COST OPTIMIZATION OF CONSTRUCTION PROJECT SCHEDULES

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The paper presents the cost optimization of construction project schedules. The optimization was performed by the nonlinear programming approach, NLP. Accordingly, a NLP optimization model for the cost optimization of project schedules was developed and applied. The nonlinear objective function of the total project costs was subjected to a rigorous system of generalized precedence relationship constraints between project activities, the activity duration constraints and the project duration constraints. The results of the optimization include the minimum total project cost and the project schedule with the optimal start times and the optimal durations of activities. A numerical example presented at the end of the paper demonstrates the advantages of the proposed approach.

KEYWORDS: cost optimization, project scheduling, construction management, nonlinear programming, NLP.

INTRODUCTION

The cost effective scheduling is one of the most important aspects of the construction project management. Traditionally used methods for the cost effective project scheduling in construction management include either the Critical path method (CPM) or the program evaluation and review technique (PERT) combined with trial-and-error procedure. In this way, the cost effective project schedules are achieved in a time-consuming cost-duration analysis of various alternatives for start times and durations of construction project activities. However, doubt always exists as to whether or not the obtained project schedule is optimal.

To surmount the mentioned disadvantages, various different optimization methods have been proposed for the cost optimization of project schedules. Considering the exact mathematical programming methods, the cost optimization of project schedules has been handled mainly by different linear programming (LP) methods, see e.g. Demeulemeester et al., (1998); Achuthan and Hardjawidjaja (2001); Möhring et al. (2001); Vanhoucke et al. (2002). Since the LP methods can handle only linear relations between the variables, the nonlinear terms of the optimization models have been formulated as the discrete relationships between the variables or they were approximated with (piece-wise) linear functions.

However, even the earliest studies in this field have recognized the nonlinear nature of the project cost-duration relationships. Therefore, the nonlinear programming (NLP) techniques have been proposed to solve project scheduling optimization problems with nonlinear cost functions, see e.g. Kapur (1973); Deckro et al. (1995) and Turnquist and Nozick (2004). Nevertheless, in most of the published works the cost optimization of project schedules was performed considering only the Finish-to-Start precedence relationships between activities.
This paper presents the cost optimization of construction project schedules performed by the NLP approach. Accordingly, a NLP optimization model for the cost optimization of project schedules was developed and applied. The nonlinear objective function of the total project costs was subjected to a rigorous system of generalized precedence relationship constraints between project activities, the activity duration constraints and the project duration constraints. The results of the optimization include the minimum total project cost and the project schedule with the optimal start times and the optimal durations of activities. A numerical example presented at the end of the paper demonstrates the advantages of the proposed approach.

**NLP PROBLEM FORMULATION**

The general NLP optimization problem may be formulated in the following form:

\[
\begin{align*}
\text{Minimize } & \quad z = f(x) \\
\text{subjected to: } & \quad h(x) = 0 \quad \text{(NLP)} \\
& \quad g(x) \leq 0 \\
& \quad x \in X = \{x \mid x \in \mathbb{R}^n, x^{LO} \leq x \leq x^{UP}\}
\end{align*}
\]

where \(x\) is a vector of the continuous variables, defined within the compact set \(X\). Functions \(f(x)\), \(h(x)\) and \(g(x)\) are the (non)linear functions involved in the objective function \(z\), the equality and inequality constraints, respectively. All the functions \(f(x)\), \(h(x)\) and \(g(x)\) must be continuous and differentiable.

In the context of the project scheduling optimization problem, the continuous variables define schedule parameters such as activity durations, start times, direct costs, etc. The objective function determines the total project cost. Equality and inequality constraints and the bounds of the continuous variables represent a rigorous system of generalized precedence relationship constraints, the activity duration constraints and the project duration constraints of the project scheduling optimization problem.

**NLP MODEL FORMULATION**

The cost optimization of the project schedules was performed by the NLP approach. In this way, the NLP model formulation consists of the total cost objective function, the generalized precedence relationship constraints, the activity duration constraints and the project duration constraints. The following total project cost objective function is defined for the cost optimization of project schedules:

\[
CT = \sum_{i \in I} C_i(D_i) + CI(DP) + P(DL) - B(DE) \quad (1)
\]

where objective variable \(CT\) represents the total project cost, set \(I\) comprises the project activities \(i, i \in I\), \(C_i(D_i)\) denotes the direct cost-duration functions of the project activities \(i, i \in I\), \(CI(DP)\) is the project indirect cost-duration function, \(P(DL)\) is the penalty-duration...
function and $B(\text{DE})$ is the bonus-duration function. The variables $D_i$, $D_P$, $D_L$ and $D_E$ denote the durations of the project activities $i, \ i \in I$, the actual project duration, the amount of time the project is late, and the amount of the time the project is early, respectively. The total project cost objective function is subjected to the rigorous system of the generalized precedence relationship constraints, the activity duration constraints and the project duration constraints.

Each project activity $i, \ i \in I$, is connected with its succeeding activities $j, \ j \in J$ by fulfilling at least one of the following generalized precedence relationship constraints:

Finish-to-Start: $S_i + D_i + L_{i,j} \leq S_j$ \hspace{1cm} (2)
Start-to-Start: $S_i + L_{i,j} \leq S_j$ \hspace{1cm} (3)
Start-to-Finish: $S_i + L_{i,j} \leq S_j + D_j$ \hspace{1cm} (4)
Finish-to-Finish: $S_i + D_i + L_{i,j} \leq S_j + D_j$ \hspace{1cm} (5)

where $S_i$ is the start time of activity $i, \ i \in I$, $D_i$ is the activity duration, $L_{i,j}$ is the lag/lead time between activity $i, \ i \in I$, and the succeeding activity $j, \ j \in J$, and $S_j$ is the start time of the succeeding activity $j, \ j \in J$.

The actual project duration $D_P$ is determined as follows:

$$D_P = S_{i_\omega} + D_{i_\omega} - S_{i_\alpha}$$ \hspace{1cm} (6)

where $S_{i_\omega}$ and $D_{i_\omega}$ represent the start time and the duration of the last project activity $i_\omega, \ i_\omega \in I$, and $S_{i_\alpha}$ denotes the start time of the first project activity $i_\alpha, \ i_\alpha \in I$.

Since the project activities must be executed between the project start and finishing time, the following constraint is set to bound the completion times of the project activities:

$$S_i + D_i - S_{i_\alpha} \leq D_P$$ \hspace{1cm} (7)

The relationship between the actual project duration $D_P$, the amount of time the project is late $D_L$, the amount of time the project is early $D_E$ and the target project duration $D_T$ is formulated as follows:

$$D_P - D_L + D_E = D_T$$ \hspace{1cm} (8)

Only one of the variables $D_L$ and $D_E$ can, at the most, take a nonzero value in any project scheduling solution. In this way, these two variables are additionally constrained by the following equation:

$$D_L \cdot D_E = 0$$ \hspace{1cm} (9)

**NUMERICAL EXAMPLE**

In order to present the applicability of the proposed NLP approach, the paper presents an example of the cost optimization of the construction project schedule. The considered construction project scheduling optimization problem is a variant of the time-cost trade-off problem for a small building project presented by Yang (2005).
The construction project consists of 7 activities. The precedence relationships and the lag times between succeeding project activities are presented in Table 1. The crash/normal points and the direct cost-duration functions of the construction project activities are presented in Table 2.

Table 1: Precedence Relationships and Lag Times between the Project Activities

<table>
<thead>
<tr>
<th>Activity</th>
<th>Succeeding activity</th>
<th>Precedence relationship</th>
<th>Lag time [day]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID Description</td>
<td>ID</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Underground service</td>
<td>2.</td>
<td>Start-to-Start</td>
<td>2</td>
</tr>
<tr>
<td>2. Concrete works</td>
<td>3.</td>
<td>Finish-to-Start</td>
<td>3</td>
</tr>
<tr>
<td>3. Exterior walls</td>
<td>4.</td>
<td>Finish-to-Start</td>
<td>0</td>
</tr>
<tr>
<td>4. Roof construction</td>
<td>5.</td>
<td>Finish-to-Start</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>6.</td>
<td>Finish-to-Start</td>
<td>0</td>
</tr>
<tr>
<td>5. Floor finish</td>
<td>7.</td>
<td>Finish-to-Start</td>
<td>0</td>
</tr>
<tr>
<td>6. Ceiling</td>
<td>7.</td>
<td>Finish-to-Finish</td>
<td>6</td>
</tr>
<tr>
<td>7. Finish work</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 2: Crash/Normal Points and Direct Cost-Duration Functions of the Project Activities

<table>
<thead>
<tr>
<th>Activity</th>
<th>Duration [days]</th>
<th>Direct cost [$]</th>
<th>Direct cost-duration function</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID Description</td>
<td>Crash Normal</td>
<td>Crash Normal</td>
<td>Direct cost-duration function</td>
</tr>
<tr>
<td>1. Underground service</td>
<td>3 6</td>
<td>4,500 1,500</td>
<td>(250 D_1^2 - 3,250 D_1 + 12,000)</td>
</tr>
<tr>
<td>2. Concrete works</td>
<td>10 12</td>
<td>7,000 5,000</td>
<td>(-10,969.6299 \ln(D_2) + 32,258.5063)</td>
</tr>
<tr>
<td>3. Exterior walls</td>
<td>8 12</td>
<td>3,600 2,000</td>
<td>(11,664 \exp(-0.1469 D_3))</td>
</tr>
<tr>
<td>4. Roof construction</td>
<td>6 8</td>
<td>3,100 2,000</td>
<td>(-550 D_4 + 6,400)</td>
</tr>
<tr>
<td>5. Floor finish</td>
<td>3 4</td>
<td>3,000 2,000</td>
<td>(-1,000 D_5 + 6,000)</td>
</tr>
<tr>
<td>6. Ceiling</td>
<td>4 6</td>
<td>4,000 2,500</td>
<td>(-750 D_6 + 7,000)</td>
</tr>
<tr>
<td>7. Finish work</td>
<td>10 14</td>
<td>2,800 1,000</td>
<td>(75 D_7^2 - 2,250 D_7 + 17,800)</td>
</tr>
<tr>
<td>Project</td>
<td>42 55</td>
<td>28,000 16,000</td>
<td>(979)</td>
</tr>
</tbody>
</table>

979
The daily indirect cost is $200.00. While the per-period penalty for late project completion is set to be $400/day, the per-period bonus for early project completion is determined to be $300/day. The targeted project duration is 47 days.

The objective of the optimization is to find a construction project schedule with the optimal activity start times and durations so as to minimize total project cost, subjected to the generalized precedence relationship constraints, the activity duration constraints and the project duration constraints.

The proposed NLP optimization model formulation was applied. A high-level language GAMS (General Algebraic Modelling System) (Brooke et al., 1988) was used for modelling and for data inputs/outputs. CONOPT (generalized reduced-gradient method) (Drud, 1994) was used for the optimization.

Since the NLP denotes the continuous optimization technique, the optimization of the project schedule was performed in two successive steps. In the first step, the ordinary NLP optimization was performed to calculate the optimal continuous variables (e.g. start times, durations, etc.) inside their upper and lower bounds, see Table 3.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Start time [day]</th>
<th>Duration [days]</th>
<th>Direct cost [$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Underground service</td>
<td>1.000</td>
<td>6.000</td>
<td>1,500.00</td>
</tr>
<tr>
<td>2. Concrete works</td>
<td>3.000</td>
<td>12.000</td>
<td>5,000.00</td>
</tr>
<tr>
<td>3. Exterior walls</td>
<td>18.000</td>
<td>8.000</td>
<td>3,600.00</td>
</tr>
<tr>
<td>4. Roof construction</td>
<td>26.000</td>
<td>6.667</td>
<td>2,733.33</td>
</tr>
<tr>
<td>5. Floor finish</td>
<td>32.667</td>
<td>4.000</td>
<td>2,000.00</td>
</tr>
<tr>
<td>6. Ceiling</td>
<td>32.667</td>
<td>6.000</td>
<td>2,500.00</td>
</tr>
<tr>
<td>7. Finish work</td>
<td>36.667</td>
<td>11.333</td>
<td>1,933.33</td>
</tr>
</tbody>
</table>

| Indirect cost [$] | 9,400.00        |
| Penalty [$]       | 0.00            |
| Bonus [$]         | 0.00            |
| Total project cost [$] | 28,666.66 |

In the second step, the calculation was repeated/checked for the fixed and rounded variables (from in the first stage obtained continuous values to their nearest upper discrete values). Table 4 summarizes the optimum rounded solution for the small building project schedule.
The minimum total project cost obtained by the NLP optimization of the project schedule was found to be $28,750.00. The gained optimal results include the minimum total project cost and the project schedule with the optimal start times and the optimal durations of activities. The example also shows that the total cost optimization of the project schedule performed by the NLP approach is carried out in a calculating process, where the start times and durations of project activities are considered simultaneously in order to obtain the minimum total project cost. The obtained maximum values for durations of the project activities 1, 2, 5, and 6 demonstrate that the cost optimization of the project schedule not necessarily minimize the project duration. Moreover, the example shows that the optimum duration of the project may also exceed the target project duration. In this way, the additional feature of the total project cost optimization represents the advantage of the proposed NLP approach to construction project scheduling over the traditionally used CPM and PERT methods.

**CONCLUSIONS**

This paper presents the cost optimization of construction project schedules performed by the NLP approach. The NLP optimization model formulation for the cost optimization of construction project schedules was developed and applied. The input data within the NLP optimization model include: the project network with determined preceding and succeeding activities, the precedence relationships and the lag/lead times between activities, the normal/crash points and the direct cost-duration functions of the activities, the project indirect cost-duration function, the penalty-duration function and the bonus-duration function. The nonlinear continuous total project cost objective function was subjected to the rigorous system of the generalized precedence relationship constraints, the activity duration constraints and the project duration constraints. For specified input data, the NLP
optimization yields the minimum total project cost and the construction project schedule with
the optimal start times and the optimal durations of activities.

The existing exact NLP methods have focused on the cost optimal solution of the project
scheduling problems which include simplifying assumptions regarding the precedence
relationships among project activities. On the other hand, the present work aims to
incorporate generalized precedence relationships between project activities and to propose the
NLP model for making optimal project time-cost decisions applicable to actual construction
projects. In addition, solving the construction project scheduling optimization problem using
the proposed NLP model avoids the need for (piece-wise) linear approximation of the
nonlinear expressions, which has been the traditional approach proposed for solving this
optimization problem using the LP models. Since the proposed optimization approach
enables an insight into the interdependence between the project duration and the total project
cost, the decision-maker can more effectively estimate the effect of the project deadline on a
total project cost before the submission of a tender.

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