Deep Beam Design Using Strut-Tie Model

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ABSTRACT

The aim of this paper is to develop nomographs and demonstration of its applicability for the simple supported deep beam design in order to simplify a number of computation steps. For this purpose, the applied basic modifications of formulation of the Strut-Tie model for the deep beam design ware briefly described. For the demonstration of the applicability of the developed nomographs, a simple supported deep beam subjected to uniformly distributed load was analyzed in which the flexural design was focused. The influence of the column was considered, simplifying by the concerning of the width, but not length of the column.

KEYWORDS: Strut-Tie model, Deep beam design, Height-span ratio

1. INTRODUCTION

Reinforced concrete deep beams are typically used as transfer members in high-rise structures due to their high resistance capacity. Also they often have a function to convert the structural system between upper and lower part of the structure. Since a deep beam has to support the whole of upper part structure, its structural behavior can influences on the stability and safety of the structure remarkably. Because the stress distribution in the section of the deep beam is nonlinear, the linear elastic theory for the general beam analysis can not be applied. Therefore ACI code requires that deep beams be designed via non-linear analysis or by Strut-Tie models.

The Strut-Tie model is formulated by straight lines expressing resultant forces of tension and compression stress in members and its section. Therefore, the merit of the concept can be that the design engineer grasps the flow of the force in the members and proposes the rational reinforcing recipe for the stress disturbance region due to the process of the representation of the natural flows of forces. With these advantages of the Strut-Tie model, the application of this method seems like quite easy and practical. However, if one will use the model for the design practice, he will immediately notice several difficulties. First of all, the user has to carry out a number of calculation steps according to the design scheme under some initial assumptions which should be checked only after several strenuous, suggested in ACI, computation process. In addition to, the result of design the using Strut-Tie model indicates in some cases remarkable discrepancy with them using other building codes, like ACI code (2005) or CEB-FIP (1990).

The aim of this paper is to develop nomographs and demonstration of its applicability for the simple supported deep beam design in order to simplify a number of computation steps. Prior to this, the applied basic modifications, discussed in detail in Noh et al.(2006), in the formulation of the Strut-Tie model for the deep beam design briefly described. For this purpose, a simple supported deep beam subjected to the uniformly distributed load was analyzed in which the flexural design was focused in the paper. The influence of the column was considered, simplifyed by the concerning of the width, but not length of the column. As the main investigation parameter, the height-span ratio of the beam was chosen.

2. MODIFICATIONS OF STRUT-TIE MODEL FORMULATION

2.1 Maximum moment

The basic concept and calculation scheme of the Strut-Tie model can be found in many literatures, such as Schaefer (1996), Rogowsky and Macgregor (1986), Marti (1985) as well as Kong (1970).

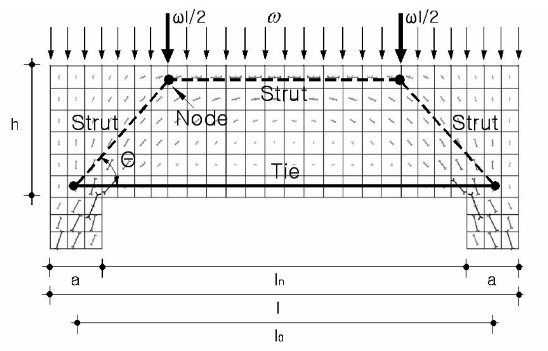

Figure 1. Concept of Strut-Tie model

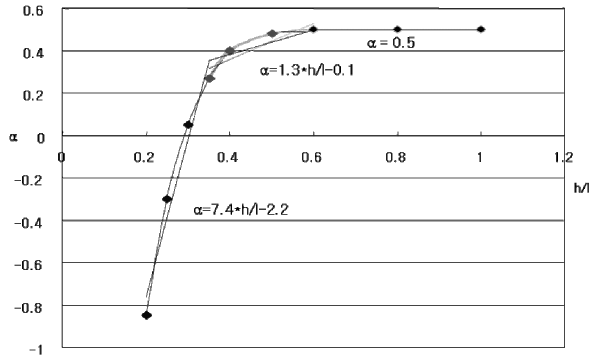

Figure 2. Span modification factor α

Figure 1 shows the principal stress paths in the simple supported deep beam subjected to the uniformly distributed load and the generally applied strut-tie model with two equivalent concentrated loads substituted from the original load. This Substitution is allowed, only if the equality of the maximum moments, the most fundamental value in the Strut-Tie model application, at the mid-span of the both systems is guaranteed. In the fact, the above concept does not correctly consider the column-width (column stiffness) and the height-span ratio which influence the flows of the compression stress and further the tension stress. The author suggested in Noh et al. (2006) the modification of the substituted concentrated load p_{mod} for the computation of the maximum bending moment in the deep beam, considering the height-span ratio and column width as follow:

$$p_{\mathrm{mod}} = w_{\mathrm{mod}} l / 2 \,, \tag{1}$$

$$w_{\mathrm{mod}} = w(l_{\mathrm{mod}} / l_0)^2 \ \text{ with } \ l_{\mathrm{mod}} = \mathrm{l_n} + a \times \alpha \tag{2}$$

Where

w	: uniformly distributed load,	w_{mod}	: modified uniformly distributed load,
p_{mod}	: modified converted concentrated load,	l_{mod}	: modified span according to Figure 2,
l_0	: column-centroid span,	α	: column width.

The span modification factor α can be computed by the Eq.(3), (4) and (5)

$$\alpha = 7.4 \times h / l - 2.2 \quad \text{for} \quad 0.2 < h / l \leq 0.35 \tag{3}$$

$$\alpha = 1.3 \times h / l - 0.1 \quad \text{for} \quad 0.35 < h / l \leq 0.6 \tag{4}$$

$$\alpha = 0.5 \quad \text{for} \quad h / l > 0.6 \tag{5}$$

2.2 Limit of angle between inclined strut and tie

ACI code defines the lower limit of the angle θ between the inclined strut and the tie as 25° based on the principal of Saint-Venant. This lower limit guarantees that B-region does not exist in the beam and trapezoidal Strut-Tie model may be applied. On the other hand, the upper limit of the angle is not stated in the code. Schlaich et al. (1987, 1991), however, suggested the limit as about 68°. In our investigation using FE-analysis, with the consideration of the column width of $1.0\,l$, the tensile force does not change, when h/l greater than about 1.2. The angle θ converges consequently to about 72°. The result indicates that in the case of the beam with h/l greater than about 1.2, the upper part of the beam should be modeled as B-region.

2.3 Limit of tie width

The horizontal strut width W_s and the tie width W_t can be computed by the equation formulated by the equality requirement of the compression and the tensile forces. Under this condition, the relationship between the widths of the strut and the tie can be found as

$$W_t = 1.25 W_s \tag{6}$$

which results from the ratio of the strength reduction factor for the strut with uniform cross-section $\beta_s = 1.0$ and for the nodal zone anchoring in the tie $\beta_s = 0.8$ according to ACI. However, the amount of W_t can be possibly evaluated smaller than the required width for the arrangement of reinforcement bar. Therefore the lower limit of W_t should be taken as the diameter of the bars plus twice the covering depth according to ACI code. For the practical application, the lower limit of the tie width W_t was suggested in this paper as $10cm$.

$${}_{Min}W_t = 2c + ds \geq 10cm \tag{7}$$

3. STRUT-TIE MODEL ANALYSIS

In the following, the design process for the deep beam using the Strut-Tie model according to ACI code was briefly described.

Step 1 : Substitute uniformly distributed load to two concentrated loads
Step 2 : Calculate reactions and maximum moment at mid-span
Step 3 : Assume widths of horizontal strut(S_{horz}) and tie (Tie) at mid-span
Step 4 : Calculate distance jd between centroid of S_{horz} and Tie
Step 5 : Calculate angle θ between inclined strut(S_{horz})and Tie
Step 6 : Calculate forces in strut, C_{horz} and in tie C_{inc}
Step 7 : Check strength in strut and tie
Step 8 : Check strength of nodes A
Step 9 : Check assumed widths of strut and tie at step 3 by control of node geometry
Step 10 : Calculate flexural reinforcement steel A_S
Step 11 : Calculate shear reinforcement steel A_{VS}

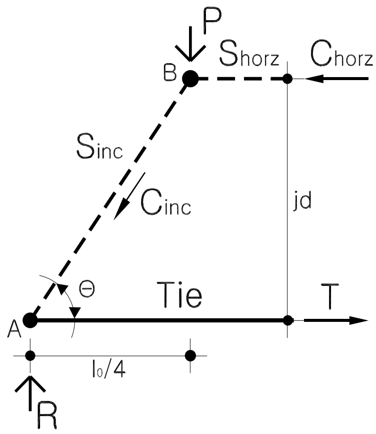

Figure 3. Degree of freedom of deep beam

The most uncertain and annoying step in the analysis process can be to assume the widths of the strut and the tie at step 3 and the recalculation with the new widths, if this assumption does not pass the check at step 9. This stress can be mitigated through the solution of second degree polynomial equation formulated by the requirement of the equality of the horizontal compression and the tension forces as well as the equilibrium condition of the moment at mid-span.

4. DEEP BEAM DESIGN USING NOMOGRAPHS

In consideration of the modification components mentioned above, nomographs of the deep beam design using Strut-Tie model were built up.

Table 1. Design data

Beam depth (h) (m)	Clear spans (l_n) (m)	Column ($a \times b$) ($m \times m$)	Load (W) (kN/m)	Concrete strength (f_{ck}) (MN/m^2)
3.0	3.0	0.4×0.5	2000	40.0

Process 1: Substituted concentrated load P_{mod}

The substituted concentrated load P_{mod} can be computed by Eq. from (1) to (5). With the height-span ratio $h/l = 0.79$ and the span modification factor $\alpha = 0.5$, P_{mod} results in $3011.7kN$. This step corresponds Step 1 in Sect.3.

Process 2: Width of strut and tie

By the requirement of the equality of the horizontal compression and the tension forces as well as equilibrium condition of moment at mid-span, a second degree polynomial equation for strut width W_s can be formulated.

$$pW_s^2 - qW_s + P_{\text{mod}} l_0 / (4 \cdot f_{ck} \cdot b) = 0 \tag{8}$$

$$p = 1.125\phi \cdot 0.85\beta \tag{9}$$

$$q = \phi \cdot 0.85 \cdot \beta_s \cdot h \text{ with } \phi = 0.75 \tag{10}$$

The solution of Eq.(8) was plotted in dependence upon h, $P_{\text{mod}} l_0 / (f_{ck} \cdot b)$ in Figure 5 from which the required width of the horizontal strut W_s can be easily read out. Then, the width of the tie can be found by Eq.(6). This process includes the uncertain and annoying step 3 and control step 7 and 9 mentioned in Sect. 3.

For the given beam, W_s indicated $7.0cm$ for $h = 3.0cm$ and $P_{\text{mod}} l_0 / (f_{ck} \cdot b) = 0.51cm^2$. W_t was calculated by Eq.(6) as $8.75cm$. However, in consideration of the lower limit of the tie width by Eq.(7), W_t was chosen as $10cm$.

Process 2: Width of strut and tie

Strength of each node is determined by the size of the strut and the tie as well as the widths of the acting load or the support width depending on the location of the node. In the case of the distributed load, the node constructed under the substituted concentrated load has not to be controlled, because the node does not exist actually. So, only the node above the support should be controlled.

With the assumption that the width of the support l_b is sufficiently designed against the support reaction, the inclined width W_s of the support node in Figure 4 should be ensured do that the reduced norminal strength ϕF_{ns} is greater than the acting force C_{inc} in the inclined strut.

$$\phi F_{ns} = \phi \left(0.85 \beta_s f_{ck}\right) b \cdot W_{SA} \geq C_{inc} = P_{\text{mod}} / \sin\theta \tag{11}$$

$$W_S = l_b \sin\theta + W_t \cos\theta \tag{12}$$

From Eq.(11,12) the minimum width of the tie W_t can be obtained as

$$_{\min}W_t = \frac{P/f_{ck} \cdot b}{\phi \cdot 0.85 \cdot \beta_s \cdot \sin\theta \cdot \cos\theta} - a\tan\theta \tag{13}$$

This Equation for $_{\min}W_t$ is plotted in Figure 6 in which the angle θ has to be pre-calculated as follows.

$$\theta = \tan^{-1}\left(\frac{jd}{l_0/4}\right) \text{ with } 25^\circ \le \theta \le 72^\circ \tag{14}$$

$$jd = h - \frac{W_t}{2} - \frac{W_s}{2} \tag{15}$$

In the applied example, $_{\min}W_t$ was found out as $0cm$, so that the width from the support $l_b \sin\theta$ is sufficiently large. Therefore, W_t of $10cm$ obtained in Process 2 is valid.

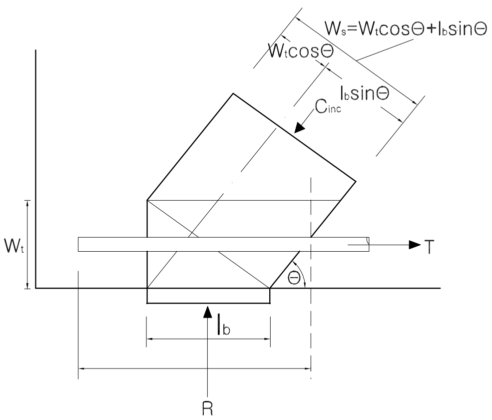

Figure 4. Geometry of support node, ACI(2005)

Process 4: Amount of required flexural reinforcement

The total amount of required flexural reinforcement for the tension force in the tie can be computed from Eq.(16), which pictured in Figure 7.

$$A_s = T/\phi f_y = P_{\text{mod}} \tan\theta / \phi f_y \tag{16}$$

The authors suggest that the strength reduction factor ϕ in Eq.(16) might be 0.9, since the problem concerns the flexural reinforcement, although ACI consider the deep beam design as shear design and recommends ϕ as 0.75.

For the given example, the required reinforcement amount can be read out from figure 7 $A_s = 27cm^2$ for $P_{\text{mod}} = 3011.7kN$ as well as $\theta = 72^\circ$.

The process for the evaluation of the shear reinforcement, which is not described in the paper can also formulated in the same way.

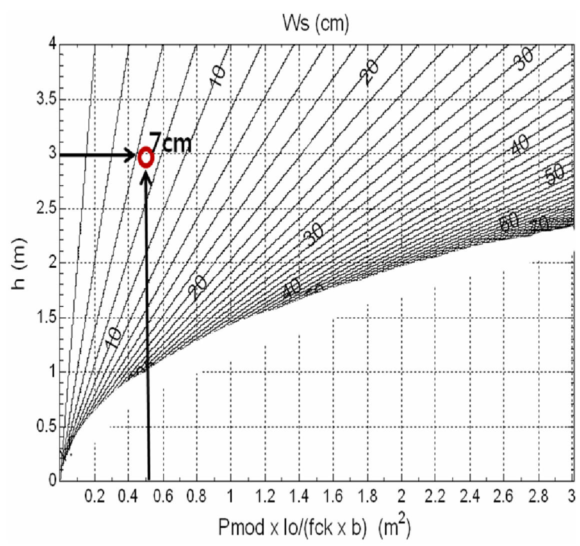

Figure 5. Width of strut

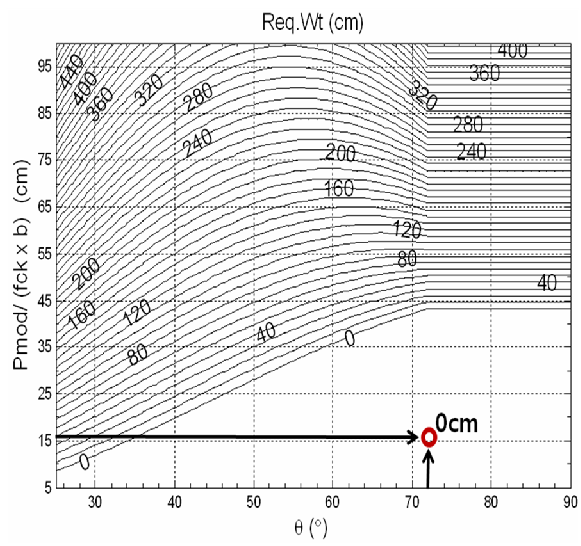

Figure 6. Minimum width of the tie

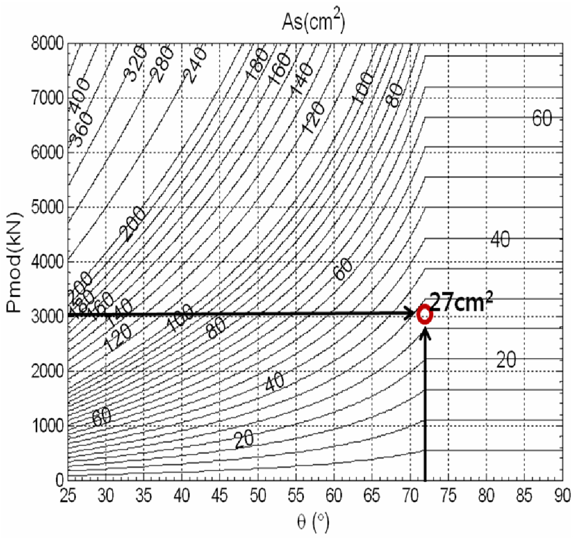

Figure 7. Amount of required flexural reinforcement

5. CONCLUSIONS

The paper demonstrated the development process of nomographs and its applicability for the simple supported deep beam design in order to simplify a number of computation steps.

Prior to this, the applied basic modifications of formulation of Strut-Tie model for the deep beam briefly described. For this purpose, a simple supported deep beam subjected to the uniformly distributed load was analyzed. With this concept, an engineer can easily design a deep beam with only 3 nomographs and some simple calculation processes, without the uncertain and annoying steps.

With the same concept, the design processes with nomographs for a concentrated load and further for a multiple scan beam can be developed.

AKNOLOFYMENT

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REFERENCES

ACI committee 318, 2005. “Building code Requirements for Structural Concrete (ACI 318-05).” American Concrete Institute, 318R-358

Comite Euro-Internation De Beton, 1990. “CEB-FIP Model Code 1990.” Thomas Telford

Kong, F. K., Robins, P. J. and Cole, D. F, 1970. “Web Reinforcement Effects on Deep Beams.” ACI Journal, December, 1010-1017

Marti, P., 1985. “Basic Tools of Reinforced Concrete Beam Design.” ACI Journal, January-February, 46-56

Noh, S.Y., Lee, C. Y., and Jang, S. M., 2006. “A deep beam analysis by applicability study of the strut-tie model.” ISAIA, Vol.Ⅱ, pp.1176-1181

Rogowsky, D. M. and Macgregor, J. G., 1986. “Design of Reinforced Concrete Deep Beams.” Concrete International, August, 49-58

Schaefer, K., 1996. “Workshop for strut-tie Models for the design of structural concrete.” 44

Schlaich, J., Schaefer, K., and Jennewein, M., 1987. “Toward a consistent design of structural concrete.” PCI Journal, Vol.32, No.3, 74-150

Schlaich , J., and Schaefer, K., 1991. “Design and detailing of structural concrete using strut-and-tie models.” The Structural Engineer, Vol.69, No.6, 113-125