

## Short Report

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### Title

„Global stability of highly stressed shear-deformable anisotropic laminate and sandwich beams under combined loads in civil engineering“

### Initial Situation

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The stability behavior of composite laminated beams still poses a challenge in terms of fast and reliable analysis methods that can be used with ease in engineering practice. All analysis methods that were developed in this research project take the typical characteristic coupling effects and transversal shear deformations into account explicitly which necessitated the employment of higher-order shear deformation theories.

### Research Topics

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The stability behavior of isotropic beams structures has been largely understood whereas the buckling behavior of shear-deformable anisotropic composite laminated beams still poses a number of open questions the treatment of which has been the primal motivation for this research project. The main aspect was the development of innovative, fast and yet reliable analysis methods that can be used for the global stability analysis of laminated beams (such as Euler-type buckling, flexural-torsional buckling, lateral buckling) as well as for the assessment of the interactive buckling behavior, i.e. the simultaneous occurrence of global and local buckling modes. The analysis methods to be developed are supposed to include the characteristic laminate coupling effects as well as transverse shear deformations wherein the latter aspect requires the use of higher-order shear deformation theories.

After establishing the theoretical basics as required in this research project (chapter 2 of the final report), chapter 3 is devoted to the construction and design of a reference part in the form of a hall frame consisting of composite laminated members. Chapter 4 includes an in-depth discussion of the effective properties of laminated beams, here especially extensional, bending and torsional stiffness properties. A comparison with FEM simulations shows that the analytical results and the numerical predictions match very well and can be used with confidence. Chapter 5 of the report is devoted to the consideration of global stability cases of laminated beam structures wherein the major focus was placed on the applicability of classical textbook formulae for the buckling assessment of laminated beams. It was shown that classical established formulae can be applied if the effective properties of the laminated beams are taken into account properly.

The topic of the local buckling behavior of composite laminated beams (i.e. the local buckling of flanges and webs of prismatic beams) is documented in chapters 6-9 of the report wherein use was made of the so-called discrete plate analysis approach (Fig. 1). Therein, the segment under consideration (i.e. a flange or a web) is separated from the beam and treated as a composite plate where at the cutting edges elastic rotational restraints are applied representing the supporting influence of the adjacent beam segments. Chapter 6 includes the development and implementation of a Lévy-type solution for such elastically restrained composite laminated plates. Such solutions are of an implicit nonlinear nature and as such not applicable in practical engineering work. However, such solutions are exact in the framework of the underlying theory and thus of high value for the generation of reference results and for the verification of approximate methods. A simple buckling solution in the form of a Rayleigh-quotient is included in chapter 7 of the final report. This kind of analysis uses an energy principle in conjunction with simple shape functions for the buckling modes, eventually enabling closed-form solutions for the bifurcational buckling loads of the elastically restrained beam segments. It was shown that such analysis approaches work with high accuracy but negligible computational effort when compared to full-scale FEM simulations (Fig. 2). An extension of this method can be found in chapter 8 where the Ritz-method is documented for the buckling analysis of shear-deformable laminated beam segments with arbitrary layups under complex loading conditions. Chapter 8 thus constitutes the basis for the investigations included in chapter 9 where a unified buckling approach for entire beams is described which is based on Ritz-approaches for all individual beam segments which are subsequently unified by the use of Lagrangian multipliers. This analysis approach was also proven to work with high accuracy at a fraction of the computational costs of comparative FEM simulations (Fig. 3).

### Summary and Conclusions

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The global stability problem of composite laminated beams has been treated completely. It was shown that classical engineering formulae concerning global stability problems can be employed also for composite beams as long as the effective properties of the composite beams are taken into account. The local buckling problem has been treated completely as well by using a variety of analysis approaches wherein besides simple closed-form analytical methods Lévy-type solutions were developed as well. A semi-

analytical approach was developed by using the Ritz-method for laminates with arbitrary layups which was shown to perform with superior computational efficiency and high accuracy. Due to time restrictions, however, the interactive stability case could not be tackled in this project. This also holds for the initially planned validation experiments. The research results contained in the final report, however, will serve as the basis for these aspects of the research topic.

## Key Project Data

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Short title: Global stability of laminate and sandwich beams

Researchers / Project management:

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Univ.-Prof. Dr.-Ing. habil. Christian Mittelstedt, TU Darmstadt (previously SOGETI)

Maxime Miget (Administration)

Total cost: 289.756,81 € €

Partial funding by the state: 202.829,77 €

Project duration: 24 months

# Figures:

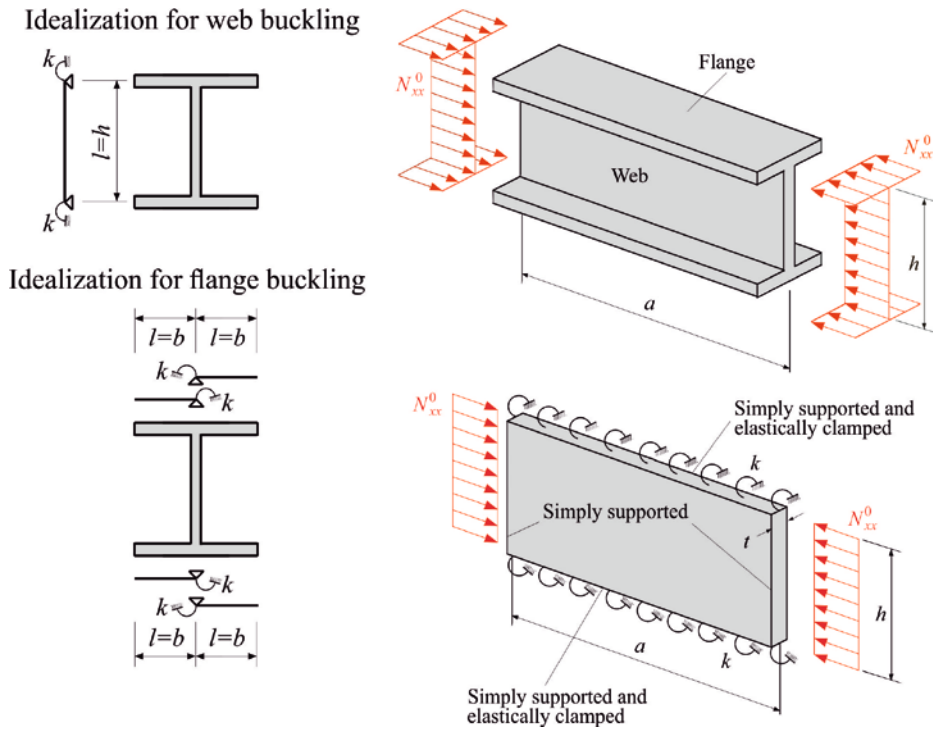


Fig. 1: DiscretePlateAnalysisEnglish.jpg  
 Local buckling analysis considering elastically restrained laminated composite plates (left) at the example of an I-beam (upper right), substitute model for the web of an I-beam (lower right).

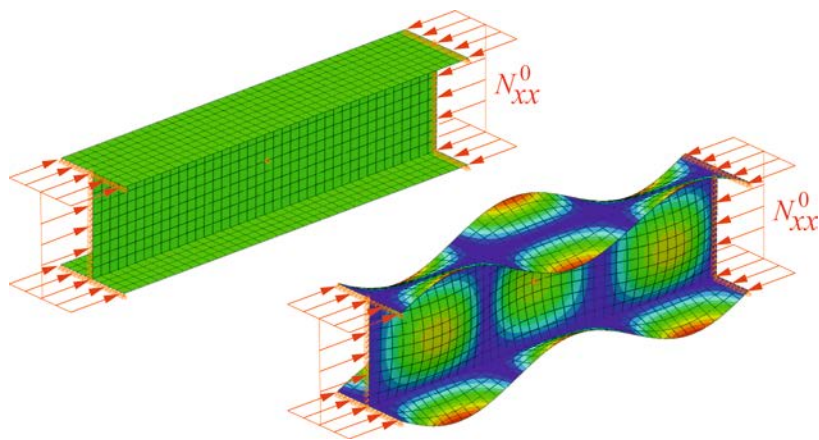


Fig 2: FEMEnglish.jpg  
 A typical FEM model for buckling analysis (left), local buckling mode under uniaxial compression (right).

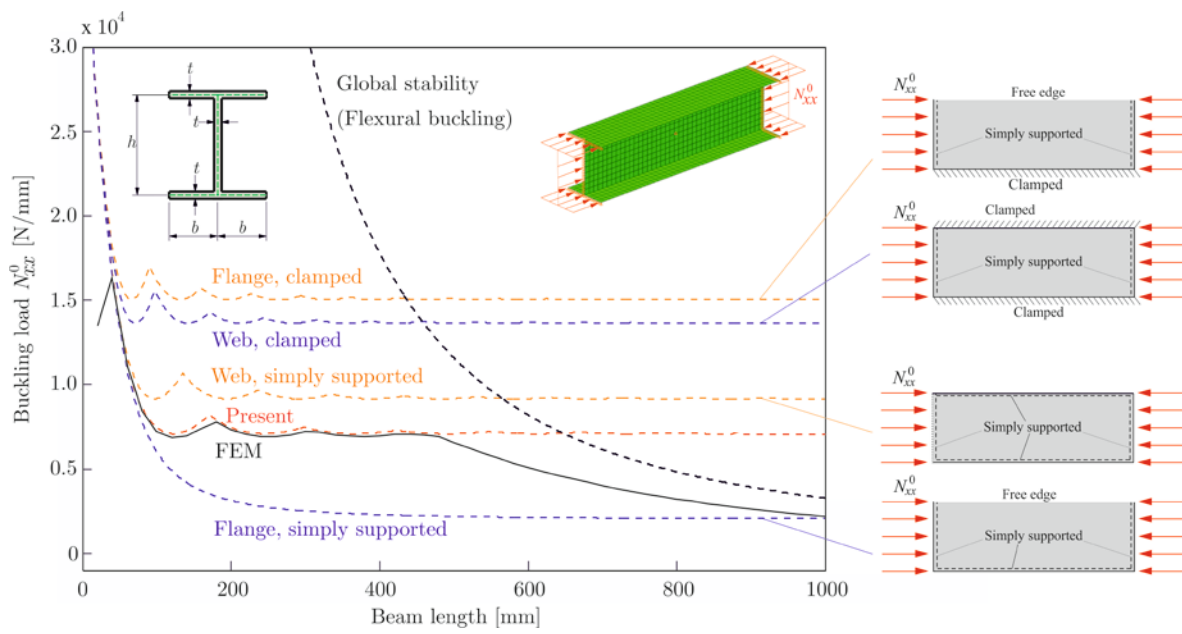


Fig. 3: MixedModeBuckling.jpg

Sample results of the analysis method according to chapter 9 of the final report; comparison with results by the Ritz-method for the limit cases "simply supported" and "clamped" and with FEM results.