

# **Simulation model of design flow rate in water submetering systems using fuzzy logic and Monte Carlo method**

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## **Abstract**

The probabilistic method has been considered by Brazilian researchers the most suitable for determining design flow rate in water supply system because it takes into account more realistic conditions of each design. Nevertheless, as the usage of some sanitary appliances, in special the shower, depends on the subjective behavior of users, the aim of this work is to present a model that uses fuzzy logic for fuzzy variables and Monte Carlo method for random variables to achieve more accurate values of design flow rate in water supply system of multifamily buildings. The methodology involved the development of a theoretical model that considers the concepts of fuzzy logic to determine the duration of the shower as well as Monte Carlo method for establishing the random start of the use of sanitary appliances. The results of the simulation model were compared with the results obtained by the method recommended by Brazilian standard and with those obtained by the probabilistic model. The simulation results indicated about 30% smaller than those obtained by Brazilian standard and similar to the probabilistic model, but with the advantage that it shows the behavior of flow rates in the peak period and not only the maximum value in a given section. Thus, this model can contribute not only to the development of more accurate water meters, but also to more appropriate sizing of water supply systems.

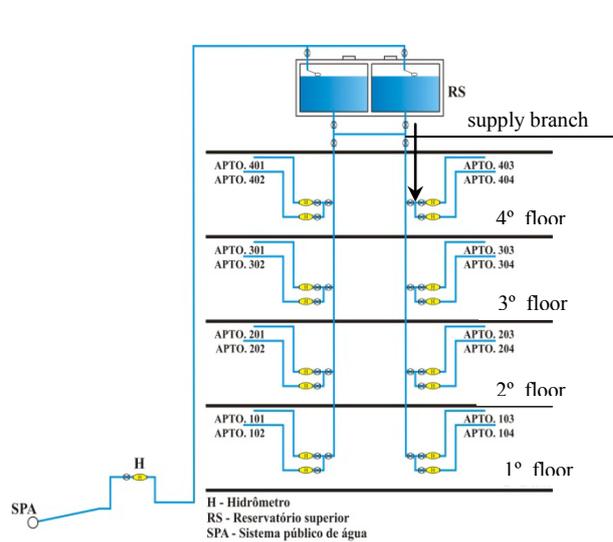
## **Keywords**

design flow rate; water supply systems; water submetering system; fuzzy logic; Monte Carlo method.

# 1 Introduction

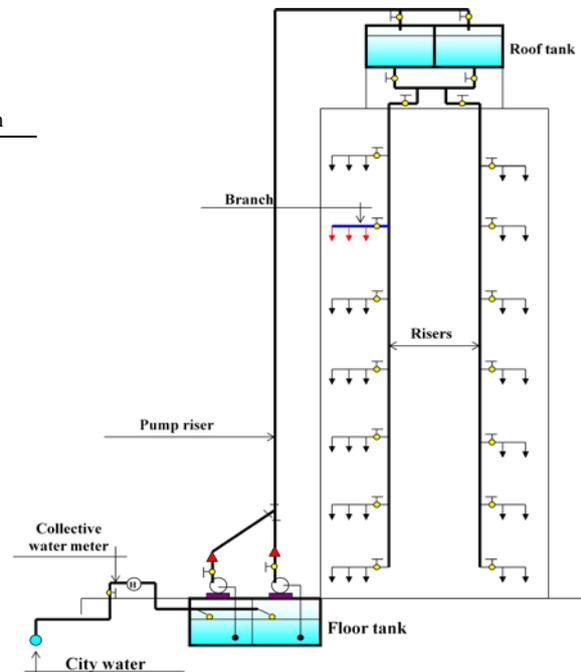
Technological evolution of building water supply systems has been influenced by water conservation concepts. This fact is made evident in new products and systems that have been developed and implemented in buildings, such as: water-saving components, water submetering and non-potable water systems. One of the impacts on building water supply systems is the reduction of design flow rates and, consequently, the need to update the models that have been used to date for determining design flow rates.

In water submetering systems, sanitary rooms with different periods of use, such as bathrooms, kitchen and laundry room are supplied by a single pipe that runs, usually horizontally, through the apartment (Figure 1). Regarding the establishment of flow rates, this is the main difference when comparing with collective metering systems (Figure 2), which have risers for supplying areas with similar usage patterns.



H – water meter  
RS – roof tank  
SPA – city water

**Figure 1 – Water submetering system**



**Figure 2 – Collective water metering system**

In water submetering systems in buildings there are several uncertainties when establishing design flow rates, since the occurrence of flows in a building system of water distribution depends on the interaction between the sanitary appliance system and the user, and is therefore influenced by subjectivity and by uncertain events. Taking this into account, the application of fuzzy logic along with the Monte Carlo method, which uses random variables of probability distributions, is promising in order to obtain better results in design flow rates.

For these reasons it is considered that a model that includes the concepts of fuzzy logic for determining users' shower duration can contribute to a better representation of the behavior of design flow rates in water submetering systems in multifamily buildings

and therefore, with more accurate design flow rates, water meters will perform better concerning the reliability requirement.

In order to reduce water wastage and provide a more accurate method to establish the design flow rate of water submetering systems, a computer simulation program for the assessment of the flow rate was developed by the authors. The aim of this paper is to present the simulation model, its implementation, the results and some discussions about the approaches adopted by the system.

## **2 Fuzzy logic and Monte Carlo method**

This section presents the basic concepts of two mathematical tools used in the simulation model of design flow rates in water submetering systems: fuzzy logic and the Monte Carlo method.

### **2.1 Fuzzy logic**

According to Yen, Langarica and Zadeh [1], fuzzy logic is an inference mechanism based on fuzzy rules, given actual propositions. In simpler terms, one can conceptualize fuzzy logic as a tool to convert subjective information, often described in natural language, to a numeric value.

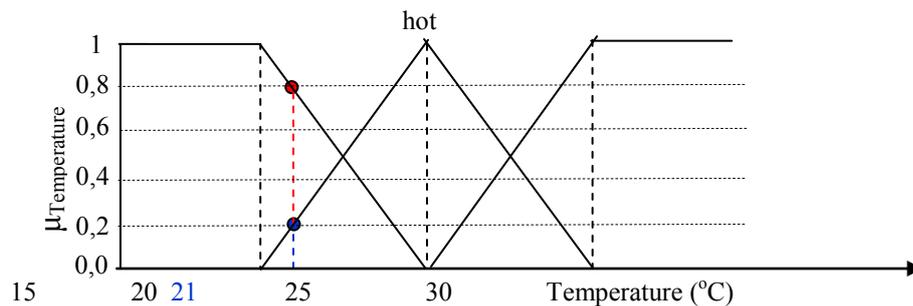
Considering that the concept of degree of membership in fuzzy logic is usually confused with the concept of probability, which results in considering the membership function as a statistical distribution function, it is relevant to explain the difference between the two concepts.

According to Simões [2] probability is the chance that an element belongs to a group, while possibility expresses the degree of membership of the element in the set.

For instance, suppose a father claims that the chance of his daughter taking a shower in the morning is 0.8. Thus, the probability that the daughter will take a shower in the following morning is 0.8, but it does not tell the quality or the duration of the shower. However, by using the different shower times on a scale (1.0 = very long shower, 0.8 = long shower, 0.5 = normal shower, 0.3 = fast shower, and 0 = no shower), the father could say that his daughter's shower would be long.

It should be observed that probability does not contribute to inform if the design flow rate is high or low, but to establish the likelihood that a certain flow rate is occurring in an event. The measurement of the shower flow rate, for example, is a fuzzy measure that ranges from minimum flow to maximum flow.

The example shown in Figure 3 also illustrates the difference between fuzzy logic and probability theory. If a person says the day is hot, it must be said how hot it is, what is the degree of the value *hot*. This value ranges from 0, representing not hot at all, to 1, representing extremely hot.



**Figure 3 – Linguistic variable *Temperature* and its membership functions**

In Figure 3, in case the room temperature is 21° C, then by means of the fuzzy set, one can say that it is 0.8 hot and not that it has an 80% chance of being hot or not, as in probability theory. Therefore, a degree is created in order to know how hot a temperature is considered to be. If the temperature were 15° C or 35° C, it would mean 0.0 hot, that is, it would not be hot. If the temperature were 25° C, then it would be 1.0 hot or extremely hot.

Thus, it may be stated that probability theory uses random variables that are dependent on the occurrence of future and uncertain events, while fuzzy mathematics uses fuzzy but certain variables about the occurrence of events.

According to Von Altrock [3], fuzzy logic translates natural language descriptions of decision policies into algorithm using a mathematical model. It is a reasoning system that involves fuzzy propositions and the procedure of the fuzzy inference consists of three major steps: fuzzification, inference and defuzzification.

**Fuzzification:** It is the initial step, where the fuzzy sets are used to translate numerical variables into linguistic variables, and the membership degree of the associated linguistic values is obtained. In binary logic a well-defined proposition would be, for example, "the shower flow rate is 0.05 L/s". In fuzzy logic the same proposition could be: "the shower flow rate is low."

**Fuzzy inference:** After translating all input numerical values into linguistic variable values, the fuzzy inferences are carried out by applying the if/then fuzzy rules that define system behavior. This step yields a linguistic value for the linguistic variable. For example, the linguistic result for "duration of shower" (linguistic variable) could be "very fast" (linguistic value).

**Defuzzification:** In this step, the fuzzy variable obtained by the inference rules is converted into a discrete numerical value that better represents the inferred values of the linguistic variable, which is the output of the fuzzy inference. There are many defuzzification methods available in the literature. However, according to Cox [4], the baricentre method and the maximum average method are the most used ones. In this work the baricentre method has been adopted.

## 2.2 Monte Carlo method

The Monte Carlo method is widely used in models to determine water demand in building systems. Some studies developed by researchers in the area are presented below.

Murakawa *et al.* [5] have developed models to simulate hot and cold water demand for different types of buildings using the Monte Carlo method. This work examines multifunctional buildings, also known as complex buildings, and for this purpose it uses the models designed for other typologies.

Mui and Wong [6] developed a model to contribute to the planning of water supply in cities with a population density similar to that of Hong Kong. Their model uses hourly frequency distributions of toilet discharges, the number of users in relation to the total in the buildings and the variation in the use of water closets in order to estimate hourly and daily water demand for water closet discharge by using the Monte Carlo method.

The Monte Carlo method is considered in the present research to take into account random events of the use of sanitary appliances such as wash basin, water closet, sink, laundry sink and washing machine. Different from the shower, as will be shown in the next sections, the use of these sanitary appliances does not show clear relation with some intervening variables.

Considering that the standard deviation of the population is not known, the number of simulations was obtained according to Shamblin and Stevens Jr. [7], using equation 1.

$$N = \frac{s^2(Z_{\alpha/2})^2}{d^2} \quad (1)$$

Where:

s = known standard deviation of the sample;

N = number of simulations;

$Z_{\alpha/2}$  = normalized standard deviation corresponding to the confidence coefficient  $1 - \alpha$  and that the true average is between  $\alpha/2$  and  $1 - \alpha/2$ ;

d = estimation error.

## 3 Methodology

Figure 2 presents the model for the determination of design flow rates in building systems with water submetering in multifamily buildings, using fuzzy logic and the Monte Carlo method. Although the model may be applied to any segment of the system, it was designed to determine the design flow rate in the supply branch of an apartment with a family made up of father, mother and teenage son.

The Monte Carlo method is employed to determine the time when all users start using sanitary appliances in the peak period.

The duration of shower use is obtained through fuzzy logic and that of other sanitary appliances can be obtained through the Monte Carlo method, when the frequency distributions of durations of sanitary appliance use in the peak period are available.

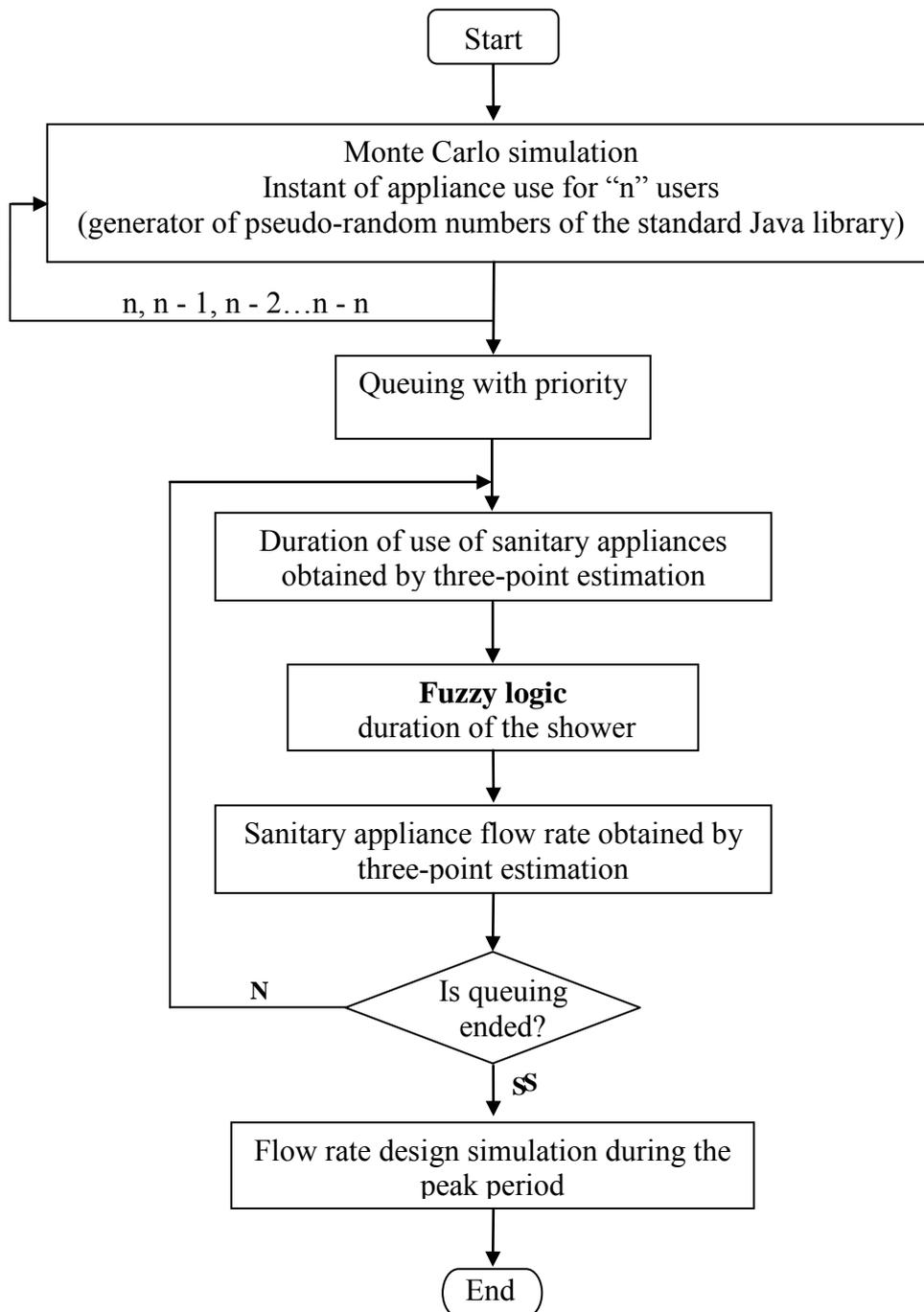
In this case, for simplicity purposes, the duration of the discharge of an appliance ( $t$ ) – except the duration of shower, which was determined by fuzzy logic was defined using *in loco* measurements and calculated by means of three-point estimation method: a minimum ( $t_{\min}$ ), a most probable ( $t_{\text{prov}}$ ) and a maximum ( $t_{\max}$ ) value, using Gamma distribution, according to Gonçalves [8], and the mean ( $\mu_t$ ) may be determined by Equation (2).

$$\mu_t = \frac{t_{\min} + 3 \cdot t_{\text{prov}} + t_{\max}}{5} \quad (2)$$

Similarly, the unitary flow rate of each appliance ( $q$ ) was also determined by means of field surveys and its mean value was also calculated by using the three- point estimation method.

When the result of the Monte Carlo method indicates an overlap of use of any sanitary appliance, that is, two users are selected to start using the sanitary appliance at the same time, a strategy that establishes priority between users is adopted to resolve the conflict.

Figure 2 shows the structure of the model to determine design flow rates in building systems with water submetering.



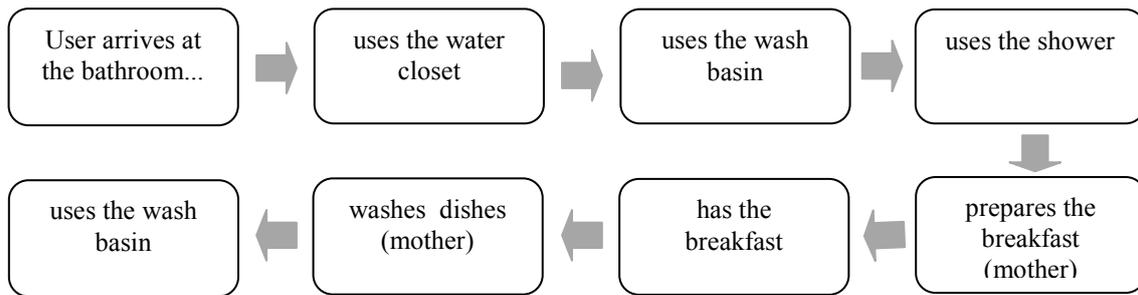
**Figure 2 – Flow chart of simulation model of design flow rates in submetering systems with the use of fuzzy logic and the Monte Carlo method**

### 3.1 Simulation model of design flow rates

The simulation model of design flow rates in building systems with water submetering system employing fuzzy logic and the Monte Carlo method consists in a logical sequence of activities related to water use, beginning in the morning peak period (in this

case from 6 to 7 am), when users start the activities of cleaning and preparing breakfast. The peak period is finished when users leave for external activities such as school and work. The model is described below.

At first, the Monte Carlo method is employed to determine the time when each user in the family starts using sanitary appliances in the peak period. There is a sequence of activities in this period, illustrated in Figure 3, which is followed by all users. In this case, the "breakfast" and "dish-washing" activities were considered only for the mother.



**Figure 3 - Sequence of activities planned for the users in the peak period**

For the simulation of design flow rates in the supply branch of the apartment submetering system, as described earlier, a program was developed in Java and its steps for determining flow rates are presented below.

**Step 1:** Random selection of users to start activities related to water use during the peak period (6 am - 7 am). The probability distribution of start times for the use of sanitary appliances by the users is presented in Table 1.

**Table 1 - Probability distribution of start times of activities related to water use by the users in the family**

Father		Mother		Son	
Time h:min	Frequency (%)	Time h:min	Frequency (%)	Time h:min	Frequency (%)
				6:10	00
6:05	10	6:00	50	6:20	20
6:10	20	6:05	20	6:30	50
6:15	50	6:10	20	6:40	20
6:20	20	6:15	10	6:50	10

**Step 2:** Users are lined up to use the sanitary appliances.

**Step 3:** The proposed sequence of use of sanitary appliances is: water closet, sink, shower and sink. If two users are randomly selected at the same time, the simulation establishes the following priority: father, mother and son. The use of the bathroom by two users is allowed only in the following cases: father with mother, and father with son.

**Step 4:** The duration of use of the water closet is three minutes for the mother, two minutes for the father and three minutes for the son. The duration of the discharge of the toilet (WC) is the same for all users, that is, 60 seconds with a flow rate of 0.15 L/s.

**Step 5:** The duration of use of the washbasin is 40 seconds for all users with a flow rate of 0.08 L/s, both after using the water closet and after breakfast.

**Step 6:** The users' shower duration is determined based on fuzzy logic in accordance with their respective matrixes of fuzzy relationships, as presented in Tables 2, 3 and 4. The shower flow rate is the same for all users: 0.12 L/s.

**Table 2 - Matrix of inference rules for the duration of the father's shower**

<b>Time Temperature</b>	<b>very early</b>	<b>early</b>	<b>on time</b>	<b>delayed</b>	<b>very delayed</b>
very cold	very fast	very fast	very fast	no shower	no shower
cold	fast	fast	fast	no shower	no shower
pleasant	normal	normal	fast	no shower	no shower
hot	normal	normal	normal	no shower	no shower
very hot	long	long	normal	very fast	no shower

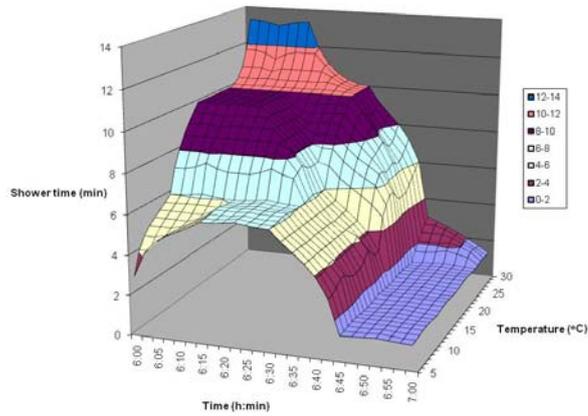
**Table 3 - Matrix of inference rules for the duration of the mother's shower**

<b>Time Temperature</b>	<b>very early</b>	<b>early</b>	<b>on time</b>	<b>delayed</b>	<b>very delayed</b>
very cold	very fast	very fast	very fast	no shower	no shower
cold	fast	fast	fast	no shower	no shower
pleasant	fast	fast	fast	no shower	no shower
hot	normal	normal	normal	very fast	no shower
very hot	normal	long	normal	very fast	no shower

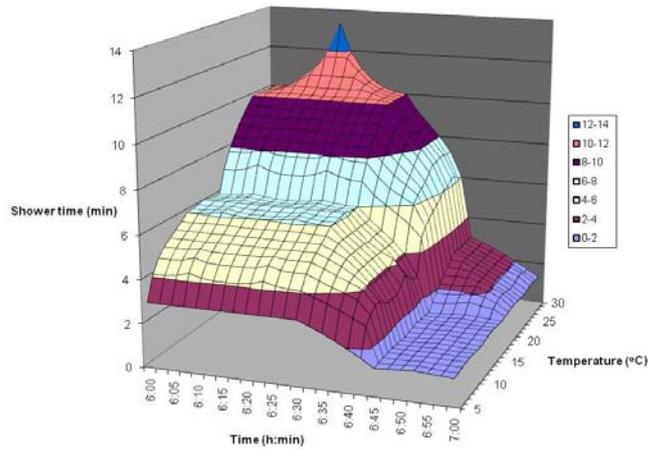
**Table 4 - Matrix of inference rules for the duration of the teenage son's shower**

<b>Time Temperature</b>	<b>very early</b>	<b>early</b>	<b>on time</b>	<b>delayed</b>	<b>very delayed</b>
very cold	no shower	no shower	no shower	no shower	no shower
cold	no shower	no shower	no shower	no shower	no shower
pleasant	fast	normal	normal	no shower	no shower
hot	long	normal	long	fast	no shower
very hot	long	long	long	very fast	no shower

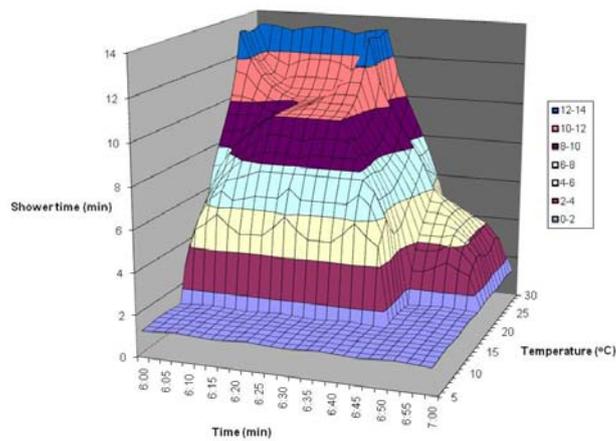
The duration of the shower calculated by fuzzy logic are presented in Figures 4, 5 and 6 for father, mother and a teenage son, respectively.



**Figure 4 – Father: duration of morning shower as a function of instant of shower and air temperature obtained by the fuzzy approach**



**Figure 5 – Mother: duration of morning shower as a function of instant of shower and air temperature obtained by the fuzzy approach**



**Figure 6 – Teenage son: duration of morning shower as a function of instant of shower and air temperature obtained by the fuzzy approach**

The son, as illustrated in Figure 6, does not worry about the duration of the shower on hot days, but he is very sensitive and does not take a shower on cold days. In addition, the son takes the longest showers in relation to the other family members.

By observing Figures 4, 5 and 6, which show the results for the duration of the showers of the father, mother and son as obtained through fuzzy logic, it should be noted that they reflect the respective roles and conditions within the family.

**Step 7:** After taking a shower, the mother prepares breakfast in five minutes and at the beginning of this period she collects water for 40 seconds at a flow rate of 0.10 L/s.

**Step 8:** After taking a shower, users have breakfast. The mother has breakfast in five minutes, the father in 10 minutes and the son in 10 minutes.

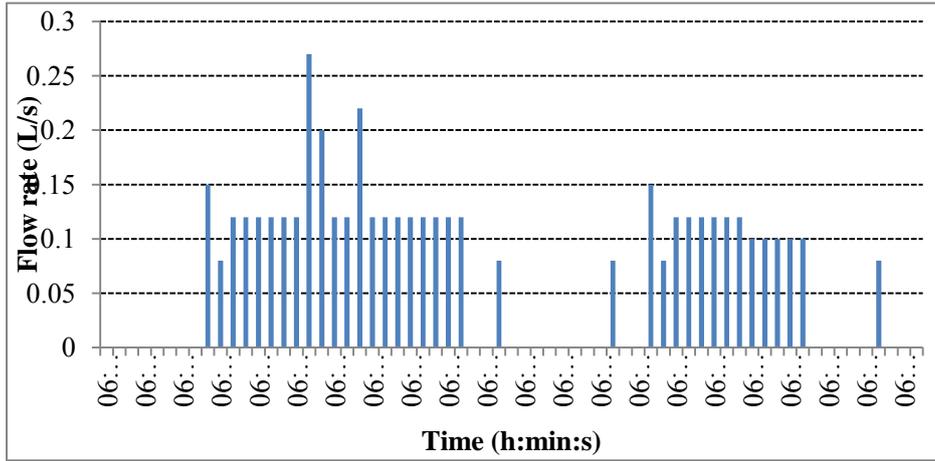
**Step 9:** After having breakfast, the mother washes the breakfast dishes for five minutes and the sink faucet has a flow rate of 0.10 L/s. In this case, only the mother washes the dishes.

**Step 10:** When all users complete their activities, the simulation is concluded, which may occur before the end of the peak period.

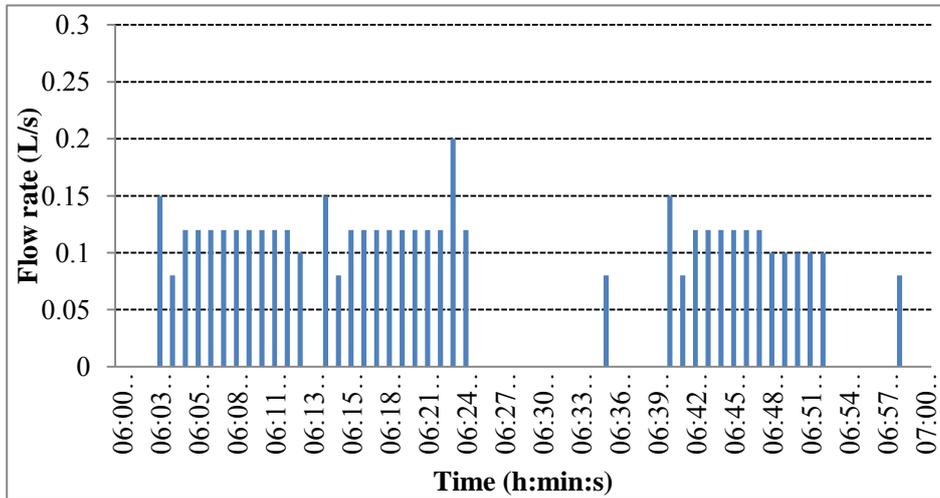
To calculate the number of simulations, the sample standard deviation was defined at 10 instances of start of shower use for each of the three users in the peak period ( $s = 0.22$ ), and a confidence coefficient of 95% ( $Z_{\alpha/2} = 1.96$ ) was established. With these data it is possible to calculate the number of simulations needed for an estimation error equal to 0.50. According to equation 1,  $N = 74$  simulations. Considering that 128 simulations were performed, the estimation error is much lower: 0.038.

## 5. Results and discussions

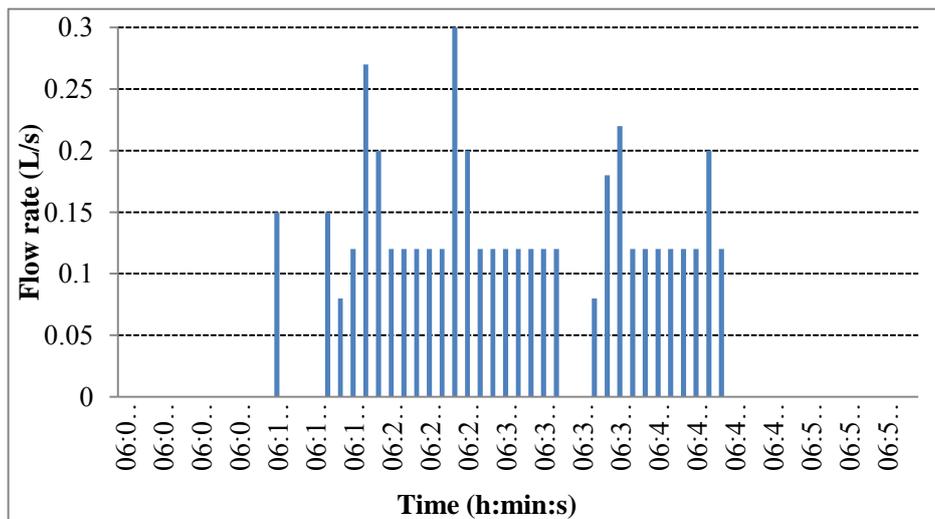
Figures 7 to 9 present the results of some of the 128 flow rate simulations in the supply branch of an apartment with water submetering for activities held during the peak period, from 6 to 7 am. The apartment has two bedrooms, a bathroom, a kitchen and a laundry. The family consists of father, mother and teenage son, as previously described.



**Figure 7 – Flow rate simulation in the supply branch during peak period**

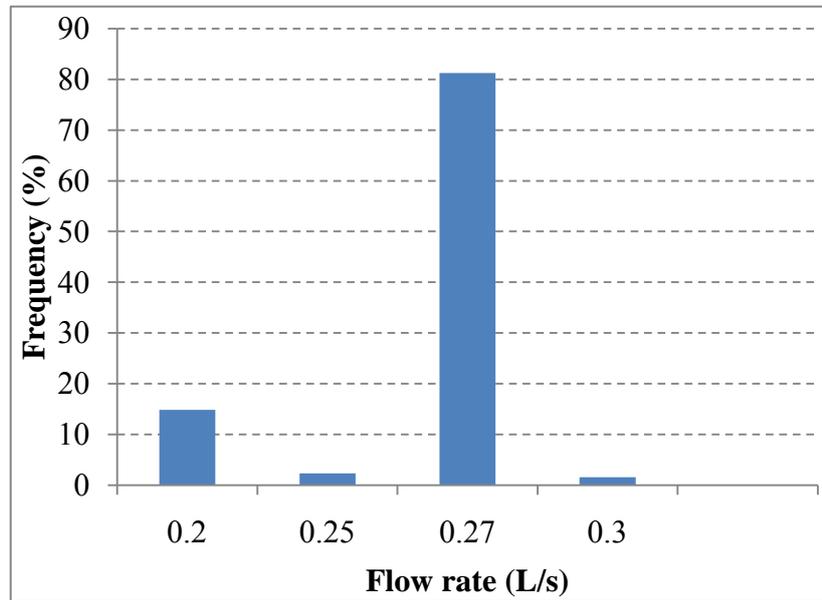


**Figure 8 - Flow rate simulation in the supply branch during peak period**



**Figure 9 - Flow rate simulation in the supply branch during peak period**

Figure 10 illustrates the frequency distribution of maximum flow rates in the supply branch of the system under analysis during the peak period, from 6 to 7 am. In 104 simulations (81%) the maximum flow rate was 0.27 L/s, in three simulations (2%) the maximum flow rate was 0.25 L/s, in 19 simulations (15%) the maximum flow rate was 0.20 L/s, and in only two simulations (2%) the maximum flow rate was 0.30 L/s.



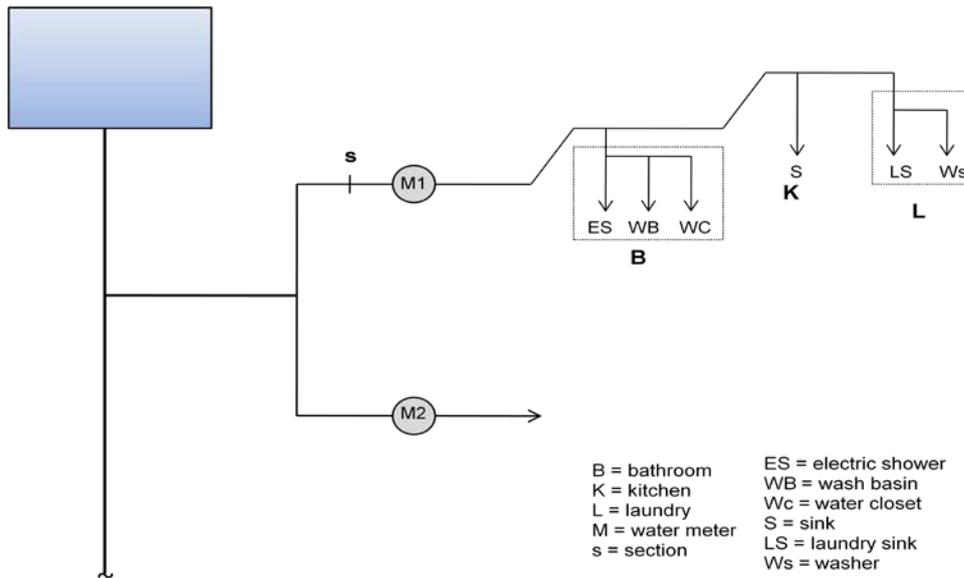
**Figure 10 – Frequency distribution of maximum flow rates in the supply branch of the system under analysis during the peak period, from 6 to 7 am**

Therefore, it can be stated that the most probable design flow rate in the supply branch of this apartment and for this family is 0.27 L/s, which represents the simultaneous use of the shower and the water closet discharge. It should be noted that the water closet discharge lasts 60 seconds, that is, a very short period during the shower.

Considering that, in general, water meters of supply branches specified for this range of values have a nominal flow rate of 1.5 m<sup>3</sup>/h (0.42 L/s), it can be stated that the water meter could have a nominal flow rate of 0.75 m<sup>3</sup>/h (0.21 L/s), since the duration of the occurrence of maximum flow rates is very brief, as verified in 104 of the simulations – three of which are illustrated in Figures 7, 8 and 9. It is known that one of the limitations of this option is the loss of high pressure in the meter, but given these results, further investigation is recommended.

## 5.2 Comparison of methods for determining design flow rates

Considering the same building water system, that is, an apartment with submetering system and a family with three users who utilize the same bathroom with electric shower (ES), washbasin (WB) and close-coupled toilet (WC), and the kitchen with a sink (S) during the 1-hour peak period, from 6 to 7 am, the design flow rate in the supply branch, upstream of the meter, was determined in section S, shown in Figure 11.



**Figure 11 – Configuration under analysis with "S" section where the design flow rate in the supply branch is determined**

Keeping in mind, on the one hand, the comparative analysis between fuzzy logic, probabilistic [8] and relative weight [8] methods and, on the other, the peak period from 6 to 7 am, neither the laundry sink nor the washing machine were considered so that the results would be compatible with the fuzzy logic model.

The results of the design flow rate obtained for section S by using the three methods are presented in Table 5.

**Table 5 – Design flow rate in section S of the supply branch obtained through the relative weight, probabilistic and fuzzy logic methods**

<b>Método dos pesos relativos Q (L/s)</b>	<b>Método Probabilístico Q (L/s)</b>	<b>Método lógica nebulosa Q(L/s)</b>
0.35	0.31	0.27

The results indicate that the design flow rate obtained by the relative weight method is approximately 30% higher than the rate obtained by the fuzzy logic method and 13% higher than that obtained by the probabilistic method.

## 6 Final considerations

This paper presented a simulation model for the determination of design flow rate in the water submetering system of multifamily building. This model considers Monte Carlo simulation for random variables and fuzzy logic to assess the duration of the shower in a residential apartment, due to it better represent the behaviour of the users. The main conclusions are:

- the simulation results show that most of the time when water flows in the supply branch, the flow rate values are below 0.17 L/s, which indicates overestimation of the flow rates that have been used for designing water meter systems similar to that simulated in this study;
- the results of the simulation model were about 30% lower than those obtained by the Brazilian standard and similar to those obtained by the probabilistic model, but with the advantage that it shows the behavior of flow rates in the peak period and not only the maximum value in a given section;

In conclusion, the model developed in this study, employing fuzzy logic and the Monte Carlo method, can be applied to simulate design flow rates in water submetering systems of multifamily buildings. A wider field database, however, is required for the consolidation and expansion of the application to other types of buildings.

### **Acknowledgments**

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