LITERATURE REVIEW OF WEB CRIPPLING BEHAVIOUR

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Abstract: A review of literature on the area of the behaviour of thin-walled cold-formed steel structural members was carried out with an emphasis on the phenomenon of web crippling of beam members. Web crippling is a common mode of failure experienced by web elements of thin-walled beams under concentrated loads or reactions. Most of the studies done on web crippling behaviour are experimental and based on compression testing of beams to determine the ultimate web crippling strength. It has been identified that the theoretical investigation of web crippling behaviour is rather complicated due to localised collapse behaviour. However, some attempts have been made to develop theoretically based models to predict web crippling behaviour, and to obtain better understanding of the failure modes. Different theoretical studies, especially on elastic and plastic behaviour of plate elements were investigated with the intention of developing an analytical model to describe web crippling behaviour. It was found that almost all of the design codes around the world make design recommendations to predict the load at which web crippling would occur, based on equations obtained from web crippling tests conducted by various researchers.

Keywords: Cold-formed steel, Concentrated loads, Collapse behaviour, Thin-walled, Web crippling.

1. INTRODUCTION

Cold-formed steel (CFS) structural members are widely used in modern construction industry due to their inherent characteristics over conventional hot-rolled thick sections. Cold-formed steel members are usually thin members with large width-to-thickness ratios. Light in weight, high strength and stiffness, accurate section dimensions, and easy of prefabrication and mass production are some of the qualities of these members that create cost savings in construction (Yu 2000). Cold-formed steel members are commonly used as purlins, cladding rails, sheeting rails, wall studs, floor joists, sheets and decks, etc. in the building industry. Unlike heavy hot-rolled steel sections, cold-formed thin-walled sections tend to buckle locally at stress levels lower than the yield strength of the material when they are subjected to various loading conditions. However, these members do not fail at these stress levels and continue to carry further loads leading to what is called post-buckling behaviour.

Flexural members such as floor joists, purlins, and decks are often subjected to concentrated loads and reaction forces. These concentrated forces result in different modes of failures depending on the loading condition in the absence of stiffeners. Basically these failure modes can be identified as shear failure, web crippling, bending failure, and failures resulting from the interaction of two or more of the above mentioned failure modes. Among these failure modes, web crippling is a significant failure mode that may be experienced by beam members under concentrated loads or reaction forces. Web crippling behaviour is identified as a localised failure of web elements just under the bearing loads (Figure 1).

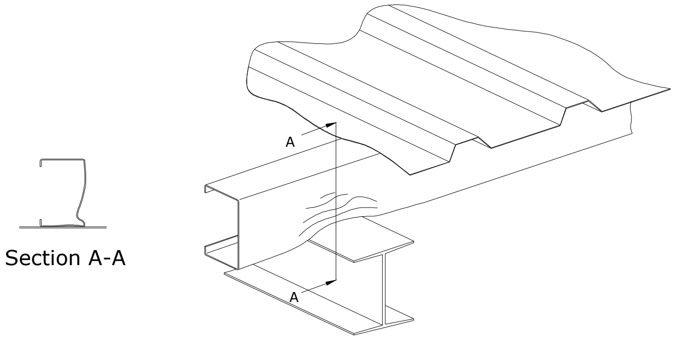

Figure 1: Web crippling at a support point (Rhodes 1994).

Web crippling may occur under various loading conditions; these loading conditions are defined based on the position of the load or reaction applied, and depending on whether the web is loaded through a single flange or both flanges. Based on the above definition, four basic loading conditions are defined in the AISI specification for a web crippling test, namely End-One-Flange (EOF) loading, Interior-One-Flange loading (IOF), End-Two-Flange loading (ETF), and Interior-Two-Flange loading (ITF) (Figure 2). Among these loading conditions Interior-One-Flange loading (IOF) condition occurs due to both concentrated loading and bending moment on the beam section that is considered.

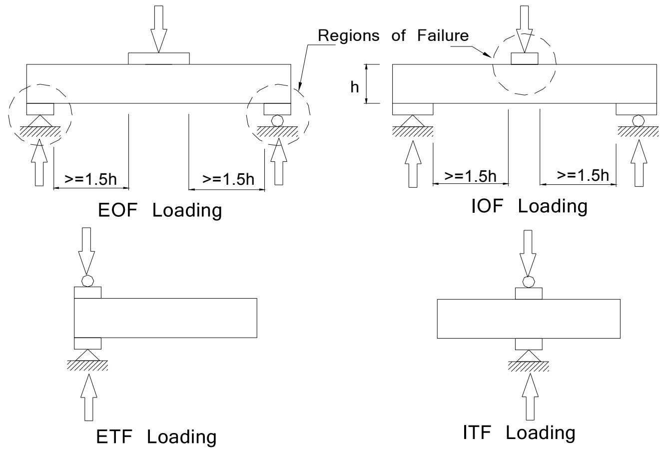

Figure 2: Loading conditions of web crippling tests (Yu 2000).

Almost all the design specifications around the world have design equations for predicting the web crippling strength based on empirical equations which were obtained by regression analysis of test results. These empirical equations are capable of predicting the web crippling strength for the range of parameters and the types of sections which were used in those tests to derive these equations, but often provide conservative or non-conservative results for other types of sections and section parameters.

The primary objective of this research is to investigate web crippling behaviour with the intention of improving the current design specifications for predicting web crippling strength. An experimental programme is intended to be carried out initially to get more understanding of the failure behaviour and study the influence of different parameters on the ultimate web crippling strength. The observed behaviour and the results will then be used to aid the development of reliable finite element models which are capable of predicting the ultimate web crippling strength. Finally, theoretical studies will be carried out based on the elastic and elastic-plastic behaviour of elements and members.

2. INVESTIGATION OF WEB CRIPPLING BEHAVIOUR

The study of web crippling behaviour of cold-formed steel flexural members has been going on since 1940s. Most of the research work that has been done on this area is based on experimental studies and the results have been used to develop the design formulae for calculation of web crippling strength. There have been some studies carried out on the theoretical analysis of web crippling behaviour despite the fact that it is extremely complicated because it involves the following factors (Yu 2000):

- Non-uniform stress distribution under the applied load.
- Elastic and inelastic behaviour of the web element.
- Local yielding in the immediate region of load application.
- Bending produced by eccentric load when it is applied on the bearing flanges at a distance beyond the curved transition of the web.
- Initial out-of-plane imperfection of plate elements.
- Various edge restraints provided by beam flanges and interaction between flanges and web elements.

2.1. Experimental Investigations

During the 1940s, the behaviour of web crippling was first examined experimentally by Winter and Pian at Cornell University in the United States. Since then there have been various experimental work done on web crippling behaviour on both single web sections, multi-web decks, and cassette sections to improve design codes and to validate various theoretical and numerical models developed by various researchers around the world. Experimental investigations provide the observations necessary to understand the failure behaviour. The present AISI design provisions for web crippling are based on the extensive experimental investigations conducted at Cornell University by Winter and Pian, and by Zetlin in the 1940s and 1950s, and at the University of Missouri-Rolla by Hetrakul and Yu in the1970s.

Winter used 136 test specimens with 18 different types of beams under all four possible loading conditions (IOF, ITF, EOF and ETF) for the experiments. The results of these tests were then evaluated and led to purely empirical formulae of a rather simple type which predicted, with reasonable accuracy, the web crippling strength for given dimensions. The test specimens were made out of channel sections by connecting channels in different arrangements using bolts and spot welding (Figure 3). The thicknesses of the sections varied approximately from 1mm to 3mm and the depth from

100mm to 200mm. Loads were applied over varying lengths of bearing ranging from 20mm to 90mm by means of steel plates. Span lengths were varied from 250mm to 900mm, but kept small enough to prevent failure by bending stress.

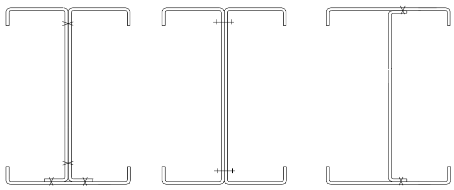

Figure 3: Typical cross section in Winter's experiment (Winter 1946).

Ultimate loads were recorded to analyse results and to obtain an equation for web crippling strength. Experiments indicated that the ultimate web crippling strength of I-section beams depends primarily on the actual bearing length ratio (N/t) and the yield strength of the material (σ_y). Further, within the test range, no influence of the depth of the beam on the crushing load of I-beams could be detected.
A general form of an equation that satisfied the test results was suggested:

$$P_{cr} = \left[A + B\left(\frac{N}{t}\right)^n \right] t^2 \sigma_y$$

Where, P_{cr} = ultimate web crippling strength, N = bearing length, t = thickness of the section, σ_y = yield strength, and A, B, n = empirical constants to be determined.

The interaction of web crippling and bending of C-shaped section steel joists was investigated by Ratliff in 1975. Ratliff proposed interaction formulae for web crippling and bending of C-shaped beams based on the results obtained from experimental investigations. In 1978, a similar experimental investigation was performed by Hetrakul and Yu at the University of Missouri-Rolla. In this research, the structural strength of cold-formed steel I-beams subjected to combined web crippling and bending was studied.

During 1982 to 1986, Wing and Schuster investigated a web crippling expression for multi-web deck sections with IOF, ITF and ETF loading conditions through experiments. The objective of the experiments was mainly aimed at determining the load resistance of multi-web deck sections under the aforementioned loading conditions. A test programme was conducted to provide experimental data to compare and evaluate the existing method of calculation of web crippling strength. More specifically, the study addressed the following important parameters: inside bend radius to web thickness ratio, bearing length to web thickness ratio and angle of web inclination. All of the specimens tested had unreinforced webs and the rate of the load application was uniform up to the failure load. Spreading of the web was prevented by bolting the lower flanges to the bottom bearing plate.

In 1986, Santaputra, Parks and Yu, investigated the web crippling strength of high strength steel beams with material yield strength up to 1310 MPa. High strength steels have been used for automotive structural components to achieve weight reduction while complying with safety standards. Because of this, additional design criteria for the use of a broader range of high strength steels were highly desirable. Tests were carried out to determine the web crippling strength of webs of cold-formed steel beams fabricated from high strength sheet steels commonly used in the automobile industry. Two types of sections were tested; hat sections were tested in order to study single unreinforced webs and I-sections to study the high degree of resistance against the rotation of the webs. Various loading arrangements were performed and new design criteria to prevent web crippling and combination of web crippling and bending were proposed based on the test results.

Another investigation was carried out by Studnicka in 1989, also aimed at predicting web crippling resistance of multi-web deck sections subjected to both one and two flange loading in both interior and end positions. The investigation was carried out experimentally and results were compared with the 1986 edition of the AISI specification and Canadian code. The test program was designed to encompass the most important parameter variations that influence the web crippling resistance of multi-web deck sections (Figure 4). For some specimens, spreading of the webs during loading was prevented by transverse tie rods which were bolted to the bottom flange of the profile. The test load was taken either as the largest load the specimen was able to sustain or the load which residual deformation of 1.0 mm developed, whichever was the lesser.

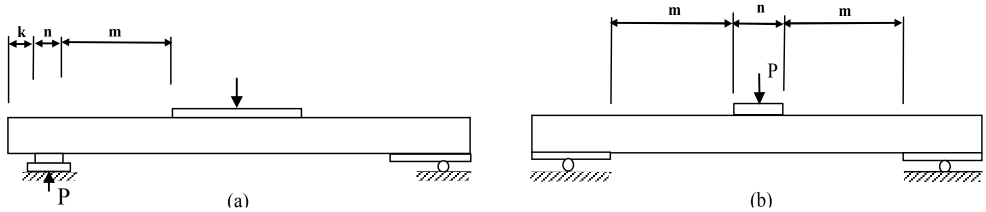

Figure 4: Test set-up for (a) end reaction (b) interior reaction.

40 specimens were tested for interior support condition and comments were made based on test results: test loads were not substantially different for normal orientation and inverted orientation of the deck sections, test results were almost linearly influenced by bearing width *n,* the test loads for specimens with ties were greater than without ties. The same comments were made for end support condition based on the test results. Further, two more comments were added to the end support loading condition: the influence of distance *m* on the test results is very small and when distance *k* is increased, the test load also increases, although the influence is not very strong.

In 1998, Young & Hancock carried out experiments to investigate the web crippling behaviour of cold-formed unlipped channels. Experiments were carried out under four loading conditions to the AISI specification. The concentrated load or reaction forces were applied by means of bearing plates which acted across the full flange width of the channel. Web crippling test results were compared with the AISI specification and the Australian/New Zealand standard for cold-formed steel structures. The design web crippling strength predictions given by the specification have been found to be unconservative for the unlipped channel sections tested.

A new design expression for web crippling strength of cold-formed flexural members was developed by Prabakaran & Schuster in 1998 based on the statistical analysis of experimental data published in various countries. All four loading conditions were considered for I-sections made of two channel sections back to back, Z-sections, channel sections and multiple web (deck) sections. Comparison was made with the expressions presented in the Canadian Standard and AISI specifications. The preliminary web crippling parameters considered were the web thickness (t), the web depth (d), the bearing length of the load (N), the inside bend radius (r), the yield strength (F_y) and the angle between the web and the bearing surface (θ). The new developed expression is nondimensional, therefore any consistent unit of measurement can be used such as empirical or SI. Certain unnecessary complexities existed were removed to simplify the web crippling expressions.

An experimental investigation of cold-formed steel stiffened C-and Z-sections subjected to web crippling was carried out by Beshara and Schuster in 2000. Two loading conditions were considered namely, ETF and ITF, with particular emphasis on large inside bend radius to thickness ratio R/t, (up to 12) and the specimens being fastened to the support during testing. They found that there was no experimental data available in the literature regarding the web crippling resistance of such members that are fastened to the support and having inside bent radius to thickness ratio greater than 2.7. Although most of the parameters of the test specimens were beyond the limits specified by the current AISI standards, the test results were compared with the calculated values of AISI web crippling design equations.

In 2003, an experimental investigation was carried out by Holesapple and LaBoube to find out the effects of overhang length on the web crippling capacity of clod-formed steel members. It was found that the current AISI design specifications for EOF web crippling capacity were conservative for overhang length ranging from 0.5*h* to 1.5*h*, where *h* is the web depth. A total of 27 specimens of channel and Z-sections were tested. All of the test specimens had an overhang or cantilevered extensions. A modified equation was obtained by analysing the test results for EOF loading condition.

2.2. Theoretical Investigations

The objective of the theoretical investigation of web crippling behaviour is to develop analytical models which can be used to determine the ultimate web crippling strength and to study the post-failure behaviour of a cold-formed section under different loading conditions. Although webs and flanges of the sections are interactive, it is also useful to study the behaviour of idealised separate rectangular flat plates loaded by localised in-plane edge forces. Elastic stability and plastic behaviour theories of plates are often used in analysis of thin-walled structures.

Elastic Behaviour of Plates and Sections:

Zetlin, in 1955, studied the behaviour of the rectangular plate which was simply supported its four edges and loaded on one edge of the plate. The energy method was used to analyse the plate and the formulated buckling load was given by $P_{cr}=K.\pi^2.D/L^2$, where, P_{cr} = critical elastic buckling load, K = buckling coefficient depending on the ratio of L/B and C/B, B = half-depth of the plate, C = half width of loading, D = flexural rigidity, L = width of the plate. In 1972, Khan and Walker also investigated similar problems of plate buckling as studied by Zetlin. They approximated the deflected shape of the plate by using a finite element solution and used them to solve the potential energy of the plate. The buckling load given by Khan and Walker was of the form $P_{cr}=K.\pi^2.D/2B$, where, B = half-depth of the plate, K = buckling coefficient depending on the ratio of L/B and C/B where L and C are length of the plate and the half-width of the plate respectively.

Plastic Behaviour of Plates and Sections:

The behaviour of plates under compression load was studied by Korol and Sherboune in 1972 using a plastic mechanism approach. According to their studies, the collapse loads of the plate can be obtained from the interaction of post-buckling loading paths and the rigid-plastic unloading lines. The elastic post-buckling path can be obtained by using the energy method while the rigid-plastic unloading or plastic mechanism can be obtained by considering the change in the plastic collapse load with geometric changes in the bent plate.

In 1980, Murray and Khoo studied the formation of local plastic mechanisms when thin-walled steel structures fail. Eight basic plastic mechanisms and three types of fully-plastic zones were observed in laboratory tests and characteristic equations were derived. Mahendran and Murray in 1991 and Mahendran in 1997 investigated the development of two types of local plastic mechanism in thin walled steel plates subjected to in-plane compression, namely the roof-shaped mechanism and the flip-disc mechanism. The type of mechanism initiated was identified from the location of first yield point shifted from the centre of the plate to the mid point of the longitudinal edge depending on the width-to-thickness (b/t) ratio, imperfection level, and yield stress of the steel. Both analysis and laboratory experiments were used to verify above fact.

Bakker in 1993 developed an analytical model to describe the web crippling behaviour and used test results to verify this model. The aim of developing this model was to obtain more reliable and more theoretical-based design formulae. From experimental research, two apparent failure mechanisms namely, yield arc mechanism and rolling mechanism were observed for hat sections under combined action of concentrated loading and bending moments. It was found that the yield arc mechanism occurs in members with small corner radii between web and loaded flange, and the rolling mechanism occurs in members with large corner radii. In this research, a model for members failing by the rolling mechanism was developed, and, according to this, the mechanism initiation load was determined as the point of intersection of an elastic curve and a rigid-plastic mechanism initiation curve. The elastic curve was taken simply as a straight line with a slope equal to the initial web crippling stiffness measured from a test. The rigid-plastic mechanism curve was derived by using generalised yield line theory.

In 1994, Setiyono developed a model to investigate the web crippling behaviour of plain channel sections under combined bending and concentrated loading. Ultimate web crippling strength was determined using an elastic loading curve based on the effective width approach and rigid-plastic unloading curve based on plastic mechanism approach.

Load capacity and post-failure behaviour of thin-walled beams were theoretically investigated by Kotelko in 2004. The problem of post-failure behaviour is solved by the rigid plastic theory taking into consideration strain hardening of the member's material. Based on the experimental results, theoretical models were developed to analyse plastic mechanisms of failure for different sections. Theoretical analysis was based on the principle of virtual work.

A new analytical model to predict the ultimate load of first-generation sheeting was developed by Hofmeyer in 2000. The model was based on two existing models, one was developed by Vaessen in 1995 on the elastic web crippling stiffness of thin-walled cold-formed steel sections and the other was based on the solution of Marguerre's simultaneous differential plate equations. Hat sections were considered for the analysis and testing instead of sheeting because they were easier to manufacture with varying dimensions. Three failure mechanisms were observed: rolling, yield arc, and yield mechanisms.

Design equations against web crippling for unlipped channels with stockier webs were proposed by Young & Hancock in 1998. It was assumed that the bearing load is applied eccentrically to the web due to the presence of the corner radii, which produces bending of the web of its plane causing a plastic mechanism, and then the plastic mechanism model was used to establish design formula. Proposed formula for channel sections were summarised as: $P_{pm} = M_p.N_m/r$, where $M_p = f_y.t^2/4$ (M_p is the plastic moment for unit length), P_{pm} is the web crippling strength predicted by the plastic mechanism model, r is the centreline corner radii, t is the thickness of the web, f_y is the yield stress, and N_m is the assumed mechanism length. The above equations for predicting web crippling strength were modified by the same researches to take the effect of the slenderness ratio into account, and the term $(1.44-0.0133(h/t))$ was introduced to the first equation by multiplying with this factor.

2.3. NUMERICAL INVESTIGATIONS

The objective of the numerical investigation is to develop computational methods to analyse the web crippling behaviour of thin-walled flexural members. Finite Element analysis (FEA) is considered as the most commonly used numerical method. FEA can be used to represent complex geometries and can be effective in treating difficult boundary conditions. Further, non-linear finite element techniques enable the inclusion of large deformation, large rotation and non-linear stress-strain characteristics (Sivakumaran 1989). FE modes are useful because they can partly substitute for expensive experiments for systematic variations of variables, they can be used to test a wide range of assumptions, and they can simulate experiments that are practically impossible to perform but worthy of studying (Hofmeyer 2000).

In 1989, Sivakumaran carried out a finite element analysis to analyse web crippling behaviour and to determine ultimate web crippling strength. He was able to analyse the web crippling behaviour extending up to ultimate load levels. The following points were highlighted when selecting the proper finite elements in his investigation: steel lipped channel sections were considered with very thin webs, flanges and lips, and were subjected to in-plane and bending action. Hence finite elements were able to represent membrane behaviour as well as the flexural behaviour. In this study, the finite element package called ADINA was used. Among the finite elements available in the ADINA library, degenerated isoparametric shell elements were considered for analysing this problem. The isoparametric shell element is a higher order degenerated isoparametric element with 16-nodes (5 degree of freedom per node). Half of the specimen was considered for the mesh generation because of the symmetry of the loading. In order to capture the local large deflection and local yielding in the region around the web crippling point, fine meshing was considered in this region. Further away from the loaded area, large elements were used. Results of the finite element analysis showed that the general deformation shape similar to experimental deflected shape and predicted ultimate loads which were well in agreement with the experimental values within a 9% tolerance.

3. CONCLUSIONS

A literature review was carried out with the intention of studying the various investigation techniques such as experimental, theoretical and numerical, for web crippling behaviour. It was found that most of the experiments were focused on testing flexural members against web crippling or combined bending and web crippling to determine the ultimate web crippling strength for various sections and parameters. However, these experiments were not capable of giving any observations or results for developing more reliable theoretically based design equations for predicting web crippling behaviour.

It is understood that there are still no totally reliable equations or models that are capable of predicting the ultimate web crippling strength. Almost all of the design specifications found predict web crippling strength based on equations obtained by empirical results. From the literature survey, it was identified that it may be possible to derive equations based on more experiments combined with accurate finite element modelling, giving more of an insight into web crippling behaviour for many different beam cross-sections. The next step of this research will be the design of an experimental program which is capable of giving the observations and results that can be further used for the development of finite element models, and analytical investigations.

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