Fuzzy Lifetime Prediction Of RC Structures Subject To Chlorides

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ABSTRACT

Pitting corrosion of the steel bars is a process that strongly jeopardizes the durability of reinforced concrete structures exposed to chlorides. This is a problem widely studied in order to evaluate structural lifetime. Nevertheless, the proper treatment of the noteworthy sources of uncertainty, that affect the numerical values of geometrical and mechanical structural parameters, is still an open question. Such properties, in fact, cannot be considered as deterministic quantities. In many real problems, very few uncertain data are available and new non-probabilistic procedures need to be defined to perform lifetime estimation. In the present study, parameters are modelled using fuzzy set theory and a time-dependent fuzzy safety factor is defined in order to indicate how the critical chloride front is distant from the bars. Moreover, the study provides a more proper mathematical analysis of chloride penetration into concrete and an improved calibrating procedure to estimate sampling model parameters, also accounting of time variability. The analysis confirms that the application of oversimplified Fick’s solutions leads to substantial conceptual errors in service lifetime estimation.

KEYWORDS

Chloride penetration model, Fuzzy Safety factor, Genetic Algorithms, Pitting corrosion, Service Life prediction.

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1 INTRODUCTION

Economic consequences due to deterioration of reinforced concrete (RC) structural elements are one of the most pressing problems, especially in industrialized nations where maintenance and repair of these types of construction are an impelling necessity. In most cases, it has been established that reinforcement corrosion is the foremost cause of deterioration [Apostolopoulos and Koutsoukos, 2007]. The corrosion process of reinforcing bars is a well known phenomenon as it is also known that the consequences of a pitting corrosion process on structural safety are more serious than that due to general corrosion. For this reason, lifetime estimation of RC subject to chloride exposure deserves particular design attention. The main difficulties that researchers face in the investigation of these problems are due to the noteworthy lack of precise information. For this reason, the service life design methodology should take into account uncertainties. In last years, probabilistic approaches have been used with the aim of conducting a time-dependent analysis of RC structures subject to corrosion under conditions of uncertainty, but it should be observed that not all the uncertainty variables can be treated following this approach. Probabilistic description is not the most adequate way to describe uncertainty for a great part of realistic conditions and sometimes its use may be inadequate for some real problems. This is particularly true when only few data about one or more models and structural parameters are available or when investigating mechanical and geometrical system properties, as well as load conditions, are economically inconvenient or practically impossible. This happens, for example, in historical constructions. In the field of non-probabilistic approaches, different methodologies have been proposed: particularly, the approach based on the concept of fuzzy sets [Zadeh 1965] also for estimating service life of RC structures [Anoop et al., 2002], seems appropriate because it is able to take in account the so-called lexical and informal uncertainties that affect both data and models. In this paper, a fuzzy service lifetime estimation of RC elements subject to chloride ingress is performed considering uncertainty on problem parameters that are modelled by using a fuzzy criterion. It should be noted that the concept of corrosion initiation time is done to coincide with the general service lifetime of the structure which ends when pitting corrosion process starts. Preliminary to this kind of investigation, it has been necessary to examine the state-of-the-art in order to recognize the potential typologies of uncertainties, in chlorides penetration process. The analysis of the bibliography on this theme, have confirmed the high uncertainty in model parameters definition. This substantially depends on the intrinsic variability of some parameters, on the insufficient or inadequate availability of data, but also on the high number of oversimplifications and neglected factors that during the years have been applied on chloride penetration model. Today, a great number of mathematical models have been developed and the application of each of them leads to different values of the main parameters. It is thus clear that sampling parameters are estimated with different confidence level of knowledge. In detail, some geometrical and mechanical parameters are well defined as deterministic or at least probabilistic, because a wide number of experimental data can be available. Other parameters instead are not well defined and therefore affected by different level of uncertainty, because for example few experimental data are available or because the assumed physical model is empirically extrapolated or is oversimplified and therefore less representative of the real process. For these reasons, all the parameters are treated under non-probabilistic assumptions. In this case, the fuzzy approach represents a more suited theory to perform other kind of analysis, properly taking into account of uncertainties. Therefore, another important preliminary point of investigation of the present work, concerns the chosen of the best deterministic sampling parameters by using a more proper calibrating procedure and then use these variable for the fuzzy treatment. With this aim, a more accurate solution of Fick’s law have to incorporate the time-dependence of the diffusion coefficient and some researchers have also integrated the diffusion law taking into account time-dependent surface concentration [Mejlbro, 1996]. Two of these improved models [Mangat and Molloy, 1994; Mejlbro, 1996] have been chosen by the authors to be tested with experimental data by mean of a genetic algorithm (GA), in order to obtain the deterministic calibration parameters of model. In the second section of the paper, some of these variables are modelled as fuzzy ones and finally, time-dependent fuzzy safety factor is obtained in order to assess the service life of RC elements in terms of...
the critical chloride front closeness to the bars during the time. This tool may be useful in order to support the decision maker (DM) in the planning of the maintenance interventions.

2 CHLORIDE PENETRATION MODELS

2.1 Conventional Model

A simplified model for the chloride penetration procedure is based on the assumption that the chlorides diffusion process can be modelled by means of the Fick’s second law, as firstly realised by Collepardi et al. [1972]. Having only results available from short time experiments, and probably also due to mathematical difficulties, they only considered the case of constant diffusion coefficient and constant surface chloride concentration. This is a diffused supposition (i.e. it is adopted by Stewart and Rosowsky [1998]). According to this law, the chloride ion content is given by:

\[
C(x,t) = C_s \left[ 1 - \text{erf} \left( \frac{x}{2 \sqrt{D_a t}} \right) \right]
\]

where \(C(x,t)\) is the chloride content (in \(\text{kg/m}^3\)) at a distance \(x\) (in \(\text{m}\)) in a generic time instant \(t\) (in \(\text{s}\)); \(C_s\) is the surface chloride concentration (in \(\text{kg/m}^3\)); \(D_a\) is the apparent diffusion coefficient (in \(\text{m}^2/\text{s}\)). Using eq. Hata! Başvuru kaynağı bulunamadı., it is possible to define the corrosion initiation time with reference to an assigned critical threshold chloride concentration on steel bars (denoted as \(C_{\text{lim}}\)). The chloride diffusion coefficient \(D_a\) is assumed a constant value even if it is time dependent. It will typically decrease as time passes since the capillary pore system will be altered as hydration products continue to form [Mangat and Molloy, 1994]. This meant that the diffusion coefficient could be written as a power function [Tang, 1996]:

\[
D(t) = D_{\text{ref}} \left( \frac{t_{\text{ref}}}{t} \right)^m = K_0 t^m
\]

where \(D(t)\) is the diffusion coefficient at time \(t\); \(D_{\text{ref}}\), the diffusion coefficient at some reference time \(t_{\text{ref}}\) and \(m\) is a constant that depends on mix proportions. \(K_0\) incorporates all constants and is defined as the effective diffusion coefficient at time \(t\) of reference. The application of equation (1) is very common in literature and also in different documents that provide guidelines for the durability design of concrete structures, as for example in documents developed by the EU DuraCrete project [1998]. The DuraCrete Model is an empirical model, which also has Fick’s second law of diffusion as a theoretical background that has the following form:

\[
C(x,t) = C_s \left[ 1 - \text{erf} \left( \frac{x}{2 \sqrt{k_c k_s D_{\text{ref}} \left( \frac{t_{\text{ref}}}{t} \right)^m t}} \right) \right]
\]

It is clear that the presence of the two multiplication factors \(k_c\) and \(k_s\), that are two constants introduced in order to take into account the probabilistic nature of the environmental aggressiveness (\(k_{\text{e}}\) – environmental factor) and the material properties (\(k_{\text{c}}\) – curing factor), does not change the mathematical characteristic of the chloride profile. Indeed, the DuraCrete model has been developed on the simplification that the diffusion coefficient is a constant value and doesn’t vary with time.
Nevertheless, in order to take into account also time-dependence effects, it has been suggested to replace the constant value of $D_a$ that appears in equation (1) with the relationship (2).

### 2.2 Evolutionary Models With Time-Dependent Parameters

Already in the 1980’s it was realised that the standard oversimplified model, on which is based equation (1), gave unrealistic rapid transport of chloride ions, especially when long term predictions are based on results from short term tests [Cairns and Law, 2003]. For the accurate prediction of chloride-diffusion in concrete, the time-dependence of diffusion rates needs to be incorporated in the analysis procedure, properly deriving the Fick’s second law. This model was developed by [Mangat and Molloy, 1994], leading to the following equation:

$$C(x,t) = C_s \left[1 - \text{erf} \left( \frac{x}{2 \sqrt{K_0 t^{1-m}}} \right) \right]$$

(4)

Other solutions of Fick’s second law exist with varying surface concentration. Some researchers observed that the chloride concentration of the exposed concrete surface of a marine structure was time-dependent. Mejlbro [1996] proposes that the chloride penetration model ought to be described by the following function:

$$C(x,t) = \left( \frac{T}{T_0} \right)^p \psi_p(z)$$

(5)

where $T$ is the transformed time variable $T = (t - t_{ref}) D_a(t)$ for $t \geq t_{ref}$, with $D_a(t)$, that is equivalent to $D(t)$ as defined in equation (2), and $T_0$ is introduced in order to to make the ratio dimensionless. The exponent $p$ depends on how fast $C_s$ increases with time, i.e. mainly on the type of binder and the environment and the function $\psi_p(z)$, where $z = x / \sqrt{4K_0 T}$, satisfy an ordinary differential equation of second order which can be solved by means of a series expansion.

### 3 ANALYSIS OF THE BEST DETERMINISTIC MODEL

Modelling the chloride ingress is an important basis for designing reinforced concrete structures and for assessing the service life of structures. Even if the present paper uses a fuzzy approach for the lifetime estimation problem, the authors firstly consider very important to overcome some conceptual limitations and also errors that may occur considering standard oversimplified models as those defined in Eq. (1) and (3) or other functions derived by using equivalent mathematical procedures and assumptions. Indeed, even if in the past years the models used in Eq. (1) and (3) have been applied in a wide number of important construction projects, some authors [Luping and Gulikers, 2007] have observed that it is necessary to evaluate the errors computed in lifetime estimation and therefore in durability design due to these mathematical oversimplifications assumed in simplified models. Several authors in fact (for example, [Maage et al., 1995; Choe et al., 2007]), seem to have simply substitute the constant diffusion coefficient in the error function solution to the Fick’s second law by the time-dependent $D$, without adequate clarifying the mathematical basis of the diffusion. The mathematical mistake so committed results in a significant difference between the diffusion coefficient $K_0$ that appears in equation (4) and the same coefficient $D_a$ that appears in equation (3), as demonstrated by
Luping and Gulikers [2007]. For the before described reasons and for the great variability whereby the diffusion law has been solved in literature, different values of the main model parameters have been obtained. Starting from this consideration about uncertainty in problem parameters definition, in the present paper an improving in service life assessment goes by two main steps. The first one regards the chosen of the best fit model in order to consider the most refined model to represent the physical process and deterministically determine the main parameters. At this stage, therefore, it tries to avoid some oversimplifications conceptually wrong in model definition, like the most common assumption of the time-independence of the diffusion coefficient. Nevertheless, parameters, which define the occurrence properties of variables, are non deterministic in nature and very often no information is available on their fluctuation. Therefore in the second part of the present paper, uncertainties have been introduced in the model parameters and a fuzzy modelization has been applied as useful tool to perform the lifetime assessment. Final results are obtained for a realistic example.

3.1 GA-Based Comparison Analysis Between Predicted And Measured Chloride Profiles

The model fitting analysis is based on the fundamental experimentation by Thomas and Bamforth [1999], applied to three concrete mixes: Portland cement concrete (PC), fly ash concrete (PFA) and slag concrete (GBS). Even if the proposed procedure followed by the authors used to determine the chloride profile, shows a very good correlation with the experimental data, it suffers of a significant conceptual limitation. This is due to the fact that firstly they assume the standard solution as written in equation (1), founding the values of $C_s$ and $D_a$ as best fit calibration parameters, by iteration. At the same time they assume the relationship (2) that describes the diffusion coefficient with time, in order to take into account time-dependence effects, not accounted in the previous used model. From this last equation, they obtain $D_{28}$ and $m$ values that best fit the data [Thomas and Bamforth, 1999]. Starting from this data, in order to determine the sampling parameters to adopt in the afterwards fuzzy analysis, a new genetic algorithm (GA) based procedure is applied. In the following, adopting the two time-dependence models of equations (4) and (5) and calibrating them also in function of the time variable, it tries to mathematically improve the deterministic estimation of the parameters, so that it is given a meaningful significance to their fuzzy treatment, later discussed. Once assumed these “evolutionary” models, the succeeding improvement in the estimation of the parameters consists in calibrating these one as time function. In this way, the chloride profile is no more defined in a bi-dimensional space that describes the chloride concentration with depth at a fixed time, but has to be dependent also on time. Therefore, the adjustment technique is developed in a three-dimensional depth-time space. Only in this manner, it is possible to rigorously estimate the diffusion coefficient and accounting for its changing with time. In order to simplify the notation, the model at the base of Eq. (4) will be called ‘Model A’ and the profile of equation (5) will be called ‘Model B’. For each of these models, the chloride profile that best fit the experimental data is plotted. For the sake of argument, the three-dimensional profile Fig.1(a) and the contour graph Fig.1(b), in function of time and depth is reported with reference to the Model A, in case of Portland cement. Results show a good agreement between the model prediction and the experimental data of Thomas and Bamforth. In Tab.1 are also indicated the calibrating parameters values, for the Model A and the Model B. In order to compare these one with the other approaches, also the values of the same parameters referred to Thomas and Bamforth’s model application are reported. For all mentioned reasons, the parameters successively treated as fuzzy variables are those obtained by the before described calibrating GA-based procedure, with reference to the mathematically improved model A, because as seen, more rigorously determined by an analytical and conceptual point of view.

<table>
<thead>
<tr>
<th>Concrete Mix</th>
<th>$C_s$ (%)</th>
<th>Diffusion coefficient $(cm^2/yr)$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC, Model A</td>
<td>0.3775</td>
<td>$K_0 = 1.864$</td>
<td>0.2528</td>
</tr>
</tbody>
</table>

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### Table 1

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model A</th>
<th>Model B</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PFA</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thomas and Bamforth [1999]</td>
<td>0.3770</td>
<td>$D_a = 6.988$</td>
<td>0.3459</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>-</td>
<td>0.1</td>
</tr>
<tr>
<td>Model A</td>
<td>0.5259</td>
<td>$K_0 = 0.283$</td>
<td>0.5807</td>
</tr>
<tr>
<td>Model B</td>
<td>0.3804</td>
<td>$D_a = 5.317$</td>
<td>0.7017</td>
</tr>
<tr>
<td>Thomas and Bamforth [1999]</td>
<td>0.5</td>
<td>-</td>
<td>0.7</td>
</tr>
<tr>
<td><strong>GBS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model A</td>
<td>0.5267</td>
<td>$K_0 = 0.076$</td>
<td>0.9941</td>
</tr>
<tr>
<td>Model B</td>
<td>0.5399</td>
<td>$D_a = 9.999$</td>
<td>0.8624</td>
</tr>
<tr>
<td>Thomas and Bamforth [1999]</td>
<td>0.5</td>
<td>-</td>
<td>1.2</td>
</tr>
</tbody>
</table>

**Figure 1.** Correlation of Chloride profile with experimental data (Model A, PFA case)

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### 4 FUZZY ANALYSIS OF STRUCTURAL SERVICE LIFE

The following treatment of lifetime prediction is conduct on the base that fuzzy sets are applied on the model variables. These variables, due to their conjectured informal or lexical uncertainties, can be treated by means of fuzzy sets theory [Zadeh, 1965]. This can be considered an acceptable assumption due to the considerable simplifications that affect the model and the large variability whereby it is defined. Moreover, it is also necessary to observe that even model parameters may suffer of variations not always capable of being assimilated to specific probabilistic distributions which are typically obtained on the basis of other models. Indeed, very often in literature it is not possible to find probabilistic univocal approaches in order to represent the variables. On the basis of all these
considerations in the following it assumes that surface chloride concentration $C_S$ and critical chloride concentration $C_{lim}$ are non deterministic variables. Besides, for the before explained reasons, in addition to these parameters, also the age factor $m$ and diffusion coefficient $k_0$ are assumed uncertain.

The application of the fuzzy set theory, on the just cited variables, offers the possibility to consider a gradual assessment of the membership grade of an element in relation to the defined set. This degree of membership is described by a so-called membership function (MF). The MF acquisition of every fuzzy variable is one of the problems of structural analysis based on the fuzzy set theory. This is a “knowledge acquisition” problem [Klir and Yuan, 1995]. Generally, it is possible to assert that an unitary approach doesn’t exist for the so-called fuzzification, but different procedures can be adopted for each situation. These methods for constructing MF can be either direct or indirect with a single expert or multiple experts [Klir and Yuan, 1995; Klir, 1996]. This work assumes that the MF is known for each fuzzy variable. In other words, we suppose that a knowledge acquisition procedure has been performed preliminary. Standard probabilistic approaches of reliability cannot be used in the presence of fuzziness and so different fuzzy reliability measures have been proposed, for instance by Shrestha and Duckstein [1998], Elishakoff and Ferracuti [2006(a), 2006(b)]. Finally, it is possible to define the central time-dependent fuzzy safety factor as follows [2006(a)]:

$$\beta(t) = \frac{C_c(t)}{C_f(t)}$$

(6)

where $C_c(t)$ denotes the abscissa of the center of gravity of the MF (centroide). $\tilde{c}$ and $\tilde{f}$ respectively denote the fuzzy cover and the position of the chloride critical front.

4.1 Numerical Application

The central time-dependent fuzzy safety factor in (6) can be assumed as a valid tool to support the DM in planning maintenance works. Geometrical, mechanical and environmental data are reported in Table 2. In order to show the effectiveness of the proposed procedure, the fuzzy lifetime analysis of a RC column with section 0.3x0.3 m of PFA concrete mix, subject chloride ingress at one side, is performed considering uncertainties. In order for it to assume an unique value, the abscissa of the center of gravity of its MF has been considered.

In Fig. 2 is plotted the fuzzy safety factor as before defined. In the upper right corner of the figure, the MF (adopted of triangular shape) for the chloride critical front distance from side $d_{cr}$ in five different years is reported with a continuous line. The corresponding safety factors and years are indicated in the principle diagram with the same symbols. The MF of the cover is also indicated with a dashed line for the purpose of comparison. For the same years the fuzzy critical front trend with time is plotted in Fig. 3 (in gray graduation from blank, for null degree of membership, to black, for unitary degree of

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type of uncertainty</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_0$</td>
<td>Fuzzy</td>
<td>$\langle 15, 25, 35 \rangle$</td>
</tr>
<tr>
<td>$m$</td>
<td>Fuzzy</td>
<td>$\langle 0.45, 0.50, 0.65 \rangle$</td>
</tr>
<tr>
<td>$C_S$</td>
<td>Fuzzy</td>
<td>$\langle 0.40, 0.45, 0.65 \rangle$</td>
</tr>
<tr>
<td>$C_{lim}$</td>
<td>Fuzzy</td>
<td>$\langle 0.15, 0.20, 0.25 \rangle$</td>
</tr>
<tr>
<td>cover</td>
<td>Fuzzy</td>
<td>$\langle 15, 25, 30 \rangle$</td>
</tr>
</tbody>
</table>

Table 2. Parameters adopted in the numerical applications
The red indicator in figure 2 shows the centroid of the MF of cover. From the analysis of Figure 2 it is possible to observe that the safety factor reaches a unitary value after about 55 years. First of all, it is possible to observe that, in time, the support of the MF tends to increase. The analysis has been extended beyond the limit state ($\beta < 1$), in order to prove that the centroid of the critical front MF changes position, during the course of time, depending on the low or high values of the fuzzy safety factor in comparison with the core. The centroid is positioned to the right, with respect to the core, in the safety region ($\beta \geq 1$). This is clearly shown with the circle and the upward-pointing triangle. This trend is not particularly visible in the safety region but becomes more appreciable in the unsafety one. This observation leads to conclude that the uncertainties are such that the deterministic analysis furnishes a more advanced fuzzy position of the chloride critical concentration front (the deterministic value is assumed in correspondence of the MF core), tending to overestimate the critical front distance $d_{cr}$ from the side, in comparison to a fuzzy approach, therefore it provides a conservative value of the lifetime estimation. Besides, this specific example shows that the effects of uncertainty, over a given time, tend to become more stable, as can be deduced by the approaching of the MF cores with time increasing and also by the stabilization of the centroid-core distance.

![Figure 2. Illustrative example for PFA mix concrete](image)

![Figure 3. Trend of the chloride critical front at 5 (a), 25 (b) and 75 (c) years.](image)

### 5 CONCLUSIONS

Service life of RC structural elements subject to chloride ingress has been estimated considering uncertainties in problem parameters. The analysis of the state-of-the-art conditions has shown that geometrical, mechanical and environmental parameters cannot be taken into account neglecting the different sources of uncertainty. Therefore, traditional probabilistic approaches cannot be performed.
This work develops a general procedure in which firstly the most refined deterministic models are selected and then, a calibration procedure GA algorithm-based is used in order to estimate the main parameters that are successively assumed as uncertain and treated as fuzzy variables. The fuzzy safety factor is obtained considering fuzzy variables for both the chloride front and the cover. A numerical example shows the effectiveness of the approach as useful tool to support the DM in economic analysis and planning of interventions, showing that the simple deterministic analysis, not properly accounting of intrinsic uncertainty of model parameters, cannot gather important aspects of lifetime prediction. The present approach is a very robust methodology, because it allows to overcome the notable difficulties that designers may face in real circumstances when non-statistical information, technical judgments and points-of-view have to be considered.

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