Reinforced concrete structural design requirements issued around the world are generally based on limit state concept. Regarding this concept, the limit state is expressed using this inequality:

$$ R - S > 0; $$  \hspace{1cm} (1)

where $R$ is the structure strength; $S$ is the forces developed within the structure.

Structure strength or stress, causing limit state, is calculated according the design codes of various countries, resulting in different design values, because load and material safety factors are not the same in different country design codes. However, reliability of structure, reflecting the first limit state and calculated according different design codes, falls usually within the same range.

In design of beamless flat floor slabs, their limit state is commonly expressed by punching shear strength, since it is obviously lower than slab shear strength. Comparing different punching shear strength calculation methods, one may find that not only material strength values and load safety factors vary, but also the sequence of the punched-out pyramid basis perimeter calculation is different. Calculation of the punching area according EC2, BS8110 and some other design codes assumes that punching plane makes $33.7^\circ$ with horizontal plane, while DIN, SNiP assume this angle to be equal to $45^\circ$, with the same situation in Australian AS3600 and American ACI318 design codes.

Also, some other codes evaluate the influence of axial forces (EC2, AS3600, ACI318) and the bending moments (ACI318, BS8110 & SABS 0100) acting within the punching zone to the punching shear strength. EC2 recommends take into account the intensity of longitudinal flexural reinforcement in the slab and column connection zone. Although, there is no common rules set for this recommendation.

In order to figure out, what affects the punching plane inclination angle, influence of reinforcement amount on the punching shear strength, and the influence of “lateral force/bending moment” ratio acting in the punching zone, theoretical research and computer-aided simulation was carried out. This job is the next stage of researching, previously done by authors [1].

2. Design for Punching Shear

Analytical methods of floor slab calculation within support zone for common cases are not clearly stated in design codes. Most national design codes deal only with shear reinforcement work evaluation recommendations, while designers are usually concerned about the influence of intensity of longitudinal reinforcement within the support zone to the punching strength, bending moment value, bending moment/lateral force ratio, influence of slab and column rigidity to the stress-strain state of joint area.

2.1 ACI318

Section 11.12 of ACI318 defines the requirements for punching shear [2]. The critical perimeter is defined to occur at $d/2$ from the column face (Fig.1), where $d$ – means the depth to the centroid of the tension reinforcement, but $d$ shall not be taken as $< 0.8D$. 

$$ 	ext{ moments (ACI318, BS8110 & SABS 0100) acting within the punching zone to the punching shear strength. EC2 recommends take into account the intensity of longitudinal flexural reinforcement in the slab and column connection zone. Although, there is no common rules set for this recommendation. } $$

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The nominal shear strength provided by concrete for non-prestressed slabs $V_c$ is taken as the smallest of:

$$V_c = \left( \frac{1}{6} + \frac{1}{3\beta} \right) \sqrt{f'_c b_0 d}$$

(2)

where $\beta$ is the ratio of the long side to short side of the column;

$$V_c = \left( \frac{\alpha_3 d}{12 b_0} + \frac{1}{6} \right) \sqrt{f'_c b_0 d}$$

(3)

where $\alpha_3$ is 40 for interior columns (30 for edge columns and 20 for corner columns);

$$V_c = \frac{\sqrt{f'_c b_0 d}}{3}$$

(4)

### 2.2 EC 2

Section 4.3.4 of EC2 defines the punching shear requirements for slabs [3]. The critical perimeter is defined at a point 1.5 $d$ from the loaded area (Fig.2.), where $d$ is effective depth of the slab (depth to the centroid of the tension reinforcement), but not less as 0.8$D$.

### 2.3 DIN 1045

Section 22.5.1.1. of DIN 1045 defines the requirements for punching shear [4]. The critical perimeter is defined to occur at $d/2$ from the column face. The shear resistance of non-prestressed slabs are calculated as follows:

$$Q_R = \tau_R \cdot u \cdot h_m,$$

(7)

where: $\tau_R$ - concrete shear strength; $u$ - the critical perimeter; $h_m = d$ - effective depth of the slab.

### 2.4 SNiP 2.03.01-84

Section 3.42. of SNiP 20301-84 defines the requirements for calculation of punching shear [5]. The critical perimeter $u_m$ is defined, as average perimeter of both top and bottom surfaces of punching pyramid.

The shear resistance of slabs are calculated as follows:

$$F = \alpha \cdot R_{0.2} \cdot u_m \cdot h_u,$$

(8)

$\alpha$ - coefficient of concrete type ($d=1$ for normal weight concrete); $R_{0.2}$ - concrete tension strength; $h_u$ - effective depth of the slab (the same as $d$ above).

As it is shown above, only Eurocode defines special design conditions for longitudinal flexural reinforcement of concrete slab inside the punching area.

### 3. Analysis models of flat floor plate-column joints

The influence of some variables on beamless flat slab punching shear resistance is hardly evaluated by analytical methods. At the same time the real testing conditions are too simplified to simulate the real stress-strain state details.
Wide possibilities opened when computer-aided modeling and analysis technologies, using finite element methods, became available. A computational experiment plan for the analysis of beamless flat slab-to-column joint was done, in order to evaluate all the factors, which may have any kind of significance for the behaviour.

For the calculations, the 2.5D model comprising two-span frame and floor slab stripe with two-floor height columns was taken (see fig. 3).

![Fig. 3. 2.5D Analysis model](image)

The cross-section of the column was 400x400 mm, while the slab depth was 200 mm. The influence of slab-column rigidity ratios was analyzed using different cross-sectional dimensions: the column cross-section was 300x300mm with slab depth 150 mm in first case, and 500x500mm column section with 250mm slab depth in the second case.

The width of effective (column) slab strip was 800 and 1000 mm respectively for 45\(^\circ\) and 33\(^\circ\) angle of inclination of punching planes. The results were compared with minimal 400mm (flat frame) and maximal recommended 1500mm (quarter of span) depth of slab strip. The considered spans were most commonly used – 6.0; 4.5; 7.5 m length.

The behaviour of non-reinforced concrete slab-to-column connection model prior to failure was analyzed in comparison with identical reinforced concrete models, when:

1. the slab was reinforced only by longitudinal tension reinforcement, which intensity varied depending on bar diameter – Ø10 mm (\(\mu_\% = 0.40\)); Ø12 mm (\(\mu_\% = 0.50\)); Ø16 mm (\(\mu_\% = 1.0\)); Ø20 mm (\(\mu_\% = 1.57\)); Ø22 mm (\(\mu_\% = 1.90\)); Ø25 mm (\(\mu_\% = 2.45\));
2. the slab was reinforced by longitudinal bars and shear ones of different quantity and distribution patterns in the bending zone.

3. the slab was reinforced by longitudinal bars and some amount of bent reinforcement of different quantity and distribution patterns in the punching zone.

In all cases, the amount of longitudinal flexural reinforcement in span was sufficient to provide strength and proof from failure in normal section.

The concrete grade used in modeling was B25, reinforcement grade – A-III. Determining the control joint strength parameters, all the concrete and reinforcement strength characteristics were taken as their characteristic values.

Computer models were loaded in stages by uniformly distributed loading in both spans until the failure of the joint due to punching shear. In order to reach the real result as close as possible, the non-linear approach was used. Modeling and analysis were performed using finite element analysis software package COSMOS/M [5].

### 4. Non-linear concrete model

The concrete model was a three-dimensional, rate-independent model with a bounding surface [6,7]. The model adopts a scalar representation of the damage related to the strain and stress states of the material. The bounding surface in the stress space shrinks uniformly as the damage due to strain softening and/or tension cracks accumulates. The material parameters depend on the damage level, the hydrostatic pressure, and the distance between the current stress point and the bounding surface.

![Fig. 4. Bounding Surface](image)
The bounding surface function is:

\[
f(\sigma_{ij}, k_{\text{max}}) = \frac{0.25 j_2 + 3.1 \sqrt{j_2}}{4 I_1 + 3.492} (\cos(\theta) + 5) - H(k_{\text{max}})
\]

where: \( H(k_{\text{max}}) = \frac{40}{39 + 2 k_{\text{max}}^2} \)

where: \( \sigma_{ij} \) is the normalized stress tensor (with respect to the ultimate compression strength \( f'_{c} \)), \( I_1 \) and \( j_2 \) are the first stress and the second deviatoric normalized stress invariants respectively, \( \theta \) is the loading angle, and \( k_{\text{max}} \) is the maximum damage coefficient.

The damage coefficient represents the damage due to strain hardening or softening. The damage coefficient value is always positive and its magnitude in conjunction with the hydrostatic pressure represents the damage due to compression and tension cracking. For instance, the damage in a uniaxial compression test at the ultimate strength is normalized to be 1.0 and its value is approximately 0.20 for uniaxial tension test. The damage is obtained by integrating the incremental damage coefficient that depends on the plastic strain and the distance from the current stress state and the bounding surface.

\[
dk = \frac{RdD}{H^p F_i(I_1, \theta)} \quad \text{for Deviatoric loading};
\]

\[
dk = \frac{d\gamma_0^p}{F_i(I_1, \theta)} \quad \text{for Post-failure};
\]

where: \( H^p = \) plastic shear modulus; \( F_i(I_1, \theta) \) is a function of \( I_1 \) and \( \theta \) and loading conditions; \( D \) - normalized distance \( r/R \); \( r \) - distance from the projection of the current stress point on the deviatoric plane to the hydroaxis; \( R \) - distance of the bounding surface from the hydroaxis along the deviatoric stress direction.

The model is defined by two material parameters which are:

\( FPC \) = the concrete ultimate strength \( f'_{c} \);
\( EPSU \) = the ultimate strain \( \varepsilon_0 \) (the strain at stress of \( f'_{c} \) in the uniaxial compression test).

The low strain elasticity modulus \( E \), bulk modulus \( (K) \), and shear modulus \( (H^p) \) are set to:

\[
E = 57,000 \frac{E_0}{f'_{c}}; \quad H^p = \frac{H^* C_i}{F_i(I_1, \theta)};
\]

where:

\[
H^* = \frac{2.4 R (1-D)^{0.65 D^2}}{(1+0.7 k_{\text{max}}^2) A_L} \quad \text{for Deviatoric loading};
\]

\[
H^* = \frac{2.4 R}{(1+0.7 k_{\text{max}}^2) A_L} \quad \text{for Deviatoric unloading};
\]

\( C_i = 1.0 \) - for \( D < 0.9 \); \( C_i = 1.0 E5 (1-D)^2 \) - for \( D < 0.9 \);

\( A_L \) = is a factor that depends on damage parameter.

The parameters are temperature independent. Moreover, the model should be used in conjunction with small strain formulation.

Under tension stresses, the model behaves as a non-linear strain hardening material until it reaches the tension strength and starts to behave as a perfectly plastic material. The maximum tensile strength for uniaxial test is considered as:

\[
f_t \approx 0.17 f_{c}';
\]

5. Analysis results

Analyzing the working conditions of slab-to-column joint, the main emphasis was to clear out the mechanism of the punched-out pyramid surface formation and the failure process propagation. This process was observed under different ratios between bending moment and shear force, also changing the slab span and the slab-column rigidity ratios.

The analysis results are represented by graphs on Fig. 5, a and b. The graphs show the principal stresses variation dynamics through the slab depth \( h \) in the slab to column contact and in the hypothetical punched-out pyramid base, defined at the level of axis of the upper longitudinal reinforcement.

At the early loading stages, the clear balance of tensile and compressive stresses in the slab-column contact zone is observed. The distribution topology for normal and principal stresses is almost identical. Increase of the loading causes the lengthening and insignificant deepening of the principal stress zone.
Neutral axis is shifted to about 0.35h level and remains quite stable until 4th loading stage (50-60% of failure loading), after which the process of progressive collapse takes place. At this stage, the formation of punched-out pyramid base is over; increase of the loading causes only the pyramid height increases. Principal stress zone progressively increases in its depth; neutral axis constantly shifts down and is at (0.15-0.2)h level at the moment of failure. The depth of the compressive zone is about 30 mm at the moment of failure.

Fig. 5. The principal stress dynamics change throughout the depth of non-reinforced slab at the slab-to-column joint (a) and hypothetical punched-out pyramid base level (b): 1 – 3.7%; 2 – 18.5%; 3 – 33.3%; 4 – 55.5%; 5 – 70%; 6 – 85%; 7 – 100% - loading stages.

First vertical cracks appear at the slab-column joint topmost corner at the 33% of the failure loading. While the loading increases to 40-50% of failure value, providing further principal stress zone length increase, the origin of collapse appears at the 1/2h. The cracks, formed at this zone are inclined at 45°-60° with respect to the slab plane. Collapse zone rapidly develops, increases in width and depth and propagates towards the lowermost angle of the support. At about 70% of failure loading, an identical kernel of maximum principal stresses appears at the slab midpoint, being separated from the original failure zone. This is how the punch-out pyramid is formed. At the failure stage, the very thin compression zone is shear-cut.

At the failure stage, the angle of inclination of the side planes of the punched-out pyramid lies within 40°-55° sector (Fig. 6a and b). From the graph shown in Fig. 5b, one may see that the edge of the base of punched-out pyramid is at the distance h, i.e., equal to slab depth, from the column side surface; this means that the inclination angle of pyramid side planes is 45°. Having 6m span, the mean value of this angle tends to increase from 40° for 160mm slab depth up to 45°-47° for 200mm slab depth, and even 55° for depth h=250mm. Changing the span from 4.5m to 6.0 or 7.5m, the ratio of bending moment and shear force at the joint also changes, varying from M/Q=0.77 to 1.03 and 1.28, respectively. Some greater mean values of this angle (52°) observed in cases of small spans, and similar results - 45°-50° – were obtained for 6.0 and 7.5m slab spans.

Analysing the behaviour of the slabs with longitudinal reinforcement at the slab-to-column joint, the influence of the flexural reinforcement intensity to the punching shear resistance of the slab was also considered. The punching shear resistance values for all of the considered analysis cases are presented in Table1.

Table 1. Punch force $F_{sh}$ values at the slab-column joint for the cases of non-reinforced and longitudinally reinforced slabs.

<table>
<thead>
<tr>
<th>$M/Q$</th>
<th>$F_{sh}/U_m*h_0$ (MN/m²), when reinforcement ratio in % is</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.44</td>
<td>0.764 0.788 0.883 1.009 1.023</td>
</tr>
<tr>
<td>0.77</td>
<td>0.590 0.627 0.724 0.814 0.823</td>
</tr>
<tr>
<td>1.03</td>
<td>0.441 0.472 0.553 0.635 0.642</td>
</tr>
<tr>
<td>1.28</td>
<td></td>
</tr>
</tbody>
</table>

Analysis results, shown in Fig. 7a and b, depict the principal stress change dynamics throughout the slab depth h at the contact zone with column and at the base of the punched-out pyramid, measured at the upper longitudinal reinforcement level from the column axis.
Even at the early loading stages, the influence of longitudinal reinforcement to the distribution of normal and principal stresses may be seen. Increase of the loading above 50% of its ultimate value causes more significant lengthening of principal stress zone than in the case of non-reinforced slab. Thus, the neutral axis shifts down just to \((0.35-0.375)h\) depth level and remains at it until structure failure. The depth of the compressive zone in this case is about 60mm, i.e. twice bigger than for non-reinforced slab. The boundaries of punched-out pyramid are not so clearly defined, but the tensile and compressive slab layers are well expressed. All this leads to conclusion that longitudinal reinforcement indeed influences the stress-strain state of the joint.

Fig. 7. Principal stress change dynamics throughout the depth of longitudinally reinforced slab at the slab-column joint (a) and hypothetical punched-out pyramid base level (b): 1 – 3.7%; 2 – 18.5%; 3 – 33.3%; 4 – 55.5%; 5 – 70%; 6 – 85%; 7 – 100% - loading stages.

Fig. 8. Normal \(\sigma_x\) (\(\text{Sigma}_X\)) and principal \(\sigma_I\) (\(\text{Principal}_I\)) stress distribution topology in the slab, reinforced by longitudinal bars in the slab-column joint at the failure stage.

At the failure stage the angle of punching surface is not well defined in the case of slab with longitudinal reinforcement. Failure zone lies within 30°-52° sector range (Fig. 8, a and b). From the graph, Fig. 7b, one may find that the edge of the punch-out pyramid base is at about 300mm distance from the column side surface. So, in that case the inclination angle of punch-out pyramid side surfaces equals 33°.

Computational simulation shows, that the longitudinal flexural reinforcement increases the punching resistance of the slab. Graph, depicted in Fig. 9 shows, that the amount of longitudinal reinforcement influences the punching resistance of 200mm thickness slab in quite wide range of reinforcement ratio \(\mu\% = 0.40-2.5\%\) depending on the balance between bending moment and shear force (M/Q ratio). However, the limit of longitudinal reinforcement amount effect may be set at about 2%, resulting in similarity to Eurocode recommendations (1.5%).

Fig. 9. The influence of reinforcement intensity to punch resistance obtained in experiment for slab of \(h=200\text{mm}\), compared to theoretical values, calculated according EC2 (B25 concrete grade, A-III class of reinforcement).

6. Conclusions

1. Theoretical experiments show, that non-reinforced and longitudinally reinforced slab punched-out concrete pyramid plane inclination angles vary: for non-reinforced slab it equals about 45° and for reinforced - 33°.

2. The value of the punching force depends on amount of longitudinal flexural reinforcement in punching shear area – with increase of amount of longitudinal reinforcement, the punching force also increases.

3. The value of punching force depends on ratio of shear forces and bending moments, acting within...
critical section; the more is the ratio, the greater punch resistance may be achieved.

4. The maximum stresses $\sigma_x$, $\sigma_{pr1}$, $\sigma_{pr3}$ in limit state within punched section reach the same values, independently on ratio of shear forces and bending moments.

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