DAMAGE AND ENERGY DISSIPATION IN CONCRETE UNDER CYCLIC UNIAXIAL COMPRESSIVE LOADING IN QUASI–STATIC TESTS

E. Schwabach, E. Raue, H.–G. Timmler
Bauhaus–University Weimar, Germany

1. Introduction

Concrete is a construction material, that’s non-homogeneous because of its mix design, its manufacturing process and its structure formed during curing. In general concrete is modeled as an homogeneous, ideal material. This is the aspect in the macro–level of the concrete. A closer inspection of the structural changes or structural damage respectively, leads at least to a view into the submacroscopic–level or meso–level of concrete.

In that level the concrete is considered as a two-component material. For that the coarse concrete aggregate (size ~> 4 mm) is embedded in an “homogeneous” matrix consisting primarily of the cement paste with its pore system and the embedded fine gravel grading.

Even before loading, concrete contains some flaws or pore–cracks or rather micro–cracks due to the hardening process. In compression, the fracture mechanical properties of concrete are governed by those of its two components, cement paste and aggregate, and the composite behavior in the contact zone between aggregate and matrix. The age–dependent properties (increase in strength and elastic modulus etc.) are primarily attributable to changes in the matrix component caused by changes in its hydration state. The strength of concrete is to a large extent dependent upon that of its cement paste fraction.

In various research papers [1, 2, 3, 4, 5], the structural changes occurring during the loading of concrete have been described with the compaction of particles in the mortar matrix (plastic deformation) and the propagation, growth and union of micro–cracks between aggregate and matrix and in the matrix itself (quasi-plastic deformation). Sporadically there are cracks through the coarse concrete aggregate too. The change in initial elastic modulus (progressive reduction in stiffness) after several unloading and reloading events is due to fact, that micro–cracks can’t close completely after unloading events [6, 7; cf. Fig. 1].

2. Elementary experimental analysis

The behavior of concrete is dependent upon its load history. The nonlinear nature of the stress–strain relationship of plain concrete subjected to a series of cycles of uniaxial compressive loading (increasing strain) and unloading (decreasing strain) is shown in Fig. 1. In general the envelope curve for a concrete subjected to cyclic axial compression coincides well with the stress–strain curve under monotonically increasing strain, especially in the ascending part. Moreover the cyclic test delivers information about the structural changes occurring com-
combined during compressive loading. The change of stiffness in the reloading stress–strain path and unloading plastic (residual) and quasi–plastic deformations are the major parameters that control general behavior of concrete under cyclic compressive loading. For investigating the pre–peak range of the stress–strain curve it’s favorable to perform the loading pattern under load control. To receive detailed information of the structural behavior up to failure, the applied load level should be gradually increased by small, constant load steps (~ 5 % cal F_u) [8]. The principle of the interaction between cyclic quasi–static loading and structural response of the concrete is shown in Fig. 2.

![Fig. 2: Load cycle and response to several cyclic loading](image)

The structural response is greatly affected by the loading rate and loading last term. It’s commonly thought that when loading periods last over 5 seconds, the measured strain has already been affected by creep, the so–called long term deformation of concrete [9]. In practice, the experiment can not mostly be achieved successfully in such a short time. It leads to the result that the short–term (instantaneous) and long–term (time–dependent) deformation are always mixed. Fig. 3 shows a typical short–term stress–strain curve of concrete in case of loading, loading last term and full unloading, in which some important deformation characteristics are exhibited.

The deformation on loading:

$$\varepsilon_{0-1} = \varepsilon_{el} + \varepsilon_{pl,0} + \varepsilon_{cr,0} = \varepsilon_1$$

(1)

contains elastic strain $\varepsilon_{el}$ (recoverable deformation), plastic strain $\varepsilon_{pl,0} - 1$ (irrecoverable deformation) and short–term creep deformation $\varepsilon_{cr,0} - 1$.

During unloading, the deformation:

$$\varepsilon_{2-3} = \varepsilon_{el} + \varepsilon_{cr,2-3}$$

(2)

consist of the recovered elastic deformation $\varepsilon_{el}$ and the reversible short–term creep deformation $\varepsilon_{cr,2-3}$.

![Fig. 3: Deformation characteristics during a full loading event](image)

Defining $\varepsilon_{cr,0-1} = \varepsilon_{cr,2-3}$, the plastic deformation not due to short–term creep may be written as:

$$\varepsilon_{pl,0-1} = \varepsilon_{0-1} - \varepsilon_{2-3} = \varepsilon_{pl,d}$$

(3)

In case of a constant loading rate and constant loading last and unloading last term respectively, $\varepsilon_{cr,3-4}$ is the reversible component $\varepsilon_{cr,el}$ of the time–dependent creep deformation $\varepsilon_{cr,1-2}$. The plastic and quasi–plastic deformation after total unloading last term consisting of irreversible short–term creep deformation $\varepsilon_{cr,pl}$ or structural changes respectively caused by plasticity and micro–cracking is:

$$\varepsilon_{pl} = \varepsilon_{0-1} + \varepsilon_{cr,1-2} - \varepsilon_{2-3} - \varepsilon_{cr,3-4} = \varepsilon_{pl,d} + \varepsilon_{cr,pl}$$

(4)

The plastic proportion $\varepsilon_{cr,pl}$, due to irreversible short–term creep then can be expressed as follows:

$$\varepsilon_{cr,pl} = \varepsilon_{cr,1-2} - \varepsilon_{cr,3-4}$$

(5)

To receive a quantitative assessment of damage occurring in concrete during loading it’s reasonable to use an energy basis, because it completely represents the interaction between imposed force or load and structural re-
sponse of the concrete. Similar to Spooner and Dougill [3] the total energy or work done during a loading–unloading cycle:

\[ E = E_d + E_{cr} + E_{el} = \int_{\varepsilon_0}^{\varepsilon} \sigma_1(\varepsilon) \cdot d\varepsilon + \sigma \cdot (\varepsilon_2 - \varepsilon_1) \]  

(6)

can be separated into three components (Fig. 4). The energy dissipated in damage or rather structural changes (micro–cracking and plasticity) is represented by:

\[ E_d = \int_{\varepsilon_0}^{\varepsilon} \sigma_1(\varepsilon) \cdot d\varepsilon + \sigma \cdot (\varepsilon_5 - \varepsilon_1) - \int_{\varepsilon_0}^{\varepsilon} \sigma_2(\varepsilon) \cdot d\varepsilon \]  

(7)

I.e. there is a directly relation between structural changes and parallel dissipated energy. \( E_{cr} \) corresponding to the energy dissipated in creep, and \( E_d \) representing the recovered elastic component of the total energy \( E \).

Fig. 4: Division of the total energy involved during a loading–unloading cycle

The experimental results for \( \varepsilon \) or \( E \) respectively are greatly affected by the loading rate and loading last term, e.g., \( \varepsilon_{0-1} \) doesn’t reach the same value as that when the loading rate is lower or \( \varepsilon_{cr,1-2} \) reaches a higher value as that when the loading last term lasts shorter. So for carry out an experiment it’s very important to have a constant, equal loading and unloading rate (~ 1/240 cal Fu [kN/s]) and constant, equal loading and unloading last terms (3 – 15 min). To achieve a steady damage state on a load level (stable equilibrium), a several repeated loading up to that load level as well as unloading to near zero load level (Fig. 5) is very useful. The registered structural changes on the next higher load level then is only due to the applied load increase [8].

To assess the structural damage effected of forces it’s useful to separate the energy–dissipating mechanisms or the energy \( E_d \) respectively dissipated by the two mechanisms. An approximate solution for that problem is described in the next section.

3. Quantitative assessment of the structural changes

The total energy dissipated in structural damage \( E_d \) is composed of the energy dissipated in micro-cracking \( E_{d,cr} \) and the energy dissipated in plastic behavior \( E_{d,pl} \):

\[ E_d = E_{d,cr} (\sigma, \varepsilon_1) + E_{d,pl} (\sigma, \varepsilon_{pl}) \]  

(8)

The progressive reduction in stiffness after unloading and reloading, indicated by the reduced initial slope of the reloading curve, corresponds to structural damage due to micro–cracking. Thus the variation of an initial secant modulus \( E_c \) (cf. Fig. 3) provides an assessment of the relative damage during loading. The change in initial secant modulus \( E_c \) on the same load or rather stress level can be written as (cf. Fig. 6):

\[ \Delta k_{Ec} = 1 - \varepsilon_{1,2} / \varepsilon_{1,1} \]  

(9a)

\[ \Delta k_{Ec} = 1 - \varepsilon_{1,3} / \varepsilon_{1,2} \]  

(9b)

between two following stress levels as:

\[ \Delta k_{Ec} = 1 - E_{c,(+1),1} / E_{c,1} \]  

(10)

with:

\[ E_{c,1,2,3} = \sigma_1 / \varepsilon_{1,2,3} \]  

(11)
The energy dissipated in internal micro-cracking can be approximately estimated as shown in Fig. 6. Thus the energy dissipated in the first cycle (primary cycle) of a stress level $\sigma_i$ can be expressed as:

$$E_{d,cri,1} = \frac{1}{2} \cdot \sigma_i \cdot (\varepsilon_{1,2} - \varepsilon_{1,1})$$  \hspace{1cm} (12)

It is noticeable that this approach provides a quantitative view of the damage occurring during loading caused by the individual mechanisms micro-cracking and plasticity. The next section deals with the evaluation of experimental results of concrete subjected to cyclic uniaxial compressive loading using the described techniques.

4. Analysis of cyclic loading tests

The following test evaluation is restricted to the recorded data of a test series performed on three plain concrete cylinders undertaken in connection with [8]. The dimension of every specimen is given in Table 1.

Table 1: Dimensions of the normal concrete cylinders

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Mean diameter $d_m$ [cm]</th>
<th>Height $h$ [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z-1</td>
<td>9.71</td>
<td>29.75</td>
</tr>
<tr>
<td>Z-2</td>
<td>9.66</td>
<td>29.80</td>
</tr>
<tr>
<td>Z-3</td>
<td>9.68</td>
<td>29.85</td>
</tr>
</tbody>
</table>

The concrete mix proportions are constant for all three test specimens. The compressive strength was tested at age 128 days on six 150 mm cubes of the same batch. The mean value of the peak stress (characteristic strength), converted to 150/300 mm cylinders, was $f_{cm} = 47$ N/mm². Considering EC 2, the concrete may be classified to strength class C35/45 ($f_{cm} = 43$ N/mm², $E_{cm} = 33500$ N/mm²).

The concrete specimens were tested under several cyclic uniaxial compressive loading and unloading as shown in Fig. 5. All tests were performed under load control up to failure of the specimens to study the pre-peak range of the stress-strain curve. The tests were carried out by a servo-controlled material testing system with a hydraulic capacity of 63t. To measure the deformation during testing, in the middle range three strain gauges were applied round the concrete cylinder. Considering the slenderness ratio of the test specimens ($h/d_m \cong 3$) in this range an uniaxial stress distribution is acceptable.

During the test a basic load of 25 kN remained to be applied. The loading and unloading rate used in the tests is 1.5 kN per second. The loading and unloading last term was constant 3 min. Two repetitions of load cycle on every load level were performed. The differential load or force respectively between the load steps was constant 25 kN.
Fig. 8 illustrates the relation between force and longitudinal compressive strain for specimen Z−1, that’s similar for all specimens. Due to carrying out the tests under force control, the plain concrete cylinders failed in a brittle manner.

Using the recorded data, the total energy of structural changes or damage respectively $E_d$ (see Fig. 4) was incremental analyzed for every primary cycle of the loading pattern. The development of $E_d$ for every specimen depending on the applied stress level up to failure is shown in Fig. 9.

Fig. 10 illustrates the deformation $\varepsilon_{0,1}$ [cf. Fig 3, Eq. (1)] of every primary loading curve ($\varepsilon_{1,1}$), following loading curve ($\varepsilon_{1,2}$) and the sum of the plastic deformation $\varepsilon_{pl}$ after every primary cycle ($\varepsilon_{pl,1}$) related to the applied stress level up to failure for specimen Z−1 (similar for all specimens).

Using Eq. (12) and Eq. (13), the proportions of the individual dissipative mechanisms in the total absorbed energy $E_d$ can be approximately determined. The result of that calculations for all specimens is shown in Fig. 11. In the current case the discrepancy of the total energy dissipated up to failure in comparison to Fig. 9 is on average 2,62 %.

Fig. 11 distinctly shows, that until failure of the specimens in essence energy is dissipated by mechanisms due to plastic effects ($E_{d,pl}$). The proportion of $E_d$ dissipated by micro-crack formation and growth ($E_{d,cr}$) is with an average value of 17,3 % comparatively small, but not negligible. Because it usually indicates the failure or rather fracture mechanism of plain concrete subjected to compressive loads.

The secant stiffness or secant modulus $E_c$ [Eq. (11)] respectively for every primary cycle depending on the applied stress level up to failure for every specimen is illustrated in Fig. 12. The initial value of $E_c$ on average correlates relatively well with the normative value ($E_{cm} = 3350$ kN/cm²) corresponding to EC 2, despite another definition and different slenderness ratio of the test specimens. However, the nearly linear degradation in stiffness with increasing stress level is more important. In the current
case the E–modulus reduces average 22.2 % up to failure of the specimens, i.e. on an average 1.48 % per load cycle (present 15 × 6.25 % of the ultimate load).

![Graph](image)

**Fig. 12: Development of the secant stiffness $E_c$ up to failure**

5. Conclusions

A technique is described which gives a quantitative assessment of the damage mechanisms occurring during the cyclic compressive loading of plain concrete. Damage is indicated by the change in initial E–modulus and of energy dissipated in damage. Most attention being given to the energy method.

Test evaluations show that the processes associated with damage are continuously effective during increasing applied stress level. A small share of the energy dissipated in damage is due to effects related to micro-cracking. Up to the failure basically energy is dissipated by effects due to the plastic behavior of the concrete.

The initial E–modulus or rather stiffness of the investigated specimens gradually falls up to failure. Thus it is no constant as supposed in standards. A caused damage on a certain stress level effects an degradation of the initial stiffness on reloading. This reduction in stiffness is basically caused by micro-cracking.

This paper is restricted to concrete behavior in uniaxial compression. Generalization to other systems of loads is possible and may be useful.

**References**