A Probabilistic-Based Durability Analysis for Reinforced Concrete Structures Subjected to Chloride Containing Environment

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ABSTRACT

Traditional durability analysis is not possible to provide a controlled durability and long-term performance of concrete structures. Recently, research works have shown that probabilistic approach based on the theory of structural reliability, would be very valuable for durability analysis and design. In this study, the probabilistic durability analysis based on a Monte Carlo simulation was carried out using sample data selected from detailed field investigation. The diffusion process of chloride ion followed the Fick’s second law, was analyzed by the finite difference method. The probabilistic properties of some durability design variables, such as diffusion coefficient of concrete and surface chloride concentration, were newly determined from experimental data. By applying a probabilistic durability analysis to an integral structural design, the durability performance of concrete structures would be remarkably improved.

KEYWORDS: Durability, Probability of corrosion, Chloride penetration, Service life, Monte Carlo Simulation, Finite difference method.

1. INTRODUCTION

Until currently, the durability design of concrete structures is based on implicit rules for materials, material compositions, working conditions, structural dimensions etc. Examples of such ‘deem-to-satisfy’ rules are the requirements for minimum concrete cover, maximum water/cement ratio, minimum cement contents, crack width limitations, air contents, cement type, and coating of concrete surface and so on. The purpose of all these rules has been to secure robustness for structures, although no clear definition for service life has been presented.

In Korea, a new bridge code needs the explicit presentation of performance not only for ultimate limit states but also serviceability limit states such as durability to give an explicit relationship between performance and service life. Recently, there was increasing interest in setting requirements for the service life of concrete structures. Also increasingly emphasized is the fact that total costs comprise both construction cost and cost of maintenance and repair. According to durability of concrete construction and the recent research that is connected\cite{Duracrete, 2000}, structure reliability theory in construction design can know effective thing for quantitative durability of construction. But they have not been fully utilized in the existing ‘deem-to-satisfy’ rules. In this paper, a new method based on probability theory is established to overcome the above problems in durability design of concrete structures. It can present the explicit relation between service life and durability and include the uncertainties of the variables in the relation. With this, the service life of concrete structures in respect to the durability can be predicted with a probability also.
2. DURABILITY ANALYSIS PROCEDURE

2.1 Serviceability limit States

The Structure design, both safety and serviceability, in most modern building codes is based on performances. The transition boundary from performing to failure is called a limit state. The durability design concerns serviceability limit states such as corrosion due to chloride penetration, corrosion due to carbonation, surface deterioration, frost attack and so on. In Korea, the most significant durability problem is the chloride penetration which results in a corrosion of steel embedded in concrete. After depassivation or onset of steel corrosion, it may take several years before any visual sign of deterioration such as cracking and spalling will occur, and it may still take a very long time before the structural capacity or integrity becomes significantly reduced. However, since the time to depassivation, the chloride ion concentration at steel reaches the critical value and the corrosion may start, represents both a reasonable and well defined stage of deterioration process, it appears appropriate to define this stage as the serviceability limit state in the durability analysis (Ferreira, 2004). With this definition, the limit state function can be written as follows;

\[ g(x, t) = R(t) - S(t) \]  (1)

in which, \( x \) is the design variable vector defining limit state function \( g \), \( t \) is time, \( R(t) \) and \( S(t) \) are time dependent variables representing resistance and load, respectively. In chloride penetration problems, resistance and load are defined as steel cover and chloride ion penetration depth whose concentration reaches critical value.

2.2 Durability Model

The rate of chloride penetration into concrete as a function of depth is normally modeled by the use of Fick’s Second Law of Diffusion;

\[ \frac{\partial C(x,t)}{\partial T} = \frac{\partial^2 C(x,t)}{\partial x^2} \]  (2)

where, \( C(x,t) \) is chloride ion concentration at a distance \( x \) from the concrete surface after being exposed for a time \( t \), and \( D(t) \) is the chloride diffusion coefficient dependent on the time \( t \). In this case equation (2) reduces to (Crank, 1975)

\[ T = \int_0^t D(t) dt \]  (3)

Also, can substitute equation (2) for equation (4) by parameter with equation (3)

\[ \frac{\partial C}{\partial T} = \frac{\partial^2 C}{\partial x^2} \]  (4)

Generally, the time dependent function \( D(t) \) can be written as follows (Thomas, 2001);

\[ D(t) = D_0 \left( \frac{t_0}{t} \right)^n \]  (t < \( t_c \))  (5a)

\[ D(t) = D_0 \left( \frac{t_0}{t} \right)^n = const. \]  (t > \( t_c \))  (5b)
where, $D_0$ is the diffusion coefficient at time $t_0$, and the exponent represents the ability of the concrete to increase the resistance against chloride penetration with time. In order to prevent the diffusion coefficient decreasing with time indefinitely, the relationship shown in equation (5) is expressed by decreasing limit time $t_c$ (= 30 years). Beyond this time, the value at time $t_c$ calculated from equation (5b) is assumed to be constant throughout the rest of the analysis period. By substituting equation (5) into equation (4),

$$T = D_m t = D_0 \left( \frac{t_0}{t} \right)^n t \quad (t > t_c) \quad (6a)$$

$$T = D_m t = D_0 \left[ 1 + \frac{t_c}{t} \left( \frac{m}{1-m} \right) \left( \frac{t_0}{t_c} \right)^n \right] t \quad (t < t_c) \quad (6b)$$

where, $D_m$ is the mean diffusion coefficient from initial time 0 to analysis time $t$.

Using the Implicit Function method proposed by Crank-Nicolson, applying the finite difference method to equation (3), is following as,

$$\left\{ \begin{array}{l}
\Delta T 
\Delta x
\end{array} \right. = - \left( \Delta x \right)^2 \left( C_{i,j+1} - C_{i,j} \right) \left( C_{i+1,j} - 2C_{i,j} + C_{i-1,j} \right)$$

where, $\Delta T$ and $\Delta x$ means increment each $T$ and $x$ axis.

2.3 Probabilistic Analysis

In this study, the probabilistic analysis was carried out by use of Monte Carlo Simulation Method (MCSM). It was verified whether MCSM gave meaningful results for each design variable. The limit state function was derived from equation (7) by defining the depth of chloride penetration at time $t$ when the $\text{Cl}^-$ concentration reaches pre-defined threshold value according to Korean Code. At every simulation, the input variables are randomly generated according to the pre-defined probabilistic distributions and the value of the limit state function is calculated. If it is positive the case is counted as SAFE, if not FAIL. The probability of failure is defined by equation (8).

$$p_f = \frac{1}{N} \sum_{j=1}^{N} I \left[ g \left( r_j, s_j \right) \right]$$

where, $N$ is the number of simulations and $I[\cdot \cdot \cdot ]$ is indicator function. The analysis program procedure by the former discussion is as follows.

1. The sampling is performed for all the design variable.
2. By use of equation (7), the value of limit state function is calculated for an individual set of the design variables.
3. The failure probability is calculated from that sum of (-) value of limit state function is divided by the number of total simulation

3. SENSITIVITY ANALYSIS

3.1 The objective of sensitivity analysis

The objective of sensitivity analysis is to observe the contributions of each design variable to the probability of failure in the probabilistic durability analysis. The design variables in the limit state function should be modeled as random variables which have their own probability density function determined by the real experimental and/or measured data. But as in many cases, it is difficult to get
enough data in Korea also, thus they are modeled based on existing reported data as normal variates and examined within some ranges of means and variance values. Table 1 shows the reference values of statistics of initial diffusion coefficient $D_0$, exponent for time-dependence of diffusion coefficient $n$, critical chloride ion concentration $C_{cr}$, surface chloride concentration $C_s$ and concrete cover of reinforcement $C$, respectively. The initial diffusion coefficients are determined based on accelerated laboratory testing after 28 days of placement.

Table 1. Statistics of design variables for each structure

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Base case</th>
<th>Range of mean values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_0$ ($\times 10^{-12}$ m$^2$/s)</td>
<td>N (6, 1.2)</td>
<td>3, 6, 9</td>
</tr>
<tr>
<td>$n$</td>
<td>N (0.4, 0.8)</td>
<td>0.3, 0.4, 0.5</td>
</tr>
<tr>
<td>$C$ (cover, mm)</td>
<td>N (70, 7)</td>
<td>30, 50, 70, 90</td>
</tr>
<tr>
<td>$C_{cr}$ (kg/m$^3$)</td>
<td>N (1.2, 0.24)</td>
<td>0.6, 1.2, 2.4, 3.6</td>
</tr>
<tr>
<td>$C_s$ (kg/m$^3$)</td>
<td>N (9, 1.8)</td>
<td>4.5, 9, 13</td>
</tr>
</tbody>
</table>

3.2 Diffusion Coefficient

The important factors which affect the chloride diffusion coefficient of concrete are cement content, types of cement used, and test methods and so on. The Korean Code specifies that the chloride diffusion coefficient should be determined based on real experimental or measured data. If it is not possible to obtain reliable data, the regression equations given in the Code may be used for ordinary Portland cement and granulated blast furnace slag cement using existing data. In this study, the variation of probabilities of corrosion initiation are examined with various initial diffusion coefficients ($D_0$) of $3 \times 10^{-12}$, $6 \times 10^{-12}$ and $9 \times 10^{-12}$ m$^2$/s and the results are shown in Figure 1 and Figure 2, respectively. In Figure 1, the probability of corrosion initiation with varying diffusion coefficients after 100 year lifetime is shown. If the diffusion coefficients doubled, the probabilities of corrosion increase more than twice. It can be concluded that one of the most effective way to prevent chloride induced failure is the reduction of diffusion coefficient and it can be achieved by use of impermeable concrete. According to Bentz (Bentz, 2003), the coefficient of variation (COV) for diffusion coefficient from test results at the same laboratory is about 17%, but that determined by test results from various laboratories becomes 37%. In this paper, the COV of diffusion coefficients are assumed as 10% and 20%, and the changes in probability of steel corrosion are examined. The required cover depths for 100 year lifetime, according to various probabilities, are shown Figure 2 and two initial diffusion coefficients and two COV of diffusion coefficients. As can be seen from that figure, the variation of diffusion coefficients gives little effect on the cover depth at 50% of corrosion. The required depth is increased by 1–2 mm as COV increases from 10% to 20%. It can be said that the effect of change in mean value of diffusion coefficient on corrosion probability is much greater than that of variance.

![Figure 1. Probability of corrosion vs diffusion coefficient at 100 years](image1)

![Figure 2. Calculated concrete cover for different mean values and COV of diffusion at 100 years](image2)

1188
3.3 The Surface Chloride Concentration

The surface chloride concentration $C_s$ has the largest variations according to geological locations and structural locations among design variables in limit state function. Especially in marine structures, the structural element, such as pier, has various values of $C_s$ depending whether it is evaluated at atmospheric zone, splash zone, and water. In this paper, the values of $C_s$ are varied and their effects on the probability of corrosion are examined. From the Figure 3, it can be seen that the surface chloride concentration has much effect on the probability of corrosion. When $C_s$ is increased from 3.0 kg/m$^3$ to 9.0 kg/m$^3$, the probability of corrosion is increased from 20% to 87%. The effect of the surface chloride concentration variation to the probability of corrosion is more than that of diffusion coefficient variation. For two values of $C_s$, 4.5 kg/m$^3$ and 9.0 kg/m$^3$ and for two values of COV, 10% and 20%, the required cover depths are evaluated at various probability of corrosion after 100 year lifetime in Figure 4. Figure 4 indicates that the probability of corrosion is little affected by the variance of $C_s$, which coincides with the tendency of diffusion coefficient.

![Figure 3. Probability of corrosion vs surface chloride concentration at 100 years](image1)

![Figure 4. Calculated concrete cover for different surface chloride concentration and COV at 100 years](image2)

3.3 The Time Dependent Factor($n$) and Critical Chloride Concentration

The time dependent factor $n$ becomes modelling in form of exponential function usually as reflecting the concrete ability increasing as time goes by. The effect of the time dependent factor $n$ on the service life calculation is considerable. This is illustrated in Figure 5. In case of the time dependent factor $n$ increasing(0.3, 0.4, 0.5), period that corrosion probability of reinforcing bar reaches 10% in equal condition is increasing by each 10 years, 17 years, 27 years and the fact means that according to the time dependent factor increasing, diffusion coefficient value is decreased, and affects corrosion probability of reinforcing bar.

In case of critical chloride concentration connected with reinforcing bar corrosion, the concentration is based concrete weight ratio 0.05% in Life-365 Program, and is present 1.2 kg/m$^3$ in Korea code. In this paper, in case of a laboratory experiment, the critical chloride concentration is 0.6~1.2 kg/m$^3$ and is in 1.2~2.4 kg/m$^3$ in case of concrete structure in damage from coastal environment. This study changed critical chloride concentration by 0.6, 1.2, 2.4 kg/m$^3$ and analyzed effect. In Figure 6, when the critical chloride concentration is in 0.6, 1.2, 2.4 kg/m$^3$, period that corrosion probability becomes 10% increase by each 9, 17, 36 years. This means it is essential that the forecast of critical chloride concentration is properly for the performance of durability analysis quantitatively.
4. CONCLUSION

In this paper, the development of a new procedure for probability-based durability analysis in chloride containing environment is established using Monte Carlo Simulation and Finite Difference Method. It is observed in the sensitivity analysis that the probability of corrosion is much affected from the mean value of both chloride diffusion coefficient and surface chloride concentration, but less from the COV’s of those. Deviations of each design variable come under the corrosion period of deterministic method in case of corrosion probability 50%. The effect of corrosion probability 10% is considerable and influence about service life of concrete structure. Therefore it may be concluded that the probabilistic procedure for durability analysis of concrete structures in chloride containing environments is useful and valuable in order to insure the reliability of service life.

5. ANKNOWLEDGMENT

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