittle elements. Such a system behaves like a series system, where failure of one element results in the system collapse, and that the safety index of the system is equal to that of the element. For a non-redundant system, $(n=1)$ $v_{nr}=v_e$. Therefore, the redundancy factor ($R_R$) can be expressed as follows:

$$R_R = r_s \left( \frac{1 - kv_e r_v}{1 - k v_e} \right)$$
7. CASE STUDY ABOUT THE EFFECTS OF REDUNDANCY ON TWO-DIMENSIONAL CONCRETE FRAMES

In order to compute redundancy indices, 16 frame samples from 2 bay to 5 bay and with two, four, six and ten stories were designated. SAP2000 software [13] and the IDARC software [14] are used for nonlinear dynamic and nonlinear static analyses. For nonlinear static analysis, 16 frame samples with high ductility and 16 frame samples with low ductility are selected (Figure 3).

The lateral load pattern applied to the structure is reverse triangular, which is
approximately in accordance with lateral force criteria of the earthquake standard 2800 of Iran. Four different cases of design and analysis are considered for comparison. In the first case, the bay length is 4 meters and the story height is 3 meters and in the second case, the story height is increased from 3 to 4 meters. In the third case, the bay length is increase to 5 meters and finally in the forth case, the gravity loading intensity is increased to 30%. Therefore, in static nonlinear analysis, one hundred twenty eight frames are designed with SAP2000 and then analyzed by the IDARC. Response curves are computed in terms of displacement at the top of structure ($\Delta t_{ar}$) with respect to base shear divided by structure weight ($C_b$). Two values, base shear coefficient during yielding and also maximum base shear coefficient are important over curve. The $r_s$ index is obtained by dividing maximum base shear coefficient to the base shear coefficient when yielding.

Using maximum number of plastic hinges formed in nonlinear static analysis, one can obtain $r_v$ index. As a result of having these two indices, resistance reduction coefficient can be obtained from redundancy according to relations in the third part.

In order to carry out nonlinear dynamic analysis with increased acceleration by the IDARC software, begins to analyze with a primary PGA value in any stage, and it continues the operation with 0.02g increase relative to the previous measure, until one of the failure conditions is reached. In this case, the value of base shear coefficient is applied for computing $r_s$ and also for computing $r_v$ index. The number of plastic hinges formed while failure is used to compute 16 frames with high ductility and 16 frames with low ductility. Eight seismic records are applied, equally, for both linear and nonlinear static analysis methods, for 4 different cases. Finally, 1024 frames were analyzed with different cases and the values of base shear coefficient while forming the first plastic hinge. Maximum base shear coefficient and the number of plastic hinges when failing are used as parameters required for computing $r_s$, $r_v$ and $R_R$ indices. It is necessary to note that the average values obtained from eight records is the basis for computing above indices.

8. CONCLUSION
Comparing the responses obtained from Static Pushover Analysis method (SPO) with Incremental Dynamic Analysis (IDA), it is concluded that in most conditions, $R_R$ coefficients obtained from static method are larger than dynamic method, but this difference is maximally 10%. According to result from Figure(4), we can conclude that the results obtained from nonlinear static method are in good agreement with results obtained by nonlinear time history method and may be used as a reliable method.

Finally, it should be emphasized that these results are only for frames modeled in this study and might not hold true for all other structural models.
Figure 4. Average of redundancy modification factor with number of story

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