SMART OPTIMISATION AND SENSITIVITY ANALYSIS IN WATER DISTRIBUTION SYSTEMS

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Abstract

Parameter uncertainty in water pipe network models are studied using newly developed simplified mathematical notions. These enable studies to be done using public domain software, including EPANET. The results obtained can be easier to use and interpret than those obtained from more general mathematical notions. The general idea is to study how a flow- and pressure-related quantity varies as a set of state parameters are varied. The quantity considered here is the average pressure, enabling smart optimisation of a water distribution system by keeping the average pressure unchanged as water demands change, by changing the speed of the pumps. Another application area considered, using the same mathematical notions, is the study of the sensitivity of parameters. Two models are analysed as examples, showing how smart optimisation works, and what the sensitivity of various sets of parameters are. The various parameter categories have very different sensitivities to a given change in the average pressure that can be tolerated. The critical state parameters to determine accurately in the models depend on the network. For the combined schemes studied as examples, variation of the pressure with reservoir depths is only related to the reservoir depths, and the pressure does not vary with the tank diameters. There is a relationship between variations of the various pipe parameters for both the Hazen-Williams and Chezy-Manning pipe major friction loss formulas. It holds for any network where there are no pipe minor friction losses. Pipe diameters are the most sensitive, pipe roughness coefficients are medium sensitive, and pipe lengths are the least sensitive.

1. Introduction

In order to build a smart water distribution system, which optimises in real time in response to water demand variation, the underlying pipe network model that represents the system needs to be a robust representation of reality. This means that the uncertainty associated with input parameters of the model which are not exactly known should be small enough to lead to model-predicted water flow rates and pressures that are in an acceptable range consistent with reality. This is referred to as *sensitivity* of parameters. Input parameters that are not known with sufficient accuracy need to be determined more accurately.

A smart water distribution system will optimise the water flow rates and pressures by changing one set of input parameters in response to a change in another set of input parameters. This is referred to as *smart optimisation* of parameters. For example, a change in water demands can lead to a change in pump characteristics, so that either the water flow rates or pressures remain unchanged (on average).

The study of the sensitivity of parameters most generally involves various techniques that are computationally complicated to implement (see “literature review” later), and hence are not available unless specialised software is purchased or written. Here we propose a simplified, although generally less flexible, method to study sensitivity that can be implemented with public domain software like EPANET (Rossman, 2000). It studies sensitivity by looking at the effect a subgroup of parameters all changing in the same way, and its effect on, for example, the average of pressures.

It will be shown that the mathematical notions defined for the study of the sensitivity of parameters can directly be used to study the smart optimisation of parameters, but are limited to cases where one set of input parameters changes in the same way in response to a another set of input parameters changing in the same way. For example, all water demands tend to rise or fall depending on the time of the day, and all pump rates can adjust by rising or falling accordingly. The advantage of the limitations on smart optimisation is that no specialised software is needed for the analysis.

In summary, it will be shown that the mathematical notions enable the study of sensitivity, as well as smart optimisation, using public domain software.

After defining the mathematical notions, both sensitivity and smart optimisation are studied for a simple network and one realistic network of a South African village. In the case of the sensitivity study, the influence of the uncertainty of various parameters sets is investigated. Specifically, reservoir water levels, tank dimensions, pump curves, as well as pipe types, diameters and lengths are considered. Such a study determines the accuracy needed for these sets of parameters, and motivates which sets of parameters should be determined more accurately.

To the best of our knowledge, the mathematical notions have not been developed elsewhere. Although more general techniques are available, they cannot be implemented with public domain software, considerably reducing their use in the community.

This research is conducted within the context of the model being an accurate representation of reality when the network is accurately constructed, and the parameters are well chosen. Weaknesses of the modelling framework itself are beyond the scope of this work. The water distribution system concepts used are those in standard models like EPANET.

2. Literature Review

Sensitivity analysis (Frey *et al*., 2002) is a tool that may be used to ascertain

- *Forward use:* how much the outputs of a given model depend on each or some of the input parameters;
- *Backward use:* how variation in the outputs of a model can quantitatively or qualitatively be apportioned to different uncertain inputs.

This work uses sensitivity analysis notions. Since nominal range sensitivity analysis (NRSA) is the easiest sensitivity analysis method to implement and understand (Frey *et al*., 2002), we develop a sensitivity analysis formalism based on this method. It is a mathematical, rather than a statistical, method. NRSA allows only a single state parameter to vary at a time. This work extends this to multiple parameters varying in a specific *correlated* way.

A recent application of mathematical (as opposed to statistical) sensitivity analysis to the hydraulics of a water distribution system is a method for calibration, pipe diameter design, and input uncertainty assessment (Möderl *et al*., 2011). This GIS-based sensitivity analysis method, which is closely related to NRSA, is obtained from the mathematical notions developed here by applying Eq. 3 below, but allowing only *one* state parameter to scale. A mathematical sensitivity analysis method that allows individual parameters to vary *independently*, i.e. not correlated as in Eq. 3 below, was used for ranking the relative importance of pipes (Izquierdo *et al*., 2008). Such a sensitivity *matrix* analysis is computationally efficient (Izquierdo *et al*., 2008), and more general than NRSA, but is also considerably more complicated.

There have been some recent studies about the hydraulics of a water distribution system, e.g. on water age via GIS-based sensitivity analysis (Sitzenfrei *et al*., 2014), and about demands via sensitivity matrix analysis (Sanz *et al*., 2015).

3. Mathematical Development

3.1 Parameter Variation

The analysis of a pressurised water distribution network involves the construction of a network with N internal nodes (junctions), and L links (or lines) joining the external and internal nodes. A link has water flowing at rate q_k through it, and each individual node has pressure p_j at the position of the node. Here

$$q_k \quad k = 1, \ldots, L \qquad\qquad p_j \quad j = 1, \ldots, N \tag{1}$$

are solutions of coupled non-linear equations. Each flow rate is taken to be positive for the base-case solution, and a pressure is usually positive. (If a flow rate is not positive, is can be made positive by reversing the direction of the corresponding link). External nodes are by definition either reservoirs or tanks.

To solve the equations, the state of the system must first be specified by a set of base-case parameters

$$x_i \quad i = 1, \ldots, M \tag{2}$$

Examples of such parameters are pipe lengths and water demands. (Note that in standard water distribution models, the demand at an internal node is a state parameter, not a flow rate q_k). The solutions of the equations can formally be denoted as a non-linear function f, such that the solution vector $(\boldsymbol{q},\boldsymbol{p}) = f(\boldsymbol{x})$.

Backward sensitivity analysis and smart optimisation both involve the inversion of f. Although complicated in general, a meaningful case where it is simple is presented in the formalism below. Consider the parameter state label set corresponding to all parameters. Let X be a subset of that parameter state label set $\{1,\ldots,M\}$ and C (the complement) be the set containing the remaining elements. Define

$$x_i(r) = r\,x_i \quad i \in X \qquad\qquad x_i(r) = x_i \quad i \in C \tag{3}$$

Hence a subset of the state parameters is scaled by a common factor r (e.g. all pipe lengths can be scaled by a common factor, while other parameters are not scaled).

Allowing scaling via a common dimensionless factor is a natural choice. For example, the reliability with which the length of a pipe is known may well be proportional to the length of the pipe; the amount with which water demand changes may well be proportional to the size of the water demand.

Applications where the state parameters of *all* the pipes change together according to Eq. 3 have particular practical use. For example, if all pipe lengths are scaled, the pipe length of the entire network changes such that the length of each individual pipe changes in the same proportion. This hence captures "the length of the network changing". Similarly change of the roughness coefficients and diameters of the entire pipe network can be studied.

According to Eq. 3 the real number r is mapped into the solution vector according to

$$(\,\mathbf{q}(r), \mathbf{p}(r)\,) = f(\,\mathbf{x}(r)\,) \tag{4}$$

Hence as r varies a line is traced out in solution space. Consider a function $g(\boldsymbol{q},\boldsymbol{p})$, mapping into a real number. Introduce the function h defined by

$$h : \Re \mapsto \Re \quad \text{where} \quad h(r) = g\,(\,f(\,\mathbf{x}(r)\,)\,) \tag{5}$$

Although the full solution of backward sensitivity and smart optimisation problems involves the inversion of f, which maps a vector into a vector, these problems will be studied here as an inversion of the much simpler function h, which maps a number into a number. Assume the function h is defined on a region around r=1.

In this work a meaningful choice of g will be studied for practical applications. Let T be a subset of the internal node label set {1,…,N}, which has N_T elements. Define the specific instance of g as g_m, and the specific instance of h as P, such that

$$g_m(\mathbf{q},\mathbf{p}) \equiv \frac{1}{N_T}\sum_{j\in T}\epsilon_j\,p_j \qquad\qquad P(r) \equiv \frac{1}{N_T}\sum_{j\in T}\epsilon_j\,p_j(\mathbf{x}(r)) \tag{6}$$

Here ε_j denotes a sign which can have the value -1 or 1. When all the ε_j are unity, g_m is the average pressure in the selected internal nodes.

If the average pressure is uncertain within certain limits, which uncertainty in the pipe lengths is allowed? To solve such *backward sensitivity* problems, the lower and upper uncertainty limits of h are specified to be respectively

$$h(r_1) - h(1) < 0 \quad \text{and} \quad h(r_2) - h(1) > 0 \quad (r_1 \text{ and } r_2 \text{ are limits of } r) \tag{7}$$

which are then solved for the limits of r. This gives the limits of the state parameters by using Eq. 3. The solution procedure involves the inversion of the function h, which is assumed to be invertible.

3.2 Linearization

If some state parameters vary around the base-case parameters with r very near to 1, i.e.

$$\begin{aligned} \Delta r \equiv r - 1 \qquad & |\Delta r| \ll 1 \\ \Delta h \equiv h(r) - h(1) = D\,\Delta r \qquad & D \equiv h'(1) \quad (\text{derivative at } r = 1) \end{aligned} \tag{8}$$

using the Taylor expansion. Practically, this equation is used as follows. Choose state parameters satisfying Eq. 3 with r very near to 1. Calculate Δh using software (including the public domain software EPANET). Then deduce D from Eq. 8.

3.2.1 Smart Optimisation

The problem of interest is to scale a set of parameters together, and then to scale another set of parameters together, so that the average pressure remains unchanged. The smart optimisation applications in this work therefore use h=P from Eq. 6, with T the internal nodes with non-zero water demands and all signs equal to unity. However, the results in this section are for general h.

The procedure for smart optimisation is as follows. Obtain D_1 and D_2 for two sets of scaling state parameters X_1 and X_2 respectively, representing deviations around the *same* base-case parameters. Then, given the scaling value r_1, determine scaling value r_2 such that

$$\begin{aligned} \Delta h_1 &= D_1\,(r_1 - 1) \\ \Delta h_2 &= D_2\,(r_2 - 1) \\ 0 &= \Delta h_1 + \Delta h_2 \end{aligned} \tag{9}$$

yielding

$$r_2 = 1 - \frac{D_1}{D_2}\,(r_1 - 1) \tag{10}$$

3.2.2 Sensitivity

Assume $h(r)$ to be within the specified uncertainty limits

$$-\Delta h, \Delta h \qquad \Delta h \text{ positive or negative} \tag{11}$$

Then applying Eq. 8 the deduced uncertainty limits are

$$\Delta r = \frac{\Delta h}{D} \tag{12}$$

This gives limits of the state parameters according to Eq. 3. The procedure for a linear backward sensitivity study is as follows. Obtain *D* for a set of scaling state parameters *X*, representing deviations around the base-case parameters. Then, given the value Δ*h*, determine Δ*r* in accordance with Eq. 12.

In order for the procedure to be meaningful, Δ*h* must represent a tolerance that is allowable for the water distribution system. The sensitivity applications in this work assume that the "average" pressure has a certain tolerance. They therefore use *h*=*P* from Eq. 6, with *T* the entire internal node label set and the signs chosen in such a way that

$$|\Delta P| = \frac{1}{N_T} \sum_{j \in T} |\, p_j(\mathbf{x}(r)) - p_j(\mathbf{x}(1))\,| \qquad |\Delta r| \ll 1 \tag{13}$$

This ensures that the pressure difference at each node contributes with the same sign, implying that the effect of some nodes cannot cancel that of others, meaning that the effect of each node is taken into account constructively.

Eqs. 10 and 12 are the main results to be used. Even though they are only exact for *r very* near to 1, they will be used for values near to 1 as well.

3.3 Time Dependence

Time is not a state parameter. Most the quantities discussed here are in principle functions of time. For example, smart optimisation can be performed at various times as the hydraulic network evolves.

For the examples discussed in this paper, the state parameters and network controls are assumed to be time-independent. Correspondingly, only the stationary solution is discussed. In practice, this means that the parameters are constant over a time scale longer than the timescale that the hydraulic network settles into a stable state (the relaxation time). Particularly, the water demands and reservoir levels are time-averaged quantities. Smart optimisation is performed over a timescale longer than the relaxation time.

4. Network Models

4.1 Idealised Model

Consider the model in Figure 1. Water is pumped from the two reservoirs by two pumps. The water pushes up into the tank, and stabilizes at a certain level in the tank. There are water demand points at nodes 3 to 6.

The model is referred to as “idealised” because many parameters are chosen to be the same to enable the results to be easily understood. The base-case parameters are as follows. All elevations above sea level are at 200 metres (m) (including the water level of the reservoirs), except for the tank minimum water surface level at 230 m. The depths of the reservoirs are 10 m. Pumps 1 and 2 produce duty flows of respectively 40 and 20 litres/second (ℓ/s) at duty head 35 m (referred to as pump speed 1). The tank has a diameter of 20 m. The non-zero demands at nodes 3, 4, 5 and 6 are respectively 9, 12, 15 and 18 ℓ/s. All pipe lengths are 1000 m, all pipe diameters are 300 mm, and all pipe Hazen-Williams roughness coefficients are 100. All parameters are expressed in the units shown.

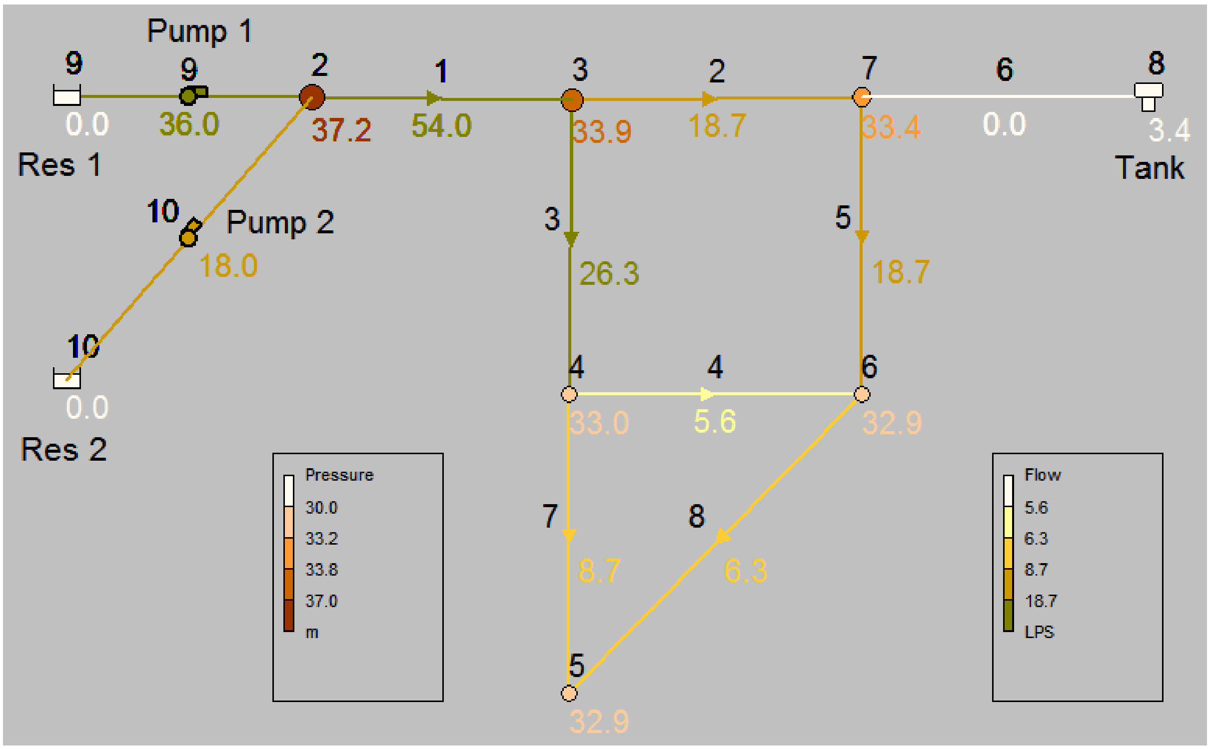

Figure 1 Idealized model. Unless shown otherwise, a link is a pipe. The flow rate along the links is displayed in ℓ/s. The pressure at the nodes is displayed as pressure head in m. The tank level stabilizes at 3.4 m.

Parameters are varied from the base-case solution to obtain the derivatives *D* from Eq. 8. These derivatives are shown in Table 1. They are measured in m, and their size should be compared to the average pressure head of the internal nodes considered.

Table 1 Results for the idealized model.

	Smart optimisation			*Reliability*	
	Demand	Pump speed		Demand	Pump speed
D (m)	-26.5	93.8		-25.2	93.8
Δr	-	-		-0.20	0.053
Δx	-	-		-1.8 to -3.6 ℓ/s	0.053
			Reliability		
	Pipe length	Pipe diameter	Pipe roughness	Reservoir depth	Tank diameter
D (m)	-3.33	15.7	6.08	10.0	0
Δr	-1.50 †	0.32	0.82 †	0.50	No bound
Δx	Insensitive	95 mm	Insensitive	5.0 m	No bound

† The deduced value of Δ*r* is too large to be consistent with the Taylor expansion approximation used to obtain Eq. 12. However if average pressure variation significantly smaller than 5 m is considered, pipe length and roughness can vary widely to be consistent with the average pressure, so that these parameters are “insensitive” to the average pressure.

Assume all water demands rise from the base-case demands by 5.0%. Using the values of *D* from Table 1 and Eq. 10, it follows that the pump speed must be increased by 1.4% to keep the average pressure of the demand nodes the same. Similarly, if the water demands falls from the base-case demands by 5.0%, the pump speed must be decreased 1.4% to keep the average pressure of the demand nodes the same.

In this way a smart water distribution system can be optimised to keep the average pressure of the demand nodes the same.

For the sensitivity study, the influence of the uncertainty of all relevant parameters sets is investigated. The derivatives *D* from Eq. 8 are shown in Table 1.

Assuming that the average pressure head at the internal nodes varies by +5 m from the base-case average pressure head of 34 m, Eq. 12 is used to obtain Δ*r*, shown in Table 1. This level of variation is a fair representation of how accurate the average pressure head should be known in a real water distribution system. Requiring that the average pressure head at the internal nodes to be known by 5 m, implies the following extreme cases:

- the pump speeds must be known extremely accurately (up to 5.3%);
- the pipe lengths do not have to be known accurately;
- the tank diameter does not have to be known at all.

From Δ*r* the uncertainty in the parameter, Δ*x,* is calculated from Eq. 3 and shown in Table 1. Requiring that the average pressure head at the internal nodes to be known by 5 m, implies that

- the pump speeds (up to 0.053) and demands (up to 3.6 ℓ/s for the 18 ℓ/s demand node 6) must be known extremely accurately;
- the pipe diameters (up to 95 mm) and reservoir depths (up to 5 m) are probably known to the required level of accuracy;
- the pipe lengths and roughness coefficients do not have to be known accurately;
- the tank diameter does not have to be known at all.

The analysis shows that the critical parameters to determine accurately are the pump speeds and the demands.

4.2 Village Model

Consider the model in Figure 1, based on data for a South African rural village. As in the idealised model, water is pumped from the reservoir by a pump. The water pushes up into the tank, and stabilizes at a certain level in the tank.

Some of the base-case parameters are as follows. Elevations above sea level vary in this hilly terrain and tend to decrease as the three pipe lines are followed away from the tank. The tank is at the highest elevation, and has a diameter of 10 m. The depth of the reservoir is 10 m. The pump produces a duty flow of 6.4325 ℓ/s at a duty head of 8 m (referred to as pump speed 1). The total demand from all the taps is 6.4325 ℓ/s. Pipe lengths vary: the total length is 7559 m. Pipe diameters vary in the range 50-90 mm, and almost all pipe Hazen-Williams roughness coefficients are 100. In the base-case solution obtained, the flow through the pipe equals 6.4325 ℓ/s.

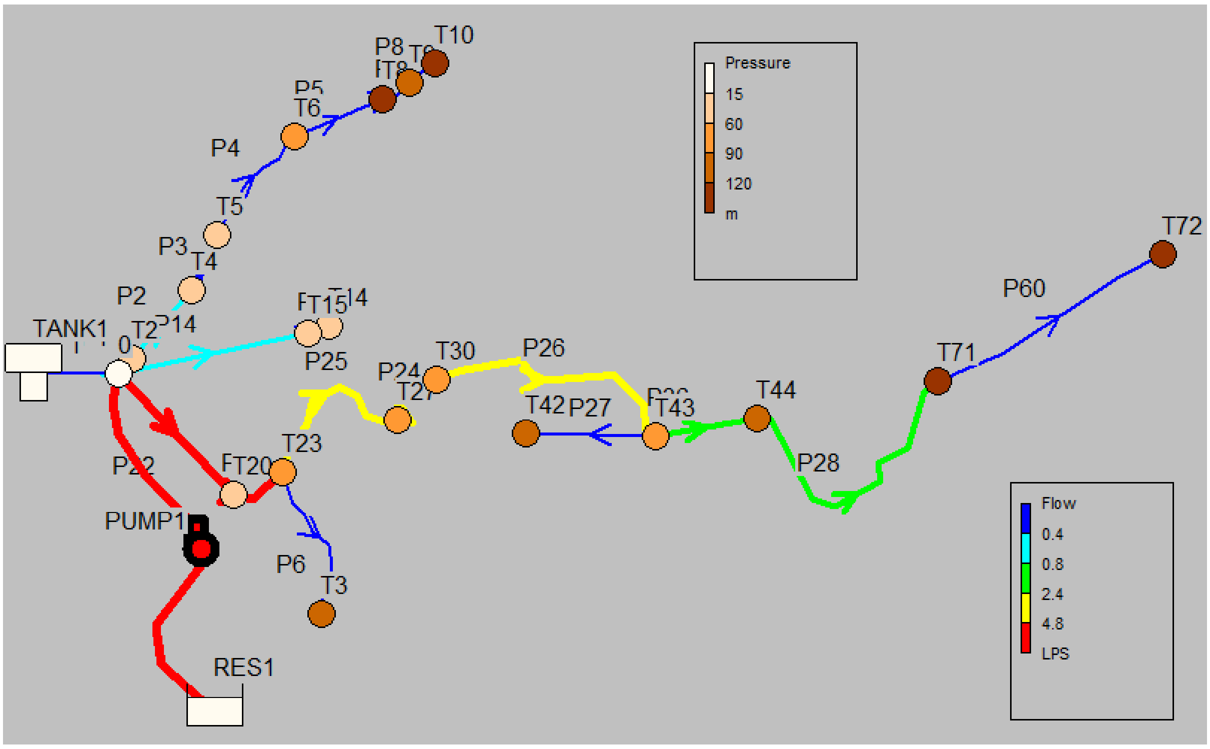

Figure 2 Village model. The taps are indicated by "T", and all have non-zero-demands. The pipes are indicated by "P". The tank level stabilizes at 3.0 m.

Assume all water demands rise (fall) from the base-case demands by 5.0%. From Table 2 it follows that the pump speed must be increased (decreased) by 8.0% to keep the average pressure of the demand nodes the same.

Table 2 Results for the village model.

	Smart optimisation			*Reliability*	
	Demand	Pump speed		Demand	Pump speed
D (m)	-34.1	21.2		-34.1	21.2
Δr	-	-		-0.15	0.24
Δx	-	-		Various	0.24
			Reliability		
	Pipe length	Pipe diameter	Pipe roughness	Reservoir depth	Tank diameter
D (m)	-15.4	72.4	27.9	10.0	0
Δr	-0.33	0.069	0.179	0.50	No bound
Δx	Various	Various	Various	5.0 m	No bound

Assuming that the average pressure head at the internal nodes varies by +5 m, the calculated values of Δ*r* are shown in Table 2*.* Requiring that the average pressure head at the internal nodes to be known by 5 m, implies the following extreme cases:

- the pipe diameters must be known extremely accurately (up to 6.9%);
- the demands must be known accurately (up to 15%);
- the reservoir depth and pipe lengths need to be known least accurately, and are probably known to the required level of accuracy;
- the tank diameter does not have to be known at all.

The analysis shows that the critical parameters to determine accurately are the pipe diameters and the demands.

4.3 Relationship between various Pipe Parameters

Assuming that there are no pipe minor friction losses in a network, we have been able to show for the Hazen-Williams pipe major friction loss formula that the friction, and hence all flows and pressures in the network, will be the same if (Page, 2015)

$$\text{H-W:} \qquad r_L = \frac{1}{r_D^{4.871}} = \frac{1}{r_C^{1.852}} \qquad \Delta r_L = -4.871\,\Delta r_D = -1.852\,\Delta r_C \quad (14)$$

Here three separate scaling possibilities are considered in Eq. 3, with *X* referring to either (1) all the pipe lengths in the network, (2) all the pipe diameters in the network, or (3) all the pipe Hazen-Williams roughness coefficients in the network. The scaling is denoted by r_L, r_D and r_C respectively. The second relationship in Eq. 14, which only holds if Δ*r* is very small, can then be used to obtain a relationship between the derivatives *D* in Eq. 8 for the three cases (noting that Δ*h* in Eq. 8 is assumed to be the same for the three scaling possibilities) [1]. In fact, if *D* is very accurately calculated in Eq. 8, and is derived from the *same* function *h* for the three classes of pipe parameters, then the values of Δ*r* calculated from Eq. 12 (examples of which are listed in Table 1 and Table 2) should exactly satisfy the second relationship in Eq. 14. It can be verified that the values of Δ*r* in Table 1 and Table 2 are well represented by this relationship.

The same considerations to be above for the Chezy-Manning pipe major friction loss formula lead to

$$\text{Chezy-Manning:} \qquad r_L = \frac{1}{r_D^{\frac{16}{3}}} = r_N^2 \qquad \Delta r_L = -\frac{16}{3}\,\Delta r_D = 2\,\Delta r_N \quad (15)$$

where *N* refers to the Manning roughness coefficient.

5. Findings and Discussion

In this paper research looking at water distribution networks from a novel angle is reported. The models demonstrate how much pump speeds need to be adjusted for the same variation of all the demands, in order to keep the average pressure the same. The various parameter categories have very different sensitivity to changes in the average pressure of the internal nodes. It is not currently known how the order of sensitivity

[1] The first relationship in Eq. 14 is consistent with the values of *D* in the idealized model in Table 1 for *r* = 1.01 where the evaluations of *D* were performed, up to numerical differences which have known explanations. The same is true of the village model in Table 2, except that here additional numerical differences are caused by the fact that different *P* satisfying Eq. 13 were used for the different pipe parameters.

varies from one network to another. The critical parameters to determine accurately are hence network dependent.

Both networks considered as examples are combined schemes (having operating pumps *and* demand balancing tanks). The idealised scheme is in a pump-network-tank configuration, while the village scheme is in a pump-tank-network configuration. Within the limits of the types of schemes analysed, the following salient features emerge about the derivative D:

- For variation in the reservoir depth, D equals the reservoir depth, independent of the network.
- D = 0 m for variation in tank diameter, independent of the network.

There is a relationship between D for the various pipe parameters that holds exactly. Such a relationship exists for both the Hazen-Williams and Chezy-Manning pipe major friction loss formulas. It holds not only for combined schemes, but for any network where there are no pipe minor friction losses. The uncertainties in the pipe parameters, Δr, corresponding to the *same* pressure and/or flow uncertainty, satisfies the second relationship in Eq. 14 for Hazen-Williams, and the second relationship in Eq. 15 for Chezy-Manning. Pipe diameters are the most sensitive, pipe roughness coefficients are medium sensitive, and pipe lengths are the least sensitive.

Further technical issues emanating from this research are:

- All parameters were taken to vary in the same direction in Eq. 3, which should be a reasonable condition when X refers to parameters of the same nature. Are there certain parameter sets of the same nature in certain networks for which this will not give reliable results?
- The case where g is the average pressure was employed in this paper (in Eq. 6). Which other choices of g have significant applications?

6. Conclusion and Further Research

The investigation originated from a desire to have a simple overview of state parameter sensitivity for the purpose of determining which parameters need to be determined more accurately during field work or from other sources of parameter data. Hence parameter uncertainty in water pipe network models is studied using newly developed simplified mathematical notions. These enable studies to be done using public domain software, including EPANET. The general idea is to study how a flow- and pressure-related quantity varies as a set of state parameters are varied. The quantity considered here is the average pressure. The sensitivity of parameters (i.e. which parameters need to be determined accurately) is studied. Smart optimisation of a water distribution system is also studied, using the same mathematical notions, by keeping the average pressure unchanged as water demands change, by changing the speed of the pumps. Two models are analysed with applications in the two areas.

The results in this paper are specifically relevant to quick smart optimisation and sensitivity studies using public domain software.

Further research could unearth how the sensitivity of various parameter sets depends on the network. It may also introduce emerging mathematical developments which lead to more robust conclusions than those presented here.

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