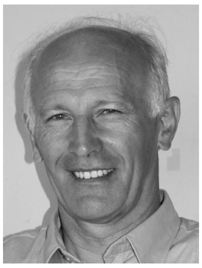
In Search of the Roundest Soccer Ball



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KEYWORDS

Soccer balls, polyhedra, isodistance, inflatables.

1 The present ball form is not ideal

Nowadays soccer balls usually have an outer skin of synthetic leather (a polyurethane coating on polyester fabric) and a flexible inner bladder which after inflation gives the ball its final spherical shape. The skin is generally composed of 12 equilateral pentagons and 20 hexagons, sewn together according a special pattern. This principle is used by many firms.

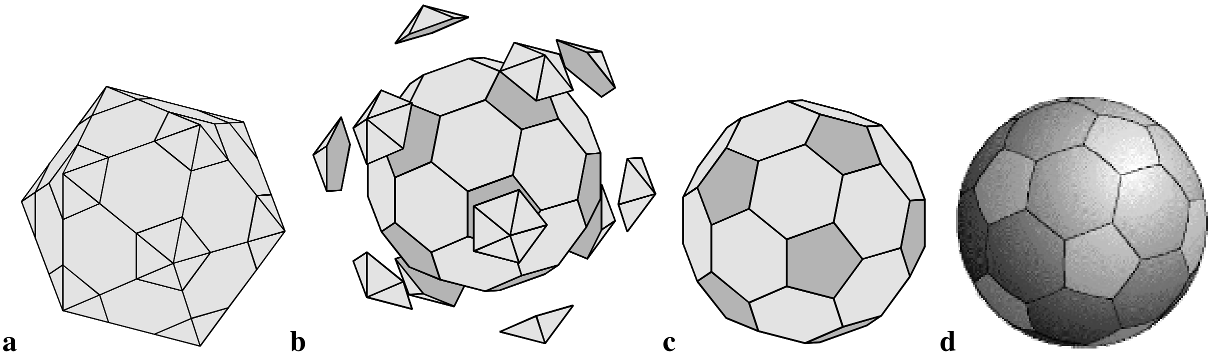


Figure 1. Subdivision pattern based on the Truncated Icosahedron.

The components are cut from a flat sheet of material and as long as they are flat, they form the faces of a mathematical solid, known as the Truncated Icosahedron. This shape is obtained by cutting off ('truncating') small pyramidal caps at the twelve vertices of the regular *Icosahedron*, that itself is composed of twenty regular triangles.

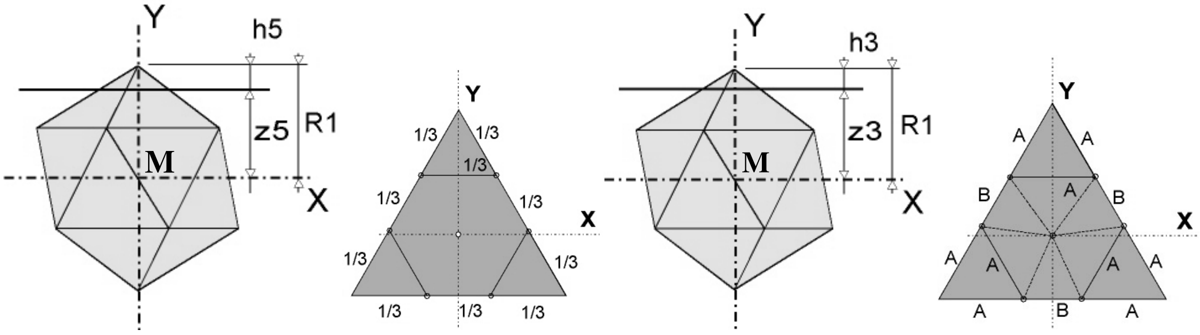


Figure 2. The principle of truncation of an Icosahedron at the distance z_5 and z_3, were the latter is similar to the distance of the triangles from the polyhedron centre M.

All vertices of this new form have the same distance from the centre M of the solid. This distance, R1 in Fig. 2, is called the *radius* of the circum-scribed sphere. The ball must receive its final round form by the inflation of an elastic internal bladder. In order to obtain the FIFA specification “Approved”, which is the highest quality standard, the ball must have a radius of 110 mm with a maximum deviation in the total sphericity of 1.5%. However, the mid-points of the faces lie in their flat state substantially under this radius at distances varying from 6.7 mm or 6.1% (pentagons) to 9.4 mm or 8.5% (hexagons). The inner air pressure and the elasticity of the skin material must equalise that difference, but this is not automatically guaranteed. It often results in inaccuracies in the form and in an uneven stress distribution in the material. It is therefore difficult to meet the official requirements. The behaviour in flight after a kick or in subtle ball handlings may also become less predictable than desired. This is aggravated by the fact that the hexagon is 1.5 times (50%) larger in area than the pentagon, so that they will react differently.

There are four possible approaches for the solution of this problem:

1. Choose a basic form, that has a *surface area* that is closer to that of the circumscribed sphere.
2. Choose a basic form, which has a *volume* that is closer to that of this sphere.
3. Change the form of the original basic mathematical figure, so that all faces get the *same distance* from the centre (become *isodistant*) and that all have the same surface area.
4. Find a form, where the faces have the *largest possible distance* from the centre: less different from the radius.

2 The *Isodistant* version of the Truncated Icosahedron

The truncation is normally done at one third of the Icosahedron faces. This reduces the triangles to equilateral hexagons. The cutting planes are regular pentagons and these are in their original flat state considerably farther away from the centre of the ball than the hexagons (103.3 mm vs 100.6 mm) and all are at a quite great distance from the circumscribed sphere. It is nevertheless possible to get the pentagons and the hexagonal faces at the same distance from the centre, if the truncation is done at the same distance from the solid centre as that of the triangles. In this case the hexagons will get a slightly irregular form and have two different side lengths A and B (see Fig. 3c). All panels will then have a distance from the centre of 101.5 mm (where R1 = 110mm). The ratio of the sides: $f_1 = B : A = \sin 24^o : \sin 36^o = 0.69198171$. The pentagons are slightly larger than in the standard ball. This can be seen from Fig. 3.

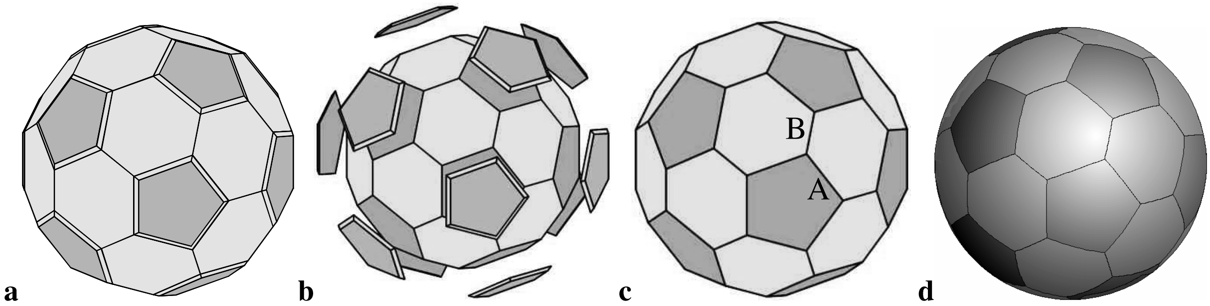


Figure 3. Derivation of the Isodistant version of the Truncated Icosahedron

A few years ago the firm NIKE introduced a new ball under the mark name *Geodesign* [Ref. 1]. They claim that this is isodistant, but the ratio of the sides of the hexagons, that was adopted in this case, is 0.84 and therefore the distances of the faces from the centre are not all alike: 102.4 mm of the pentagons and 101.1 mm of the hexagons (Fig. 4). Even if this truncation is done properly, the Isodistant Truncated Icosahedron still has the disadvantage that the face centres are relatively far away from the sphere surface (8.5 mm), so that the faces have to be stretched considerably in order to become sphere caps.

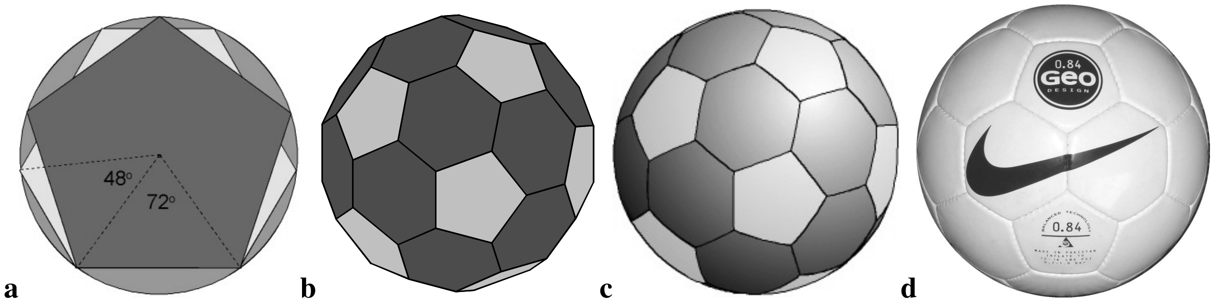


Figure 4. a) The faces of the Isodistant Truncated Icosahedron in their circumscribed circle (B/A = 0.69). b,c,d) The Geodesign ball of NIKE with B/A = 0.84.

3 More faces give a better approximation of the sphere!

Within the group of polyhedrons with regular faces, the so-called Archimedean solids, the Snub Dodecahedron is the one with the greatest number of faces. A Dodecahedron itself is a regular figure, that consists of 12 pentagons. “Snubbing” means in a mathematical sense that all corners and connection lines of this Dodecahedron are chamfered. The snub version consists of 12 pentagons and 80 triangles, 92 equilateral faces in total [Ref. 3]. The fact that it has many more faces than the Truncated Icosahedron (32), shows already that it is closer to the circumscribed sphere. In order to reduce the number of composing parts, for production reasons four triangles at a time can be combined into one triangle of the double side length, so that the final form can be composed of 12 pentagons and 20 large triangles: again 32 in total. G. Obermann applied for a German patent on this idea [Ref. 2].

4 The Isodistant Snub Dodecahedron

Although this Snub Dodecahedron approximates the circumscribed sphere more closely than the Truncated Icosahedron, it implicates great difference in the central distances of the triangles (106.0 mm) and of the pentagons (101.1mm). But it is again possible to develop an isodistant version. In this case A : B : C = sin 84^o : sin 36^o : sin 30^o (see Figs. 5c and 5d). The ratio between the side length of the pentagon and that of the large triangle is normally 0.5 but, if for this ratio the factor f_2 = A : (A+B) = 0.37147355 is chosen, a situation is created where the pentagons and all small triangular parts are at exactly the same distance from the centre (all at 104.2 mm).

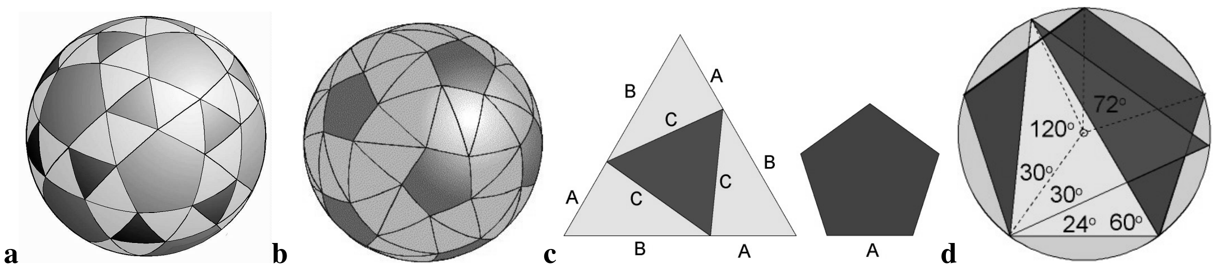


Figure 5. The inflated versions of the Archimedean and the isodistant Snub Dodecahedron (all faces fit in one circumscribed circle)

It consists of 12 regular pentagons and 20 larger, regular triangles. A few prototypes of the Isodistant Snub Dodecahedron have been made (Ref. 4). Fig.6b shows one of these. This form has however two drawbacks: the stitching length is larger than that of the standard ball (4596 mm in stead of 3995 mm) and the meeting of the sharp (60^o) angle of the triangles against the straight side of its adjacent triangle appeared a bit difficult to make. It also has two different orientations, right-handed and left-handed, which fact might influence the behaviour of the ball. This version was tested but disapproved in the end.

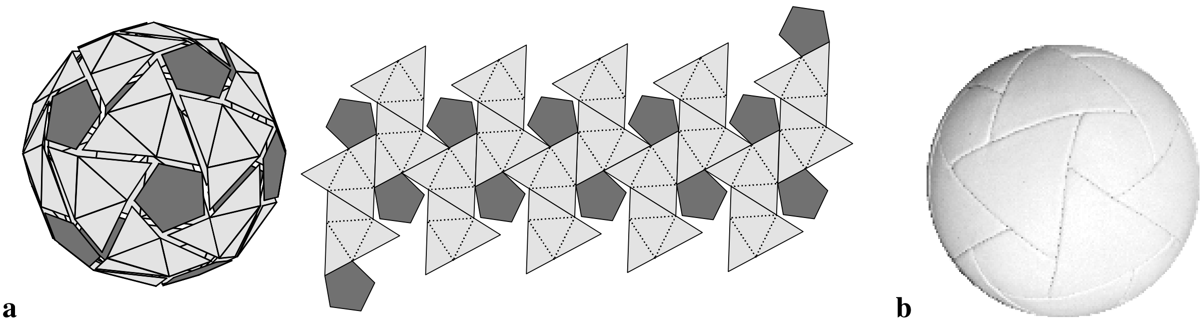


Figure 6. Layout and prototype of the 'Isodistant Snub Dodecahedron'

5 The Isodistant Rhombicosidodecahedron: a simplifation!

A better solution is found, if the Isodistant Truncated Icosahedron of Figs. 3c and 7b is truncated once more at the same distance from the centre as the other faces, but this time parallel to the edges. A figure is found which can be called: Isodistant Bistruncated Icosahedron (Fig. 7e). It can also be considered as an isodistant version of the Rhombicosidocahedron, which is an Archimedean solid, consisting of 12 pentagons, 20 triangles and 30 squares, all regular (see Fig. 7a).

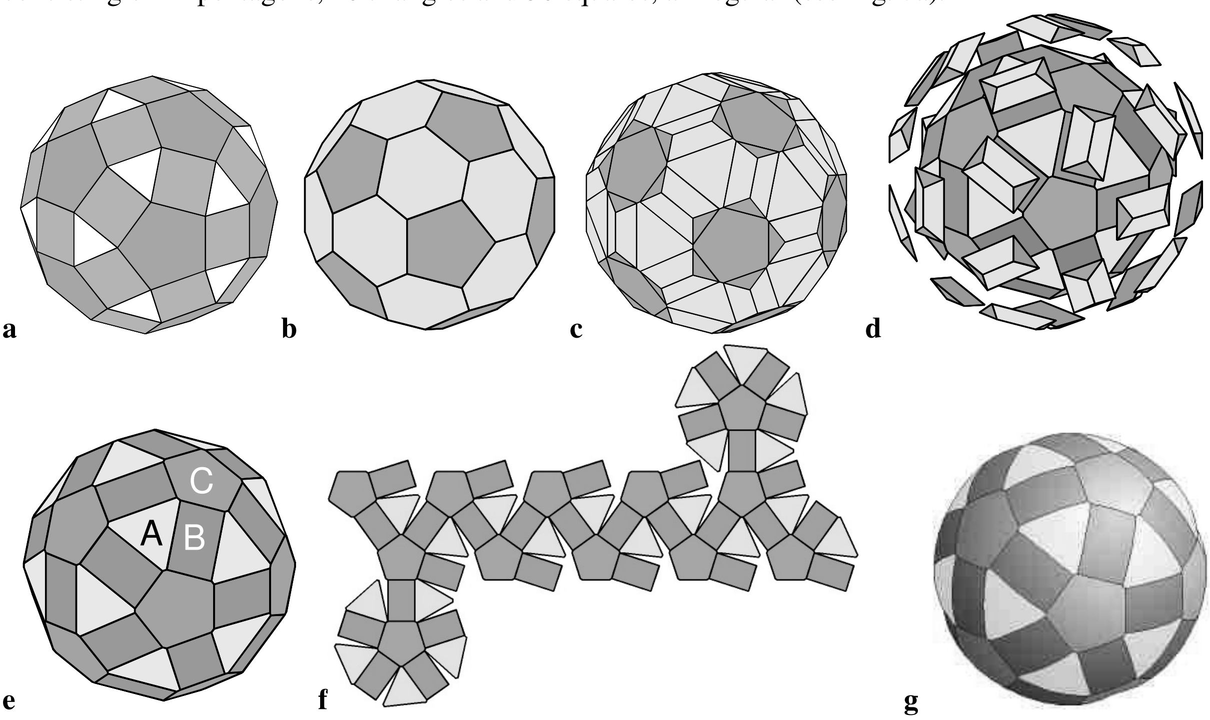


Figure 7. The Isodistant Rhombicosidodecahedron, composed of 3 kinds of elements

The new solid has 12 pentagons, chamfered at their corners (distorted decagons so to say), 20 chamfered triangles (or hexagons) and 30 rectangles (see Fig. 8). This figure has quite similar characteristics to those of the previously described ball type. Its geometric properties are also given in the table 1, where it can be compared with the other ball concepts. All its 62 faces are at the same distance from the ball centre, which is virtually the same as that of the previous ball type, 104.2 mm, and the centres of these faces thus are 5.8 mm distant from the sphere surface. It approximates the sphere more closely than the other balls in the table. It can be calculated, that the ratios of the sides are equal to C : D : E = sin 57^o : sin 36^o : sin 3^o (Fig. 8). Although this ball type is not difficult to make, because of the chamfered vertices (E) of the hexagons and pentagons, it still has the disadvantage, that the total stitching length is comparably long: 5814 mm versus 3995 mm of the standard ball.

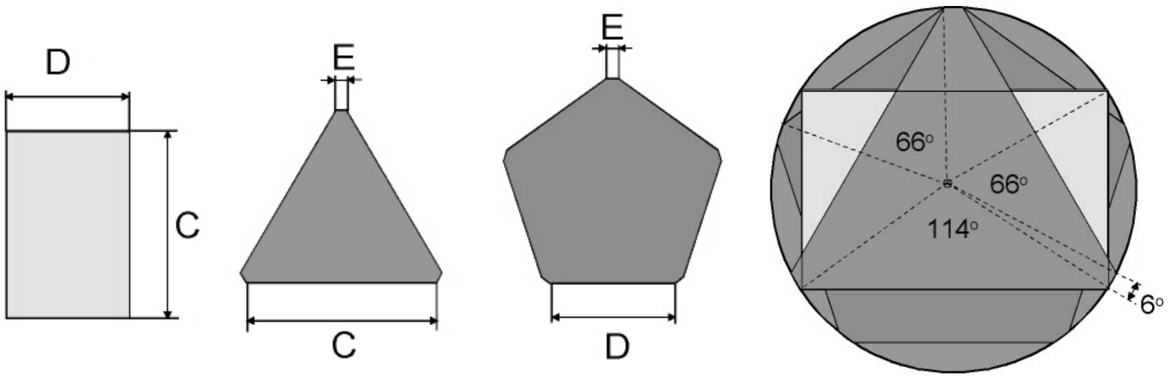
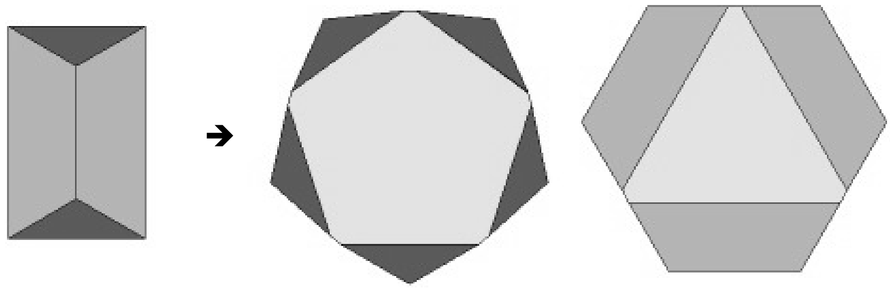


Figure 8. Relative dimensions of the three different faces

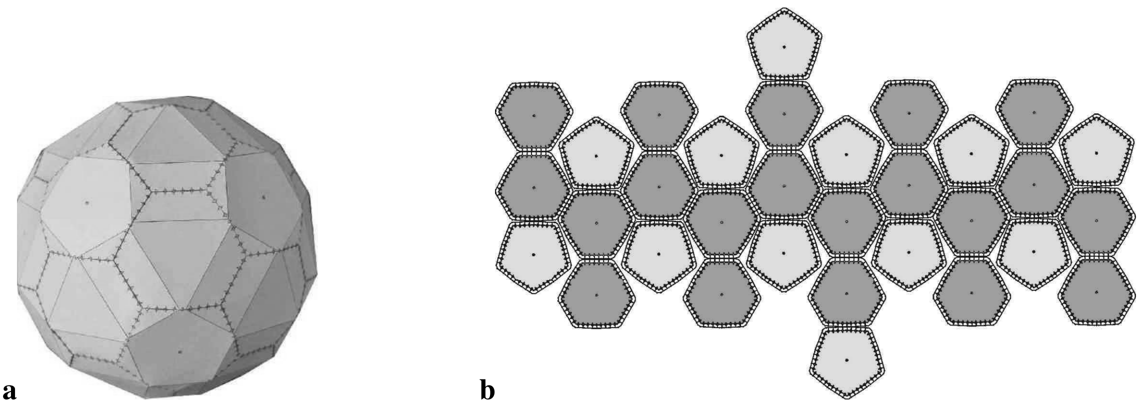
This can be overcome by cutting the rectangles appropriately in four parts: into two isosceles triangles that have the short sides of the rectangle as their bases and into two trapezoids with the longest sides as their bases (Fig. 9). If five of the triangles are coupled with their basis to every pentagon and three trapezoids with their longest side to the hexagons, then 12 larger ‘pentagons’ are obtained with slightly bent sides and 20 ‘hexagons’ with alternate bent and straight sides (ref. 5).



‘hexagon’ **‘pentagon’**

Figure 9. The new types of faces, formed by the combination of parts from the rectangle with the original chamfered triangles and pentagons.

Thus a construction method is found, which is quite well comparable to that of the standard ball, but now with a number of apparent advantages. As already indicated in the foregoing, all original 62 face parts are at the same distance from the centre and they too are more close to the circumscribed sphere. But the new ‘pentagons’ and ‘hexagons’ are also mutually almost identical: in the flat situation they both have the same inscribed and circumscribed circle (see Fig. 11d). Furthermore their surface areas are very similar: the ratio hexagon/pentagon is 1.05 (in stead of 1.51 in the case of the standard ball). The difference now is only 5.2%. The circumference of the two panel types is almost identical: only 3.7 mm difference. All faces can have the same number of stitches (see the paper model in Fig. 10a).



a **b**

Figure 10. Paper model and layout of new construction.

This implies that both panel types in this ball by the inflation of the bladder are stretched to the same amount and that the stress distribution in the material is more uniform then in all other cases. The required deformation to become spherical is identical and smaller than in other balls. When any of these faces are kicked, they will feel and act similarly, so that the behaviour of the ball will therefore be more predictable. The meeting of three faces in the vertices is flat (360^{o}), which means that the ball will presumably feel softer when headed than the standard ball, where this meeting has the form of a shallow pyramid. This might reduce the danger of head injuries. As the sides of most panels are slightly bent, the seams will follow the curvature of the ball. The total seam length is similar to that of the standard ball.

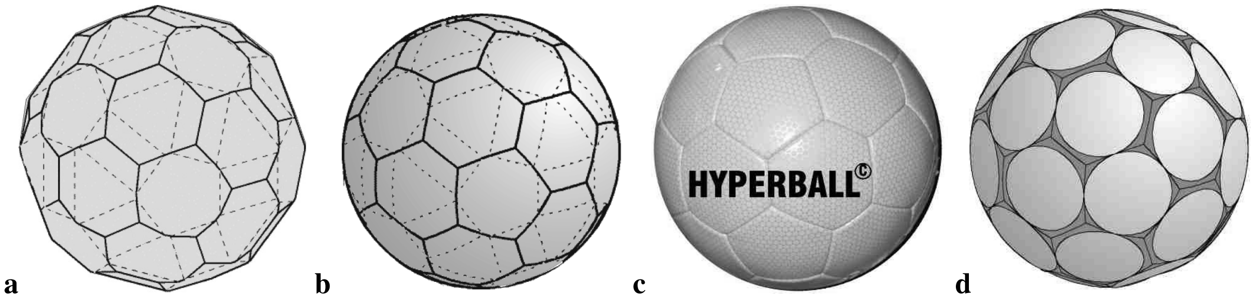


**Figure 11. The new construction method with 12 ‘pentagons’ and 20 ‘hexagons’:
a) The panels in the flat state, b) The inflated situation, c) prototype, called Hyperball,
d) A printing pattern of 32 equal circles in densest packing (inscribed circles of panels)**

Possibilities for the production of this new ball are being studied. The relevant data of the various ball types, discussed in this context, are given in the accompanying table 1 in Fig 13.

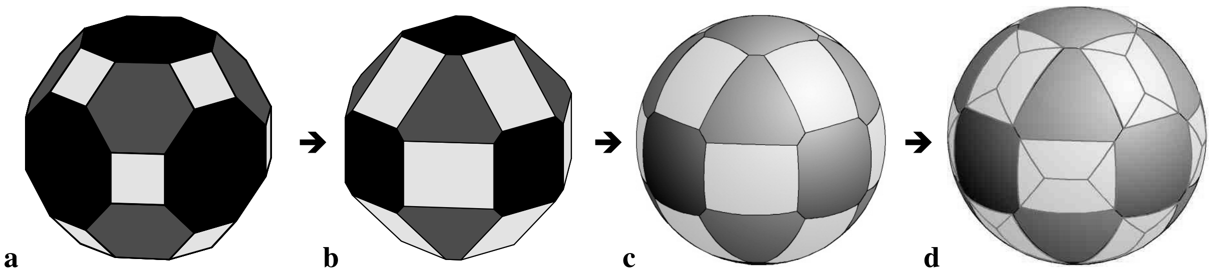


Figure 12. A similar principle can be followed for the Truncated Coboctahedron, consisting of 6 octagons, 8 hexagons and 12 squares. This number of 26 panels can be reduced to 14 by the redistribution of the rectangles (blue lines in d).

6 References

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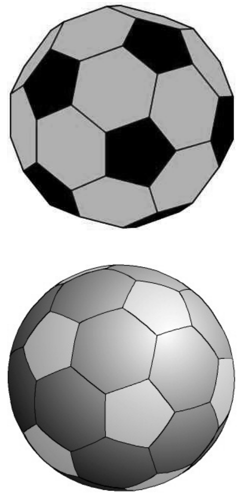
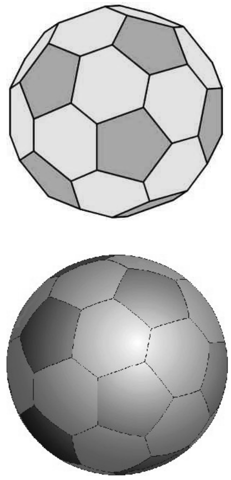
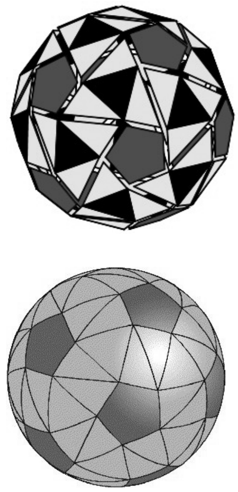
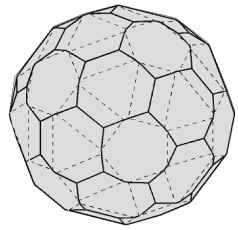
Table 1	**1**	**2**	**3**	**4**
				
	Truncated Icosahedron	**1st Isodistant truncation of Icosahedron**	**Isodistant Snub Dodecahedron**	**2nd Isodistant truncation of Icosahedron**
Distances of faces to centre in mm	5) 103.3 6) 100.6 Ave) 101.6	All) 101.5	All) 104.2	All) 104.2
Area of polygons in mm^2	5) 3390 6) 5119	5) 4281 6) 4576	3.1) 1619 3.2) 1262 5) 2963	5) 4369 6) 4595
Ratio Max./Min. Area	1.510	1.069	2.348	1.052
Difference Max./Min. Area	51%	6.9%	134.8%	5.2%
Total Area in mm^2	143072	142887	143636	144338
Total Volume in mm^3	4836076	4833688	4988076	5012655
$Volume_{tot}$/ $Volume_{theor.}$	0.867	0.867	0.896	0.899
Circumference of polygons in mm	5) 221.9 6) 266.3	5) 249.4 6) 253.2	3*(3.1+3.2) =1068.9 5) 351.1	5) 247.6 6) 251.3
Total seam length in mm	3995	4029	4596	3998
Difference of circumference in mm	44.4	3.8	717.8	3.7
Inscribed circles in mm	5) 30.5490 6) 38.4431	5) 34.3288 6) 38.7643	5) 8.5529 3 Large) 96.7474	All) 35.2419

Theoretical values:

$$Radius = 110mm$$

$$Area_{theor.} = 4\pi R^2 = 152053mm^2$$

$$Volume_{theor.} = \frac{4}{3}\pi R^3 = 5575280mm^3$$

Figure 13. Comparison of 4 ball concepts all transformed to a radius of 110 mm, both in flat and in inflated situation