MULTI CRITERIA DECISION MAKING AND THE PRINCIPLE OF REFLECTION

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ABSTRACT: Apart from the methodology constructed by Barzilai (2005), the methodologies underlying multi criteria decision making tools lack the foundation to allow for the application of the mathematical operations of addition and multiplication. Consequently, numerical output derived using these methodologies has no meaning. In this paper I will demonstrate this problem using two examples. It will be shown that the methodologies that underlie the decision process in these examples lack the required foundation for the application of mathematical operations. Reference will also be made to a methodology that does have a foundation and that enables the mathematical operations of addition and multiplication. The result of this paper can be of use for future projects involving multi criteria decision making.

Keywords – Multi criteria decision making, operations research.

1. INTRODUCTION

In this paper I will show that the methodologies underlying two cases of Multi Criteria Decision Analysis (MCDA) of projects lack a foundation required for the construction of a mathematical system which is a reflection of the empirical system, namely, the preferences stakeholders have for different (design) alternatives. I will also give reference to a methodology that does have a foundation and does allow for the mathematical operations of addition and multiplication.

2. MULTI CRITERIA DECISION ANALYSIS AND MEASUREMENT

Although I assume that the reader is somewhat familiar with MCDA, I will start with a brief description of MCDA. MCDA is aimed at supporting a decision maker or a group of decision makers in choosing, from a number of decision alternatives, the most preferred alternative. Usually different properties (decision criteria) of the decision alternatives are taken into account. MCDA involves measuring the decision maker's preferences per alternative per criteria. This paper focuses on this measuring procedure.

Determining the decision maker's preference per alternative per criteria is a measuring process involving the use of scales. Because there are requirements with regards to the scale type used for measuring the preference per criteria in order to allow for mathematical operations, the next section will focus on the following two issues concerning MCDA:

- The application of mathematical operations;
- The types of scales used in measuring.

3. STEVENS' SCALE TYPES

As stated before, determining the decision maker's preference per alternative per criteria is a measurement process involving the construction of scales. The purpose of measurement is to enable the application of mathematical operations to the objects under measurement. Measurement scales can be classified by the mathematical operations that are enabled on the resultant scales and scale values. Although several scale types have been identified in the literature of classical measurement theory, it will suffice for the purpose of this paper to focus on the following four types of measurement as proposed by Stevens (1946):

1. Nominal measurement;
2. Ordinal measurement;
3. Interval measurement;
4. Ratio measurement.

Nominal measurement involves assigning numerals to objects as labels or names. This measurement procedure only allows comparisons between objects on equality or inequality.

Ordinal measurement involves assigning numbers to objects to represent the rank order (1^{st}, 2^{nd}, 3^{rd}, etc.) of the objects measured. This measurement procedure allows 'less than' or 'greater than' comparisons.

Interval measurement assigns numbers to objects that have all the features of ordinal measurement but also, equal differences between measurements represent equivalent intervals. Measuring the temperature of objects using the Celsius or Fahrenheit scale are examples of interval measurement. The zero point on interval scales is arbitrary.

Ratio measurement assigns numbers to objects in a similar way as interval measurement but the zero value is non-arbitrary. Physical measurements such as mass, or length are measured using ratio scales.

The operations of addition and multiplication are not applicable to measurement scales as described by Stevens, including to ratio and interval scales. In fact, these operations are not applicable to any measurement scales that are based on the models of classical measurement theory. For a construction of scales that permit these operations see Barzilai (2005).

In MCDA projects ordinal scales nearly always play a role. There is, however, a problem with ordinal scales when used in MCDA. They lack a foundation that makes further mathematical operations possible. Such operations would result in modelling errors.

I would like to illustrate this problem using the following two cases.

4. FIRST CASE: THE 'RIJNLANDROUTE'

Consider the 'Rijnlandroute', a project aimed at linking two Dutch motorways. There are several possibilities where these two motorways could be linked. Figure 1 shows one such possibility. The project's design team considered six alternative ways to link the motorways. In order to make a choice between the different alternatives the project team took into account different criteria like investment costs, travel time gain, emissions, etc. The results of the study are summarised in Table 1. The table shows that the measurement makes use of different scale types and units. Some measurements make use of a ratio scale and are expressed in monetary units, and some make use of an ordinal five-point scale.

In order to make a choice based on the different alternatives, the design team must arrange the alternatives in order of preference, thus allowing it to select the one that is most preferable.

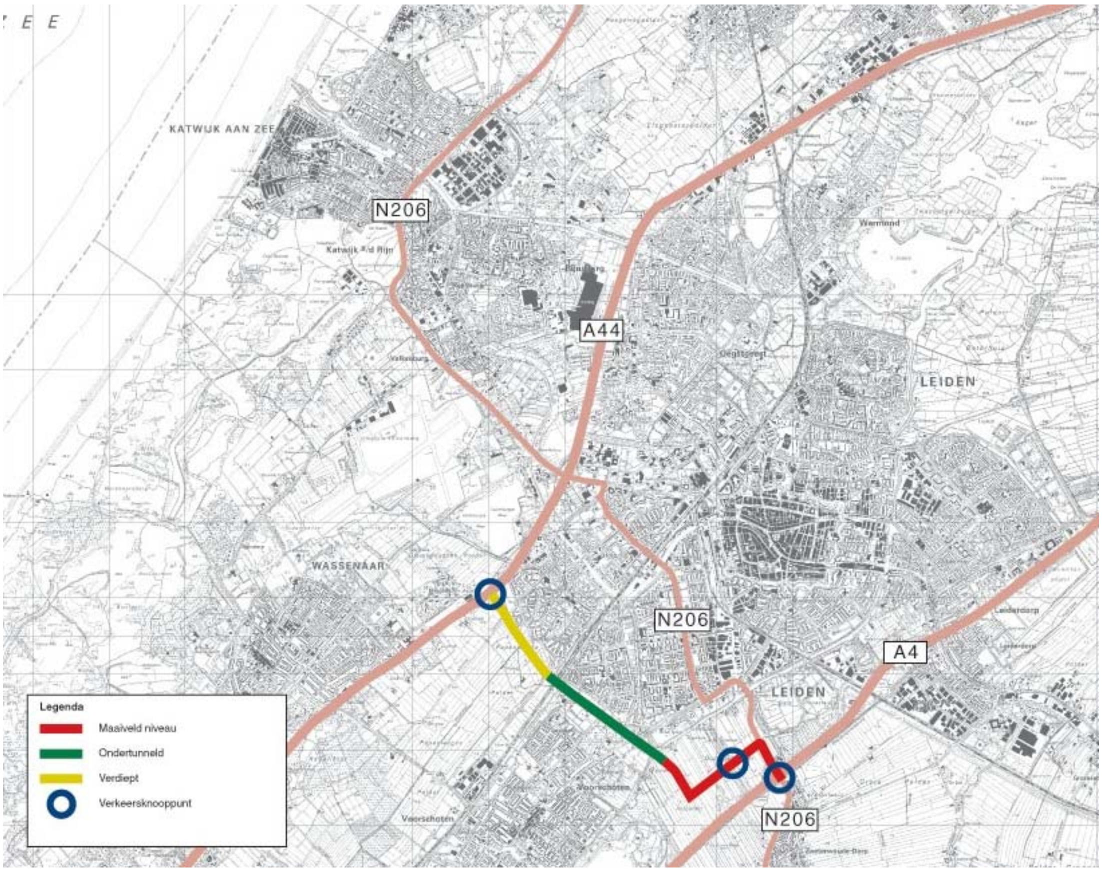

Figure 1. Map showing one of the alternatives to link the two motorways

Table 1. The different alternatives to link the two motorways and the criteria considered

	Alternatives					
Criteria	1	2	3	4	5	6
Investment costs [M€]	520	464	779	377	293	261
Maintenance costs [M€]	166	148	249	120	94	83
Exit value [M€]	57	51	85	41	32	29
Traveltime gain	538	661	815	1347	1165	1731
Shipping reliability gain	11	14	17	28	24	36
Spatial development	++	++	++	++	++	++
Internal safety	0	0	0	0	0	0
External safety	0	0	0	+	+	+
Emissions	-	0	0	-	0	-
Noise	--	--	0	-	-	-
Nature and landscape	-	0	0	-	-	-

The measurements regarding the criteria ‘spatial development’, ‘internal safety’, ‘external safety’, ‘emissions’, ‘noise’ and ‘nature and landscape’ are made, using a five-point scale (--

/-/0/+/++). So, in essence, these measurements are using a symbolical representation that is only *order* preserving, so-called ordinal scales, and cannot be used in further mathematical operations. This could mean two things:

- The empirical system is ordinal and therefore, if the mathematical system is a reflection of it, then the mathematical system should be ordinal too;
- The empirical system is not ordinal and therefore the mathematical system should not be ordinal either.

Some of the measurements, for instance the measurements regarding the emissions, were originally expressed using a ratio scale (NO_x concentrations measured in $\mu g/m^3$). In the final table of the study, however, these measurements were made using an ordinal scale thereby reducing the information that can be gathered from them and the possible mathematical operations that can be carried out upon them. Therefore it is very likely that the empirical system is not ordinal and that the mathematical system should not be ordinal either. It can therefore be concluded that the methodology used in this project cannot be used as a proper base for multi criteria decision making because it does not fulfil the basic requirements that make the construction of a mathematical system possible that is a *reflection* of the empirical system (the Principle of Reflection).

5. ANOTHER EXAMPLE: THE DYNAMIC ACTOR NETWORK ANALYSIS TOOL

The Dynamic Actor Network Analysis (DANA) is a software package aimed at supporting policy analysts in some policy situation:

> "The aim of the DANA project is to construct a workbench to support policy analysts in their representation and analysis of information on actors (organizations, stakeholder groups, or individuals) that play a role in some policy situation.
>
> The design of the workbench is largely determined by the underlying method of actor network analysis. This method (dynamic actor network analysis = DANA) leads the analyst to think in terms of actors who all have their own problem perception. By making these perceptions explicit in a qualitative, conceptual language and then perform different types of comparative analysis, the analyst sharpens her insight not only in the policy situation at hand, but also in her own reasoning (analyst as reflective practitioner). The representations of actor perceptions may also serve as (organizational) memory and as a basis for discussion amongst analysts and/or actors." (www.dana.tudelft.nl)

The use of the words 'qualitative' and 'comparative' points to the use of ordinal scales for measurement. If this is the case then the mathematical operations that are enabled on the resultant scales and scale values are limited.

In its measurement procedure the method makes use of a seven-point scale. The use of this seven-point scale is only order preserving, because it only makes use of a similarity of that property and symbols, not numbers. As mentioned before, these ordinal scales limit the mathematical operations that can be carried out upon them.

The maker of DANA acknowledges this and uses the term 'semi-quantification' which is very confusing. Quantification is a variable binding operation and one cannot 'semi-bind' a variable. Although there is no foundation for carrying out extraneous operations on ordinal scales and scale values, DANA does internally convert ordinal measurements into numerical values and uses these values to compute the required analysis results. Table 2 shows how the ordinal scale is translated into numeric values. Because this conversion is based upon ordinal scale values it is in fact a modelling error and does not permit the mathematical operations of addition and subtraction. The resulting numerical output is therefore meaningless.

One of our master students used DANA and concluded that DANA was very useful in identifying the different stakeholders and the way they are linked. The numerical output, however, suffers from inconsistencies and was not useful.

Table 2. The ordinal scale used in DANA and the corresponding numeric values

	Changes	Multipliers	Utilities
---	-4	-2	-4
--	-2	-1	-2
-	-1	-½	-1
0	0	0	0
+	1	½	1
++	2	1	2
+++	4	2	4

6. CONCLUSIONS

In this paper I showed that two current multi decision making methodologies lack a foundation that enables the construction of a mathematical system that is a reflection of the empirical system. The use of ordinal scales in both examples prohibits further mathematical operations and therefore prohibits the construction of a mathematical system.

However, this does not mean that it is not possible to construct a mathematical system that is a reflection of the empirical system. Barzilai (2005) constructed a strong model to which the operations of addition and multiplication are applicable.

7. REFERENCES

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