A discrete stochastic model for performance prediction of roofing systems

Z. Lounis, D. J. Vanier and M. A. Lacasse
Institute for Research in Construction, National Research Council, Ottawa, Canada

Abstract
A stochastic modeling of the performance of roofing components and systems is proposed using a discrete Markov chain that predicts the incremental deterioration of roofing components. The capability to predict the performance of roofing-components and systems throughout their service lives is essential and constitutes the first step towards the development of a comprehensive building envelope life-cycle asset management system. The proposed Markovian model allows the simultaneous consideration of the time-dependence and randomness of the performance of roofing components and systems taking into account the uncertainty and variability associated with material properties, environmental degradation factors, quality of workmanship, and maintenance conditions. Depending on the controlling explanatory variables, transition probability matrices are generated for roofing components and systems using in-field performance data obtained from inspections. This probabilistic model can then be coupled to an optimization algorithm for the development of a risk-based optimal maintenance management system for roofs at both project and network levels.
Keywords: Condition rating, deterioration, Markov chain, performance, roof system, roof components, service life, stochastic process.

1 Introduction

Low-slope and more specifically built-up roofing systems are and have been extensively used in the North American building industry. For example, the volume of work in built-up roofing constructed in 1995 in the USA rose to 18.35 billion dollars (US) of which 79.1 % was comprised of low-slope roofing [1]. Similar statistics regarding the extent of use of low-slope roofing were derived from a survey undertaken by the Canadian Roofing Contractors Association (CRCA); it indicated that well over 56% of the roofs constructed in 1993 were built-up-roofing (BUR) [2]. Premature roof failures are nonetheless common place and the associated costs of repair can be a substantial portion of the yearly maintenance budget, especially in large single-story industrial buildings. Preventative maintenance strategies for roofs are increasingly being used to help mitigate these harmful effects. Nevertheless, without well established methodologies for verifying the in-service performance and expected remaining service life of the roof, the risk of failure and the consequences of premature roof deterioration cannot be readily ascertained using the existing maintenance methods.

In an industry-wide survey of built-up roofing membranes, it was found that no type of roofing membrane had more than a 50% probability of lasting 20 years [3]. The main causes of roof failures are water ponding, untested new materials, membrane splitting, water vapor condensation within the roof due to insulation use, membrane blistering, and leakage or destruction of the insulation due to vapor barrier use [4].

The performance of a roof system and its components deteriorates with time due to aging effects such as aggressive environmental factors, excessive loads as well as poor workmanship and lack of maintenance. The uncertainty and variability associated with these deterioration factors result in considerable fluctuations of the performance and service life from the mean values. Therefore, a realistic predictive model of roofing system performance should be time-dependent and probabilistic.

In this paper, a stochastic modeling of the performance of roofing components and systems using a discrete Markov chain is proposed. A Markov chain is a discrete stochastic process that enables the simultaneous consideration of the time-dependence and uncertainty of the system performance. This approach has been widely applied to solve different time-dependent probabilistic problems [5,6,7]. This probabilistic performance prediction model for low-slope roofing systems and their components constitutes the first step towards the development of an effective building envelope life-cycle asset management system (BELCAM) [8]. The development of the BELCAM system includes the following phases: (i) Collection of roofing performance data from in-field inspections and grouping according to material and roof system type, geographical location, and building type; (ii) Generation of transition probability matrices and development of Markovian models; (iii) Assessment of the risk of failure for different roofing systems and building types, and; (iv) Development of a risk-based life-cycle roofing maintenance management system.

2 Performance of built-up roof components and systems

A modern built-up roof system has in general five basic components, namely a waterproofing membrane, thermal insulation, flashings, structural deck and possibly a vapor or air barrier [9-10]. There is a strong correlation between the performance of different components, which in turn has a direct impact on the performance of the entire roofing system and its risk of failure. Generally speaking, two types of roofing configurations can be identified depending on the location of the membrane, namely: (i) Conventional roofing system in which the membrane is at the top and is directly subjected to environmental stresses, and; (ii) Protected or inverted membrane roof system in which the membrane is protected from environmental stresses by the insulation. In the following section, a brief description of the degradation mechanisms of various components is given. The performance requirements of a roofing system can be summarized as follows [9-10]: (i) *Water tightness:* prevention of water leakage into the building; (ii) *Heat transfer control:* prevention or minimization of heat (or cooling) exchange between the interior and exterior; (iii) *Condensation control:* prevention of water vapor condensation within the roof system; (iv) *Load accommodation:* ability to sustain dead and live loads, and; *(v) Maintainability:* capability of economic repair.

2.1 Failure modes of roofing components

- *BUR waterproofing membrane:* The built-up roofing (BUR) membrane is predominant on low-slope roofs and is manufactured by adhering three or more felt plies to each other with hot bitumen. BUR membranes are reinforced with organic, asbestos or glass-fiber felts. The service life of BUR membranes increases with the number and strength of felt plies, and as mentioned earlier, no type of roofing membrane had more than a 50% probability of lasting 20 years [3]. It was also found that due to a considerable scatter in the life data of these roofing systems, a normal distribution was used to approximate their service life [3]. The main degradation factors are temperature, solar radiation, water and wind. The major modes of failure are blistering, splitting and ridging. Slippage, delamination, alligatoring and surface erosion are less widespread [4]. Blisters, and to a lesser extent splitting, are the most common distress affecting BUR membranes. Splitting is the most serious failure mode and occurs more frequently in cold climates, more often on large roof areas. The causes of splitting include thermal contraction, insulation movement, water absorption in felts, and deck deflection.
- *Thermal insulation:* It reduces heating and cooling costs and has become an indispensable component of roofing systems for occupied buildings given the rapid increase of energy costs. The thermal resistance of the insulation depends on the performance of the vapor barrier and membrane, and its effectiveness is seriously compromised by moisture. Notwithstanding its positive attributes, the insulation may increase the probability of membrane splitting and water vapor condensation within the roof system. The presence of moisture in the insulation is the result of either condensation or water penetration through defects in the membrane and flashing. Moisture reduces the thermal performance of insulation and this reduction is a function of the material type.
- *Flashings:* They seal the joints at vertical elements, curbs, chimneys, vents, expansion joints and wherever the membrane is either interrupted or terminated. Base flashings are a continuation of the BUR membrane at the upturned edges, while

counterflasings shield the exposed joints of the base flashings. Flashings are considered to be the most common source of roof leaks [4]. The basic failure modes of flashings include [4]: Sagging, ponding leakage, leakage around (through) the flashing, separation of flashing materials, diagonal wrinkling, and post-construction damage.

- *Vapor / air barriers:* A vapor barrier is used to prevent the water vapor condensation within the roof system. The air barrier is used to minimize air leakage through the roof system [9].
- *Structural deck:* It provides structural resistance to the sustained dead and live loads and transmits the loads to the roof framing. The failure modes of a roof deck include excessive deflection, cracking (concrete deck) under service loads and flexural and shear failures under ultimate loads.

2.2 Reliability of roofing systems

The roof system is a multi-component system with multiple failure modes that can be modelled as a hybrid system that is a combination of series and parallel subsystems. The probability of failure of each roofing component is time-variant and increases with time due to the time-dependent degradation of its performance. The probability of failure can be determined using systems reliability approach taking into account the correlation between different components and failure modes. In addition, the corresponding risk of failure of the roofing system could be evaluated once the consequences of failure are established. Two types of failures can be identified: (i) Envelope failure defined by the loss of the envelope main functions (loss of water tightness and energy control), and; (ii) Structural failure (outside the scope of this paper), defined by the deck failure that includes collapse and loss of serviceability. The risk of envelope failure should be kept below a certain threshold value, which depends on the value and vulnerability of the building contents under the roof, costs of repair and other incurred costs.

3 Existing approaches to roofing performance prediction

The prediction of the performance or service life of a roofing system and its components is a very complex problem as it depends on several factors including the loading, environment, quality and frequency of maintenance, quality of materials and workmanship. Despite the large number of publications related to durability and service life prediction of building materials and components, there is a lack of reliable and practical quantitative approaches to performance and service life prediction of materials, components and systems. Moreover, the existing systematic methods, including accelerated testing [11], have yet to be successfully applied to roofing systems.

Accelerated testing is carried out by imposing higher than normal stress levels on building components in order to increase the rate of degradation and thereby shorten the component life. The purpose of such testing is the generation of performance or lifetime data in a short period of time, thus overcoming the time and money constraints associated with field exposure tests. Such data are analyzed by fitting a degradation model to the data to estimate the relationship between performance, age and stress level. The principal objective is the determination of the relationship between failure time and stress magnitude (of degradation factor) in order to extrapolate to actual in-service stress magnitudes. The main accelerating stresses used to increase the deterioration process of BUR roofing membranes include temperature (60°C to 150°C), ultraviolet radiation, water and mechanical stress [10]. Although accelerated testing can be useful in ranking the performance of different roof materials or components, it suffers from many shortcomings [12, 13], which include: (i) Difficulty to simulate the degradation mechanisms of the various materials and their interaction; (ii) Difficulty to reproduce the stochastic nature of the degradation factors affecting the roof performance, and: (iii) Difficulty to correlate accelerated laboratory test results to the actual in-service results.

Given the time-dependence, large uncertainties and correlation of the performance of roofing components, a stochastic model is proposed in this initial research stage and is described in the following section. At a later stage when considerable performance data are collected, simplifications may be made to derive practical semi-probabilistic approaches to performance and service life prediction of roofing systems.

4 Markov chain modeling of roofing system performance

In this paper, a stochastic modelling of the performance of roofing components and systems is proposed using a discrete Markov chain. This model forecasts the future performance of roofing components and systems throughout their entire service lives. This performance prediction capability is essential for the development of an effective roofing maintenance management system. The Markovian prediction model represents an important class of stochastic processes with wide engineering applications (queuing, inventory and maintenance models) [5,6,7] This model takes into account the randomness of performance and its time-variance. The roof component (or system) performance is not a deterministic quantity but rather presents a large scatter, which is due to the variability of material properties, degradation factors, workmanship and maintenance conditions. The performance deterioration may be the result of excessive environmental conditions (temperature, water, moisture, wind), poor materials and workmanship and inadequate maintenance. In addition, there is in general a strong correlation between the performance of the different roofing components (membrane, insulation, flashing, vapor barrier, deck). The combination of the above may result in different and complex degradation mechanisms with cumulative damage effects. These irreversible damages accumulate until the component reaches a threshold performance level that defines failure of the component or system. The service life of the component or system is defined as the time corresponding to this minimum performance level.

In this paper, condition ratings are used as a quantitative measure of the performance of roofing components and systems. Seven integer ratings are adopted to estimate the performance of the membrane, insulation, flashing, and deck, with 7 and 1 representing the “excellent” condition and “failed” conditions, respectively as shown in Table 1. The use of condition ratings for the assessment of facility performance is very widespread given their practicality and cost-effectiveness as they allow the development of optimal maintenance management strategies. They constitute an integral part of various maintenance management systems used for different facilities, including bridges, pavements, and roofs [7,12]. The initial condition ratings of the roofing components are estimated from in-field inspection and evaluation of the **type,** quantity and severity of distresses, by a procedure similar to that used in “ROOFER’ [12].

Table 1. Condition assessment of roof components and systems

Condition	Excellent	Very good	Good	Fair	Poor	Very poor	Failed
Deterioration	None	Negligible	Low	Medium	High	Major	Extensive
Rating	7	6	5	4	3	2	1

The Markov chain is a stochastic process whose state space is finite or countable, that may be described by: $\{S(t_m) = s_k, k = 1, 2, 3, 4, 5, 6, 7\}$, such that the probability of a future state of the system, $S(t_{n+1})$, at time t_{n+1} is governed solely by its present state $S(t_n)$ at time t_n, and not its entire history, i.e.:

$$P[S(t_{n+1}) = s_j \mid S(0) = s_k, S(t_1) = s_{k-1}, \ldots, S(t_n) = s_i] = P[S(t_{n+1}) = s_j \mid S(t_n) = s_i] = p_{ij} \quad (1)$$

The transition probability p_{ij} represents the likelihood that the condition of a roof system will change from state i at time t_n to state j at time t_{n+1}. Equation (1) indicates that the conditional probability distribution of any future state $S(t_{n+1})$ is independent of the previous states, $S(t=0), S(t_1), \ldots, S(t_{n-1})$, and depends only on the present state $S(t_n)$. This assumption represents the first-order type of stochastic process correlation underlying the Markovian process [6]. The states 7, 6, 5, 4, 3, and 2 are called transient states, whereas state 1 is called an absorbing state, which is a state that cannot be vacated, once entered if no repair is made. The Markovian modeling of the performance requires only limited data on the condition of the roofing system (or components) at two or more points in time, in order to derive the probabilities of transition from one state to other states having lower condition ratings.

The performance of different roofing systems and components is dependent on several explanatory variables, including age, environmental conditions, material and system types, loading magnitude, workmanship quality and maintenance level. In order to validate the adopted Markov chain model, it is necessary to develop transition probability matrices for roofing components and systems according to their classification with regard to the above explanatory variables. A stationary stochastic process may be assumed which implies the time-invariance of the transition probability matrix. The transition time is generally taken as one year, but other time intervals can be chosen depending on the time between consecutive inspections. Given the adopted condition rating scale and short transition time, the probability of a roofing component decaying by more than one state in one year may be assumed negligible. As a result, the performance of a roofing system (or its components) of a certain class can be modeled using the following transition probability matrix:

$$\mathbf{P} = \begin{vmatrix} p_{77} & p_{76} & 0 & 0 & \ldots\ldots & 0 \\ 0 & p_{66} & p_{65} & 0 & \ldots\ldots & 0 \\ \cdot & \cdot & \cdot & \cdot & p_{22} & p_{21} \\ 0 & 0 & 0 & 0 & \ldots\ldots & 1 \end{vmatrix} \quad (2)$$

in which p_{ij} is the probability of transition from state i to state j in one step or transition time. Hence, the transition probability matrix is greatly simplified and consists only of two terms in each row, namely: p_{ii}, and p_{ij} ($p_{ij} = 1 - p_{ii}$), representing the probability of remaining at the same state and that of deteriorating by one state in one transition period, respectively. Each drop in the condition rating may correspond to a different degradation mechanism. The Markov chain modeling and the associated transition probabilities for a deterioration process are illustrated in Fig. 1, for a roofing component or system with the seven condition states defined in Table 1.

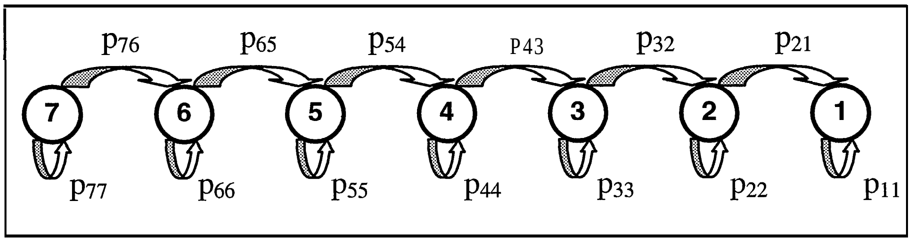

Figure 1 Markov chain modelling of roofing performance deterioration (with no repair)

Once the one-step transition probability matrix is generated, the future performance of a roofing component can be predicted using the n-step transition matrix as follows:

$$\mathbf{P}\{S(t_n)\} = \mathbf{P}\{S(0)\}\mathbf{P}^n \qquad (3a)$$

which can be written in the following simplified form :

$$\mathbf{P}(n) = \mathbf{P}_o\mathbf{P}^n = [p_,(n),\ p_6(n),\ldots..,\ p_1(n)] \qquad (3b)$$

in which $P\{S(t_n)\} = P(n)$ is the state probability matrix at time t_n after n transitions; $\mathbf{P}\{S(0)\} = \mathbf{P}_o = [p_7(0), p_6(0), \ldots, p_1(0)]$ is the initial state probability matrix; P is the transition probability matrix; and $p_k(n)$ and $p_k(0)$ are the probabilities that the component is in state k at times $t = t_n$ and $t = 0$ (initial condition), respectively.

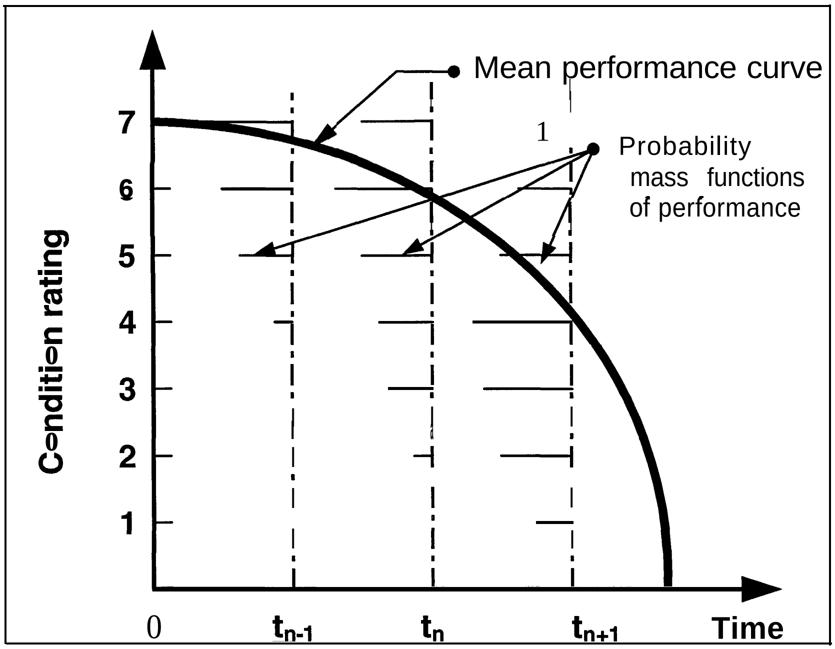

Figure 2 Markov chain prediction of roofing system performance

The probabilistic prediction of the performance using Eq. (3a) is illustrated in Figure 2, which indicates the evolution with time of the probability mass function of roofing performance. In general, at the start of the roof service life, the probability mass is close to condition rating 7; after aging and deterioration, this probability mass shifts from states having high condition ratings to those states with lower condition ratings. Ultimately, if no repairs are made, all the probability mass accumulates in the absorbing state with a condition rating 1. The transition probability matrix is determined as the solution of the following nonlinear programming problem, where the objective function is the minimization of the difference between the performance data obtained from in-field inspections and Markov chain predictions, i.e.:

$$\min \quad \sum_{k=1}^{7}\sum_{n=1}^{N}[p_k(n) - p_k^*(n)]^2 R(n) \tag{4a}$$

$$\text{subject to:} \quad 0 < p_{ii} \leq 1 \qquad (i = 1, 2, \ldots\ldots 7) \tag{4b}$$

in which $p_k^*(n)$ = relative frequency of components in state k at age n from inspection data; $p_k(n)$ =Markov chain prediction of probability of being in state k at age n, obtained using Eq.(3b) as a function of p_{ii}, (i = 1, 2, ...,7); N= number of inspections or years of available data; and R(n) = number of components at age n.

The underlying assumption of the first-order Markov chain model referred to as the "Markovian property" is the dependence of the rate of deterioration on the current stress and cumulative damage and not on the entire stress history. The validity of this property will be checked through statistical inference testing. The transition probability matrix can be further updated using the Bayesian technique if additional performance measurements become available. The development of this Markovian model will be based on condition data of roofing components and systems collected throughout Canada using "ROOFER' [12] as a data acquisition system, in which many roofs, (grouped according to their age, material type, geographical location, and operating conditions) will be inspected at two or more points in time.

5 Conclusions

This paper illustrates the possibility of modeling the performance of roofing envelope systems using a discrete Markovian stochastic process that takes into account the time variance and randomness of this performance. The transition probability matrices for roof components and systems will be derived using in-field inspection data, for roofs grouped according to the controlling explanatory variables. This probabilistic modeling of performance enables a rational evaluation of the actual risk of roof failure that depends on the probability of failure of the entire roof system (membrane, insulation, flashing, and vapor barrier) and the consequences of this failure. The consequences of roof failure depend on the cost of repair, value and vulnerability of the building contents under the roof, and cost of disruption. This time-dependent probabilistic model constitutes the first phase towards the development of an effective risk-based roofing maintenance management system at either project- or network-levels that minimizes the life-cycle cost of the roof including the costs of maintenance, repair, replacement and failure.

6 References

1. Kane, K., (1996) NRCA's Market survey indicates business was better than expected, *Professional Roofing* 26(3), pp. 16-20
2. Anonymous (1995) CRCA conducts roofing survey, *Canadian Roofing Contractor*, July, pp.23-26.
3. Cash, C.G. (1980) Durability of bituminous built-up roofing membranes, in: *Durability of Building Materials and Components*, American Society for Testing and Materials STP 691, Sereda, P.J, and Litvan, G.G. eds., pp. 741-754.
4. Griffin, C.W. (1982) *Manual of Built-up Roof Systems.* 2nd Ed., McGraw-Hill.
5. Bogdanoff, J.L., and Kozin, F. (1985) *Probabilistic models of cumulative damage.* J. Wiley & Sons, New York.
6. Ross, S.M. (1996) *Stochastic processes,* J. Wiley & Sons, 2nd Ed., New York.
7. Saito, M. (ed.) (1997) Infrastructure condition assessment -Art, science, practice, *Proc. ASCE Conference,* American Society of Civil Engineers, Boston.
8. Vanier, D.J., and Lacasse, M.A. (1996) BELCAM project: service life, durability and asset management research, in: *Durability of building materials and components 7,* V.2, pp.848—856.
9. Baskaran, B.A. (1996) Roofing systems and their performance requirements, in: *Building Better roofs,* NRCC/IRC Seminar, 20p.
10. Cash, C.G., and Bailey, D.M. (1993) Predictive service life tests for roofing membranes: Phase 1. *USACERL Interim Report FM-94/03*.
11. ASTM E632 (1988) Standard practice for developing accelerated tests to aid prediction of the service life of building components and materials, *American Society for Testing and Materials,* V4.07, ASTM, West Conshohoken, USA.
12. Bailey, D.M., Brotherson, D.E., Tobiasson, W., and Knehans, A. (1989) ROOFER: an engineered management system (EMS) for bituminous built-up roofs. *USACERL Technical Report M-90/04.*
13. Masters, L.W. (1987) Service life prediction: a state of the *art,* in: *Durability of building materials and components, Singapore,* pp. l-9.