Further steps towards a quantitative approach to durability design

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Abstract
This paper presents further steps in the development of reliability-based approaches for the durability design and service life prediction of building components which integrate the requirements of safety, serviceability and durability. In general, the load and resistance should be modelled as stochastic processes and the resulting durability problem is formulated in a time-dependent probabilistic format. Using the classical reliability approach, the resulting time-dependent reliability problem is transformed into a time-independent reliability problem through the adoption of an extreme-value probability distribution for the maximum lifetime load. The resistance degradation and its variability are included in the model, and the probabilistic design problem is transformed into a deterministic (or semi-probabilistic) problem using the first-order second moment theory. This semi-probabilistic integrated approach to durability design and prediction overcomes the shortcomings of the empirical factorial approach and the complexities of a fully time-dependent probabilistic method. An alternative approach using stochastic process theory is proposed to formulate the durability design problem as a crossing problem for which the probability of failure within the component lifetime is obtained from the first-passage probability for the stochastic process. In addition, a service life-based formulation of the durability design and prediction problems is presented in order to illustrate its equivalence with the performance-based formulation. It is shown that in principle the same probabilistic approaches used for the development of structural design approaches for safety and serviceability are also applicable for durability design. The durability design objective is to keep the probability of failure within a specified time interval (or service life) below a certain threshold value that depends on the consequences of failure of the component or system. It is expected that in the near future, further simplifications of the proposed approaches will be made leading to practical and reliability-based methods to durability design or service life prediction. These simplified methods will be implemented in the design of durable new structures and optimal life-cycle maintenance management of existing structures.

Keywords: Crossing problem, deterioration, durability, performance, random variable, semi-probabilistic, service life, stochastic process, time-dependent reliability.
1. Introduction

The existing construction practice based on minimum initial cost and the lack of practical and reliable methods for service life prediction and life-cycle maintenance management have contributed to the significant backlog of deficient infrastructures and have inhibited any competition based on life-cycle cost optimization. This situation is worsened by the limited funds allocated for the maintenance and rehabilitation of the various infrastructure systems. Furthermore, the current design philosophy for safety and serviceability adopted by major design codes may be inadequate in certain conditions (e.g. aggressive environments) because of the assumption of the time-invariance of the resistance. Moreover, the existing qualitative (“deemed-to-satisfy”) code requirements for durability are empirical with no explicit formulation in terms of performance or intended service life.

In general, the resistance of a building component is a random variable that decreases over time, which in turn, reduces the reliability and may accelerate the risk of failure and shortens its service life. In addition, the load (or action) is also a time-dependent quantity that is also random in both magnitude and time of occurrence. Therefore, a realistic modelling of both the resistance and load, and a solution to the corresponding time-dependent reliability problem require the use of stochastic processes theory. In a second stage, this time-dependent reliability problem should be simplified into a time-independent probabilistic problem. It should be emphasized that a probabilistic modelling of the durability problem will enable a rational determination of the service life or “end of life” of a component by considering both the service and economic life through life-cycle cost optimization including the costs of inspection, maintenance, repair, failure and replacement. In the last stage of the development, the approach for durability design and prediction should be time-independent and deterministic (or semi-probabilistic), in which the randomness and time-dependence of the load and resistance have been considered through appropriate probability distributions and partial safety factors (Fig. 1).

The development and implementation of such a practical approach to durability design and prediction will require calibration through extensive performance and service life data in various environments for different building components. For design and maintenance purposes, the objective is to keep the probability of failure within a given time interval (e.g. service life) to an acceptable level that depends on the consequences of failure of the building component or system. The durability design and prediction problems can be solved using either the component performance or component service life formats.

2. Existing approaches to service life prediction and durability design,

Despite the fact that thousands of papers related to material and component durability have been published over the past two decades, they had a negligible impact on the development of an effective approach for durability design [1]. Three guides for the durability of buildings have been proposed in Britain [2], Japan [3] and Canada [4]. The British and Canadian guides address qualitatively the issues of durability of structural and building envelope elements throughout the building life-cycle, while the Japanese guide (AIJ) proposes a factor-based or “factorial” approach for the prediction of the service life of building components. However, the use of these guides is very limited
given the lack of reliable quantitative durability design and prediction procedures, in addition, no proof of the correctness of these guides has been given yet. Further, it can be added that these methods are not fully developed at a practical level. The designer/engineer using these methods may be liable for the damage due to the durability design made on the basis of these guides. For this reason, the BS 7543 is hardly used in Britain, however, the attempt of producing a durability guide is a positive development. The feasibility of the Japanese “AIJ” factorial approach for the prediction of the durability of building components has been examined [5]. The factorial approach is a simplified methodology for service life prediction in which an assumed “standard life” of a building component is adjusted by different factors that take into account the quality of the component and its incorporation within the building [3,5]. An international standard on the “Design Life of Buildings” is being developed by ISO/TC59/SC3/WG9, which is intended to provide a methodology for the prediction of the durability of building components based on the factorial approach [5]. In the literature, there is a limited body of work related to the formulation and solution of the time-dependent reliability design problem [8-18].

The prediction of the performance or service life of a building system and its components is a very complex problem as it depends on several factors including the loading, aggressivity of the environment, quality and frequency of maintenance, quality of materials and workmanship. Given the time-dependence and large uncertainties of the performance of building components and the associated risks of failure, it is necessary to adopt a stochastic model and use the stochastic process theory [6] for the prediction of the performance or service life at the first stage (Fig.1). At a later stage, it will be necessary to simplify this time-dependent probabilistic model into a time-independent deterministic model that has been calibrated through extensive performance and service life data (Fig.1). A practical and reliable durability design approach should overcome the shortcomings of the “AIJ” empirical factorial approach and the complexities of a full time-dependent probabilistic approach. It should be similar in format to the existing semi-probabilistic structural design approach adopted by various standards [7], i.e. the basis is probabilistic but the design approach is deterministic.

![Figure 1. Development process for service-life design and prediction methodology](image-url)
Such a probabilistic format will enable a rational determination of the “end of life” for a system or its components by evaluating both the service life (minimum performance) and economic life through life-cycle cost optimization including the costs of inspection, maintenance and failure. In addition to life cycle costing, building regulations should define the minimum reliability level for durability design based on economic and/or legal obligations.

3. Time-dependent reliability analysis

Generally, the load (action) S and resistance R (capacity) are time-dependent random variables. Very often the load tends to increase, while the resistance tends to decrease due to deterioration induced by aggressive environmental factors, higher than expected loads or fatigue damage as shown in Fig.2. For the case of reinforced concrete structures, the most damaging environmental factors for concrete are freeze-thaw cycling, sulfate attack, alkali-silicate reactions and temperature, while corrosion due to chloride attack or carbonation is the most damaging factor for steel reinforcement. Therefore, a realistic modelling of the load and resistance requires the use of stochastic processes (Fig.2). The solution of the corresponding time-dependent reliability problem can be derived using stochastic process theory. Models for such processes may include discrete Poisson or Markov processes or continuous Gaussian processes.

Figure 2: (a) Representation of load and resistance as stochastic processes; (b) Probability density functions of point-in-time and lifetime maximum loads
The instantaneous probability of failure $P_f(t)$ is defined as:

$$P_f(t) = P[R(t) \leq S(t)]$$

in which $P_f(t)$, $R(t)$, and $S(t)$ are the instantaneous probability of failure, resistance and load effect at time $t$, respectively. If the instantaneous probability density functions (pdf) $f_R(r,t)$ and $f_S(s,t)$ of the point-in-time resistance $R(t)$ and load $S(t)$, respectively, are known, and if $R(t)$ and $S(t)$ are independent, (1) can be calculated from the following convolution integral:

$$P_f(t) = \int_{-\infty}^{\infty} F_R(s,t) f_S(s, t) ds$$

However, in time-dependent reliability analysis, the quantity of interest is not the instantaneous probability of failure, but rather the probability of failure over an interval of time $[0-t]$ or $[0-t_L]$, where $t_L$ may represent the lifetime or service life of the structure. The determination of this probability of failure is not a straightforward task. This probability of failure can be obtained by integrating the above instantaneous probability of failure over the interval $[0-t_L]$. However, this is not possible owing to the correlation between the instantaneous probability of failure values at times $t$ and $(t+\delta t)$ as $\delta t \rightarrow 0$, which is due to the correlation of the stochastic processes themselves [9].

### 3.1 Time-dependent reliability analysis using classical reliability approach

The classical reliability approach to this stochastic problem is based on the lifetime maximum load concept. It was adopted in the derivation of the probability based limit states design codes [7]. In this approach, the resistance is assumed time invariant, while the load is considered as a stochastic process. The approach is based on the fact that the structure will not fail, if it survives the maximum load $S_{\text{max}}$ that will occur during the time interval $[0-t]$, which is represented by an extreme-value distribution. If the load history is modelled as a sequence of $n$ identically distributed and statistically independent random variables (Fig.2) with cdf $F_S(s)$, and pdf $f_s(s)$, then the cdf and pdf (shown in Fig.2) of the maximum load $S_{\text{max}}$ in the interval $[0-t_L]$ are given by [11]:

$$F_{S_{\text{max}}}(s) = [F_S(s)]^n$$

$$f_{S_{\text{max}}}(s) = n[F_S(s)]^{n-1} f_S(s)$$

If the number of loads $n$ is very large, $F_{S_{\text{max}}}$ asymptotically approaches an extreme-value distribution (e.g. Type I or II asymptotic forms). Therefore, the probability of failure within the interval $[0-t_L]$ denoted $P_f[0-t_L]$ is given by [11]:

$$P_f[0-t_L] = \int_0^{\infty} [1 - F_{S_{\text{max}}}(r)] f_R(r) dr = \int_0^{\infty} F_R(s) f_{S_{\text{max}}}(s) ds$$

The above integral does not contain the factor time or the number of loads $n$ which have been included in the derivation of the distribution of the lifetime maximum load $S_{\text{max}}$. Hence, the original time-dependent reliability problem has been transformed into a much simpler time-independent reliability problem through the use of an extreme-value representation of the probability distribution of the maximum load that will occur during the structure lifetime. The objective in design is to keep the above probability of
failure during the service life below some maximum acceptable threshold value \( P_{f,\text{max}} \). The value of \( P_{f,\text{max}} \) depends on several factors related to the consequences of failure such as cost of repair or replacement, risk of loss of life, importance of the component in the system, and type of failure. To simplify the computation of the above limit states probabilities in Eq. (4), first-order, second moment reliability analysis methods referred to as FOSM or FORM have been developed since the late 1960’s \[7,8,9,10\]. The FOSM methods were used to derive the load and resistance safety factors for limit-states design of major structural design codes in North America and Western Europe. Generally, in structural design, the pdf of resistance is often considered as lognormal, while that of the load is taken as an extreme-value Type I. However, to illustrate the practicality of the FOSM approach, both \( R \) and \( S_{\text{max}} \) are assumed as having lognormal distributions. Therefore, the overall safety factor \( R/S_{\text{max}} \) has also a lognormal distribution and the limit state probability can be expressed as:

\[
P_{f}[0-t_L] = P \left[ \frac{R}{S_{\text{max}}} < 1 \right] = P \left[ \ln \frac{R}{S_{\text{max}}} < 0 \right] \quad (5a)
\]

\[
P_{f}[0-t_L] = \Phi \left[ \ln \left( \frac{\mu_R}{\mu_{S_{\text{max}}}} \right) \sqrt{\left( V_R^2 + V_{S_{\text{max}}}^2 \right)} \right] \leq P_{f,\text{max}} = \Phi(-\beta) \quad (5b)
\]

in which \( \mu_R, \mu_{S_{\text{max}}} \) are the mean values of \( R \) and \( S_{\text{max}} \), respectively; \( V_R, V_{S_{\text{max}}} \) are the coefficients of variation of \( R \) and \( S_{\text{max}} \), respectively; \( \Phi \) is the standard normal distribution function; and \( \beta \) is the so-called reliability index. The above approximation is valid if the coefficients of variation of \( R \) and \( S_{\text{max}} \) are less than 0.3, which is a relatively low value for most practical problems in building design. The above equation can be transformed to yield:

\[
e^{-\beta V_R} \mu_R \geq \left( e^{\beta V_{S_{\text{max}}}} \right) \mu_{S_{\text{max}}} \quad (6a)
\]

or

\[
\Phi \mu_R \geq \gamma \mu_{S_{\text{max}}} \quad (6b)
\]

The above equation is similar to the current standard equations for limit states design, where \( \Phi (e^{-\beta V_R} < 1) \) is the understrength safety factor and \( (\gamma e^{\beta V_{S_{\text{max}}}} > 1) \) is the overload safety factor and \( a \) is a separation function \[9\]. Equation (6b) represents the current semi-probabilistic load and resistance factor design (LRFD) method adopted by the structural concrete and steel design codes. This design approach is very practical because it is totally deterministic, and reliable, since the partial safety factors have been derived probabilistically by taking into account the uncertainties in the resistance and load and the consequences of failure. For the time-dependent resistance problem, where the resistance decreases with time due to material degradation, the probability distribution of interest for the resistance should be the lifetime minimum value. However, it is highly unlikely that the occurrence of the lifetime maximum load will coincide with the lifetime minimum resistance \[9\]. Within the above framework, it may be possible to model the uncertainties related to the lifetime resistance that will account for the resistance reduction with time due to material degradation. Assume that the lifetime resistance \( R \) can be expressed as follows:

\[
R = D R_0 \quad (7)
\]

in which \( R= \) lifetime resistance; \( D \) = degradation factor due to aggressive environmental factors (independent of the load history); and \( R_0 \) = initial resistance of the non-deteriorated component. If we assume that both \( R_0 \) and \( D \) are lognormally distributed, then, \( R \) is also lognormally distributed, with a mean value \( \mu_R \) and a coefficient of variation \( V_R \) given by:
in which \( \mu_D \) (\( \leq 1 \)) is the mean deterioration factor; \( \mu_{R_0} \) is the mean initial resistance; \( V_D \)
\( V_{R_0} \) are the corresponding coefficients of variation. The above equations show that the mean lifetime resistance is lower than the initial value, while the uncertainty associated with it increases due to the added variability of the deterioration factor. Using Eqs. (5) and (6), and assuming the same \( \alpha \) factor, the following design equation can be derived:

\[
\psi \mu_D \times \phi \mu_{R_0} \geq \gamma \mu_{S_{\text{max}}}
\]

in which \( \psi = e^{-\beta a^2 V_D} < 1 \) = partial safety factor that accounts for the uncertainties in the resistance degradation due to environmental factors; \( \phi = e^{-\beta a^2 V_{R_0}} < 1 \) = initial resistance understrength safety factor; and \( \gamma = e^{\beta a V_{\text{max}}} > 1 \) = overload safety factor. Reliable data on the degradation of the resistance and its variability in different environments are required to derive reliable values for the above safety factors. The above design equation is similar in format to the current standard design expressed in Eq. (6). This is a semi-probabilistic design approach that integrates the requirements of safety (or serviceability) and durability.

### 3.2 Time-dependent reliability analysis using stochastic process theory

In this section, an alternative reliability-based approach to the durability design and prediction problems is proposed using stochastic process theory. As mentioned earlier, models of stochastic processes include discrete Poisson, Markov processes and continuous Gaussian processes. A problem of great importance in time-dependent reliability analysis using stochastic process theory is that of the first crossing by a stochastic process of a given barrier or curve. The stochastic process may represent a load \( S(t) \), and the curve may represent a strength \( R(t) \) as in Fig. 2. This constitutes a so-called “crossing problem” [9]. The time at which \( S(t) \) crosses the barrier \( R(t) \) for the first time is the time to failure and is a random variable. The probability that \( R(t) \) is less than \( S(t) \) occurs within \([0-t]\) is called the “first passage probability” [8,9]. The first passage probability is equivalent to the probability of failure within \([0-t]\) expressed by Eq. (4). As pointed out in the previous section, the solution of the crossing problem is rather difficult, because the complete history of the stochastic processes within \([0-t]\) should be considered. If the number of loads \( N(t) \) within the interval \([0-t]\) is a random variable described by a Poisson counting process, then:

\[
P[N(t)=n]= \frac{(vt)^n}{n!} e^{-vt/n!}
\]

in which \( v \) is the intensity of the Poisson process (or mean of occurrence of the loads). The waiting time \( t_k \) for the occurrence of the \( k^{\text{th}} \) load is a random variable and is gamma distributed [8]. Therefore, the cdf \( F_m(t) \) for the time \( t_n \) which must elapse before the occurrence of the \( n^{\text{th}} \) load is obtained as follows:

\[
F_m(t) = P[t_n \leq t] = 1 - P[t_n > t] = 1 - P[N(t) < n] = 1 - \sum_{k=0}^{n-1} (vt)^k e^{-vt/k} / k!
\]

Of particular interest is the waiting time \( t_1 \) before the occurrence of the first load within \([0-t]\), which is exponentially distributed and is given by:

\[
\mu_R = \frac{\mu_D \mu_{R_0}}{\sqrt{V_D^2 + V_{R_0}^2}}
\]

\[
V_R = \sqrt{V_D^2 + V_{R_0}^2}
\]
\[ F_{t_1}(t) = P [ t_1 \leq t ] = 1 - e^{\nu t} \]  
(12)

The above equation represents the *first passage probability* [9]. If we consider a Poisson spike process as shown in Fig. 2, which is defined by its intensity \( \nu \) (constant) and rectangular pulses of magnitude \( S_k \) and duration \( \tau \). The load \( S_k \) is a random variable independent between pulses with a cdf \( F_S(s) \). The first passage probability can be obtained for the limit case as \( \tau \to 0 \). The probability of an upcrossing \( \nu_R^+(t) \) of level \( R(t) \) is given by [9]:

\[ \nu_R^+(t) = \lim_{\Delta t \to 0} \frac{P \{ [S(t) \leq R(t)] \cap [S(t+\Delta t) > R(t)] \} \nu}{\Delta t} \]  
(13a)

\[ \nu_R^+(t) = F_S[R(t)] \{ 1 - F_S[R(t)] \} \nu \]  
(13b)

The above equation expresses the intensity of a Poisson process and is referred to as “upcrossing rate” [9]. If \( R(t) \) is large, the upcrossing rate becomes:

\[ \nu_R^+(t) = \{ 1 - F_S[R(t)] \} \nu \]  
(13c)

Using Eq. (12), the first passage probability or the probability of failure within \([0-t_L]\) is:

\[ P_{[0-t_L]} = 1 - \exp[-\nu_R^+(t) t_L] = 1 - \exp[-\{ 1 - F_S[R(t)] \} \nu t_L] \]  
(14)

Thus, the probability of failure is a function of the lifetime or service life \( t_L \) of the structure. If \( \nu \) is a function of time \( \nu(t) \), then \( \nu t_L \) should be replaced in Eq. (14) by \( \int \nu(t) dt \). If the resistance is assumed to deteriorate with time according to Eq. (7), and if the degradation function \( D=D(t) \) is assumed independent of the load history (which is the case of deterioration due to environmental factors), and \( R_o \) is a random variable with a pdf \( f_{R_o}(r) \), then Eq. (14) becomes [11,141:

\[ P_{[0-t_L]} = 1 - \int_0^\infty \exp \left[ -\nu \{ t_L - \int_0^t F_S[D(t)R_0] dt \} \right] f_{R_o}(r) dr \]  
(15)

An approach for durability design and maintenance is obtained by limiting the above probability to an allowable level that depends on the consequences of failure. The implementation of the above requires further simplifications to derive a practical design and prediction procedure similar to the semi-probabilistic approach in section 3.1.

### 3.3 Performance- vs. service life -based durability design and prediction

The probability of failure in the time interval \([0-t]\), \( P_{[0-t]} \) defined above is equivalent to the event: the service life \( T \) (or \( t_L \)) is less than \( t \) [11,12,13], i.e.:

\[ P_{[0-t]} = P \{ R(t) \leq S(t) \text{ in } [0-t] \} = P \{ T \leq t \} = F_T(t) \]  
(16)

in which the service life \( T \) is a random variable, and \( F_T \) is the corresponding cumulative distribution function. The probability density function of \( T \), \( f_T(t) \), also called the unconditional failure rate function [8,12], is given by:

\[ f_T(t) = dF_T(t)/dt \]  
(17a)

thus

\[ f_T(t) dt = P \{ t < T \leq t+dt \} = P \{ R < S \text{ in } [t, t+dt] \text{ and } R > S \text{ in } [0-t] \} \]  
(17b)

The hazard or conditional failure rate function \( h_T(t) \), is defined as the probability of failure per unit time conditional upon survival to time \( t \), i.e.:
\[
\frac{\partial h_T(t)}{\partial t} = P(t < T \leq t + \Delta t \mid T > t) = P\{R < S \text{ in } [t, t + \Delta t] \mid R > S \text{ in } [0-t]\}
\]

thus: \[
\frac{\partial h_T(t)}{\partial t} = f_T(t)\left[1 - F_T(t)\right] = d\left(\ln\left[1 - F_T(t)\right]\right)/dt
\]

For low values of \(F_T(t)\) (in the order of \(10^{-4}\)), \(f_T(t)\) and \(h_T(t)\) can be made equal. Using (17a) and (Mb), the following relationships are obtained:

\[
F_T(t) = 1 - \exp\left[-\int_0^t h_T(\tau) \, d\tau\right]
\]

and \[
f_T(t) = h_T(t) \exp\left[-\int_0^t h_T(\tau) \, d\tau\right]
\]

The above equations show the equivalence of the performance-based and service life-based formats for the formulation of the durability design and service life prediction problems. The application of either of the two formats will yield the same solution. If failures occur purely at random, the hazard function is constant and represents an exponential distribution. However, if the failure is due to deterioration and wearing out, the hazard function increases with time, and if \(h_T(t)\) increases with time as follows:

\[
h_T(t) = \frac{(\beta/A)(t/\alpha)^{\beta-1}}
\]

then \[
F_T(t) = P\{O-t\} = 1 - \exp\left[-(t/\alpha)\beta\right]
\]

which represents a Weibull distribution, where \(\alpha\) and \(\beta\) are the scale and shape parameters, respectively. As discussed earlier, the derivation of a reliable design is based on limiting the above lifetime probability of failure to some acceptable threshold value that depends on the consequences of failure.

4. Conclusions

Reliability-based approaches for durability design and prediction of building components are proposed. The durability design and prediction of service life of building components has been formulated as a time-dependent reliability problem. The time-dependent reliability problem has been simplified into a time-independent reliability problem using an extreme-value probability distribution for the maximum lifetime load. The resistance degradation and its variability are included in the model, and the probabilistic design problem is then transformed into a deterministic (or semi-probabilistic) design problem using the first-order second moment theory. This semi-probabilistic integrated approach to durability design overcomes the shortcomings of the empirical factorial approach and the complexities of a fully probabilistic design method. A second approach based on stochastic process theory is developed, in which the durability design or prediction problem is formulated as a crossing problem, for which the lifetime probability of failure is obtained from the first passage probability. Durability design and service life prediction are based on keeping the lifetime probability of failure below a certain target value that depends on the consequences of failure of the component or system. The proposed approaches constitute further steps towards reliability-based durability design and prediction.
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References