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Sichtung und Aufbereitung der Kenntnisse zur Schalenstabilität für die Teile 3 und 4 des Eurocode 3

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Postfach 80 04 69
70504 Stuttgart

Nobelstraße 12
70569 Stuttgart

Telefon (07 11) 9 70 - 25 00
Telefax (07 11) 9 70 - 25 08

E-Mail irb@irb.fraunhofer.de

www.baufachinformation.de

Universität
Gesamthochschule
Essen

Fachbereich 10
Bauwesen

Stahlbau
Prof. Dr.-Ing. Herbert Schmidt

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Sichtung und Aufbereitung der Kenntnisse zur Schalenstabilität für die Teile 3 und 4 des Eurocode 3

Schlußbericht
zum DIBt-Forschungsvorhaben
IV 1-5-782/95

Inhalt:
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Essen, 24.01.1997

Dieser Bericht hat 2 Seiten Text und 27 Blatt Anlagen.

1 Zielsetzung des Vorhabens

Zur Schalenstabilität gab es bis Ende der 70er Jahre in der Bundesrepublik Deutschland kein allgemein gültiges Grund-Regelwerk. Erst die 1980 erschienene DASt-Richtlinie 013 [1] schloß diese Lücke. Ihr folgte 1990 die Stahlbau-Grundnorm DIN 18800 Teil 4 [2], die inzwischen von der Stahlbaupraxis weitgehend angenommen worden ist. Da mit den Regeln in DIN 18800-4 aber noch nicht alle Stabilitätsfälle schalenartiger Stahlkonstruktionen bearbeitet werden können, erschien 1992 als eine Art Ergänzung für spezielle Fälle der Entwurf für die DASt-Richtlinie 017 [3]. Sie steht demnächst zu einer Bestandsaufnahme, Überarbeitung und Erweiterung an.

Etwa zeitparallel zur DASt-Ri 013 waren auf europäischer Ebene die ECCS-Recommendations "Buckling of Steel Shells" erarbeitet worden und in erster Ausgabe 1980 erschienen. Sie sind vom Ansatz her mit DASt-Ri 013 vergleichbar; die derzeit letzte Ausgabe (4th edition) stammt von 1988 [4].

Im Rahmen der Arbeiten am Eurocode 3 war vorgesehen, für die Teile, in denen das Schalenbeulen einer der zu beachtenden Grenzzustände sein würde (Part 3: Chimneys, Masts, Towers; Part 4: Silos, Tanks, Pipelines), einen gemeinsamen Anhang zur Schalenstabilität zu erarbeiten. Inzwischen wurde dieser auf die Gesamtthematik "Strength and Stability of Steel Shell Structures" erweitert und wird derzeit als Grundnorm EC3-Part 1.6 konzipiert. Der Linksunterzeichnende ist Mitglied in beiden EC-Project Teams und Sprecher einer vierköpfigen Ad-hoc-Gruppe, die Part 1.6 erarbeitet.

Da hier im Rahmen der EC3-Arbeit das Thema Schalenstabilität erstmals angefaßt wurde, mußte zunächst Vorarbeit geleistet werden. Sie bestand darin,

(a) alle international einschlägigen Regelwerke zur Schalenstabilität in ihren derzeit gültigen Versionen und

(b) alle seit 1992, d.h. seit Erscheinen der E-DASt-Ri 017, neu erschienenen Veröffentlichungen über Schalenstabilität

zu sichten und auf ihre Verwendbarkeit für die geplante EN-Grundnorm "Strength and Stability of Steel Shell Structures" zu überprüfen. Hierüber wird nachfolgend in knapper Form berichtet.

2 Sichtung von Regelwerken

Anlage 1 enthält die Liste der uns bekannten Regelwerke, die nennenswerte Abschnitte zur Schalenstabilität enthalten. Es wurde zunächst sichergestellt, daß uns die jeweils neuesten Ausgaben dieser Regelwerke vorliegen. Die Regelwerke wurden dann erneut daraufhin durchgesehen, ob sie bisher nicht beachtete Ansätze, Rezepte oder Regeln enthalten, die für die Eurocode-Arbeit beachtet werden sollten.

3 Sichtung neuerer Literatur

Anlage 2 enthält eine Literaturliste zur Schalenstabilität, die mit Arbeiten aus dem Jahre 1993 beginnt. Das ist das Erscheinungsjahr der beiden Arbeiten [30] und [47], denen der seinerzeitige Essener Wissensstand über die "regelwerkrelevante" Schalenbeul-Literatur zugrundelag - in [30] über die Grundlagen zu DIN 18800-4 und in [47] über die Grundlagen zur E-DASt-Ri 017.

Die aufgelisteten Arbeiten wurden darauf durchgesehen, ob sie Vorschläge oder Erkenntnisse enthalten, die für die Eurocode-Arbeit beachtet werden sollten.

4 Arbeit an EC3-Teil 1.6

Anlage 3 enthält einen Auszug aus dem CEN/TC250/EC3-Document N630E, das als "2nd draft" den Bearbeitungsstand des Teils 1.6 "Strength and Stability of Steel Shell Structures" im August 1996 wiedergibt. Der Auszug enthält - neben dem Gesamtinhaltsverzeichnis - die beiden Teile (Chapter 7 und Annex D), die sich primär mit der Schalenstabilität befassen.

Das Document wurde von der eingangs erwähnten Ad-hoc-Gruppe unter der Federführung des Linksunterzeichnenden erarbeitet. Alle aus den unter Ziff. 2 und 3 beschriebenen Sichtungsarbeiten gewonnenen Erkenntnisse sind, soweit relevant, in die Arbeit am "2nd draft" eingeflossen.

Bearbeiter:

(Univ. Prof. Dr.-Ing. H. Schmidt)

(Dipl.-Ing. D. Velickov)

Regelwerke zur Schalenstabilität

[R1] DASt-Ri 013: Beulsicherheitsnachweis für Schalen. Hrsg.: Deutscher Ausschuß für Stahlbau. Köln: Stahlbau-Verlag, 1980.

[R2] DIN 18800-4: Stahlbauten. Stabilitätsfälle, Schalenbeulen. Hrsg.: Deutsches Institut für Normung. Berlin, Köln: Beuth Verlag, November 1990.

[R3] Entwurf DASt-Ri 017: Beulsicherheitsnachweis für Schalen - spezielle Fälle -. Hrsg.: Deutscher Ausschuß für Stahlbau. Köln: Stahlbau-Verlag, 1992.

[R4] ECCS-Recommendations: Buckling of Steel Shells - European Recommendations. 4th ed. Brussels: ECCS General Secretariat, 1988.

[R5] AD-Merkblätter: a) B2: Kegelförmige Mäntel unter innerem und äußeren Überdruck. Hrsg.: AD. Berlin: Heymanns-Beuth, Januar 1995. b) B3: Gewölbte Böden unter innerem und äußerem Überdruck. Hrsg.: AD. Berlin: Heymanns-Beuth, Oktober 1990. c) B4: Tellerböden. Hrsg.: AD. Berlin: Heymanns-Beuth, Mai 1991. d) B6: Zylindrische Mäntel unter äußerem Überdruck. Hrsg.: AD. Berlin: Heymanns-Beuth, Januar 1995.

[R6] API 2A: Recommended Practice for Planning, Designing and Constructing. Fixed Offshore Platforms - LRFD. Hrsg.: API. Dallas, Oktober 1993.

[R7] ASME Boiler and Pressure Vessel Code - Section III, Code Case N-284. Hrsg.: ASME. New York, 1995.

[R8] BS 5500: Specification for Unfired Fusion Welded Pressure Vessels. Hrsg.: BSI. London, 1988.

[R9] CODAP: Code Francais de Construction des Appareils a Pression non soumis a l`Action de la Flamme. Hrsg.: SNCT. Paris, 1990.

[R10] DNV-Rules: a) Classification Notes - Buckling strength analysis of mobile offshore units, Note No. 30.1. Hrsg.: DNV. Hovik, Mai 1992. b) Rules for Certification - Diving Systems. Hrsg.: DNV. Hovik, 1988.

[R11] ÖNORM B 4650-4,5,7: Stahlbau. Beulung von Kreiszylinderschalen, Beulung von Kreiszylinderschalen mit abgestufter Wanddicke. Hrsg.: ÖN. Wien, 1977/1980.

[R12] TGL 19348: Örtliche Stabilität gekrümmter Flächentragwerke. Hrsg.: TGL. Leipzig: Verlag für Standardisierung, 1965.

[R13] API Standard 650: Welded Steel Tanks for Oil Storage (9th ed.). Hrsg.: API. Dallas, Juli 1993.

[R14] BS 2654: Manufacture of Vertical Steel Welded Storage Tanks with Butt-Welded Shells for the Petroleum Industry. Hrsg.: BSI. London, September 1989.

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[3] *Calladine C. R.*: Understanding imperfection sensitivity in the buckling of thin-walled shells. Thin-Walled Structures, vol. 23 (1995) no. 1-4, p. 215 - 235.

[4] *Chen, Q./ Kulak, G. L./ Elwi, A. E.*: Flexural tests of longitudinally stiffened fabricated steel cylinders. Can. J. of Constr. Steel Research, vol. 34 (1995) no. 1, p. 1 - 25.

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Anlage 8 zu DIBt-IV 1-5-782/9

Document CEN/TC 250/SC 3

N 630E

Whitegates
Blackacre Road
Theydon Bois
Essex CM16 7LT
England

Tel/Fax: 01992 812 542
Int: +44 1992 812 542

Secretariat of CEN/TC 250/SC3

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PT4 SILOS, TANKS AND PIPELINES

PART 1.6 : GENERAL RULES : SUPPLEMENTARY RULES FOR SHELLS (FORMERLY PART 1.Y)

SECOND DRAFT

[redacted]

Contents

Annex A (informative): Membrane theory stresses in shells

Annex B (normative): Additional formulas for plastic collapse

Annex C (informative): Formulas for linear elastic membrane and bending stresses

Annex D (normative): Formulas for buckling design

Annex E (normative): Fatigue details

Drafting note: For Annex E see Annex 3.2..x of EC-part 3.2.

7 Buckling Limit State (LS3)

7.1 Design values of actions

(1) All relevant extreme values of actions causing compressive membrane stresses or shear membrane stresses in the shell wall and their relevant combinations shell be considered.

7.2 Special definitions and symbols

7.2.1 Definitions

(1) **Critical buckling resistance:** The smallest bifurcation or limit load determined by elastic theory assuming the idealized conditions of perfect geometry, perfect load application, perfect support, material isotropy and absence of residual stresses (LA or GNA analysis).

(2) **Critical buckling stress:** The nominal membrane stress (nominal membrane stress resultant divided by t) associated with the critical buckling resistance.

(3) **Characteristic buckling stress:** The nominal membrane stress which, unlike the critical buckling stress, is reduced in order to cover the geometrical and structural imperfections and the nonelastic material behaviour inevitable in practical steelwork.

(4) **Design buckling stress:** The design value of the buckling stress, obtained by reducing the characteristic buckling stress by the partial safety factor for resistance.

(5) **Key value of the stress:** That vale of a stress field to be introduced into the LS3 assessment equation.

7.2.2 Symbols

(1) In general, the symbols defined in section 1.6 are valid. Differing or additional symbols are given below.

(2) **Stress resultant and stress quantities**

n_x, σ_x design values of the acting buckling-relevant meridional membrane stress resultant and stress, positive when compression

n_θ, σ_θ design values of the acting buckling-relevant circumferential membrane (hoop) stress resultant and stress, positive when compression

$n_{x\theta}$, τ design values of the acting buckling-relevant shear membrane stress resultant and stress

(3) **Structural stability parameters**

σ_{xRc}, $\sigma_{\theta Rc}$, τ_{Rc} critical buckling stresses

σ_{xRk}, $\sigma_{\theta Rk}$, τ_{Rk} characteristic buckling stresses

σ_{xRd}, $\sigma_{\theta Rd}$, τ_{Rd} design buckling stresses

$\bar{\lambda}$	non-dimensional shell slenderness parameter
α	elastic imperfection reduction factor (knockdown factor)
χ	stability reduction factor

7.3 Buckling-relevant geometrical tolerance

(1) Unless specific buckling-relevant geometrical tolerances are defined in the relevant EC3 application parts, the following tolerance limits shall be observed if LS 3 is one of the ultimate limit states to be considered.

(2) The out-of-roundness U shall satisfy the following condition where nom d is the nominal diameter and max d and min d are the actual maximum diameters (see fig. 7.1).

For $d \le 500$ mm:
$\Delta d = \max d - \min d \le \boxed{0.02}$ nom d.
For $500 \text{ mm} < d < 1250$ mm:
$\Delta d = \max d - \min d \le [\boxed{0.02} - \boxed{0.01}$ (nom d -500)/750] nom d.
For $1250 \text{ mm} \le d$:
$\Delta d = \max d - \min d \le \boxed{0.01}$ nom d.

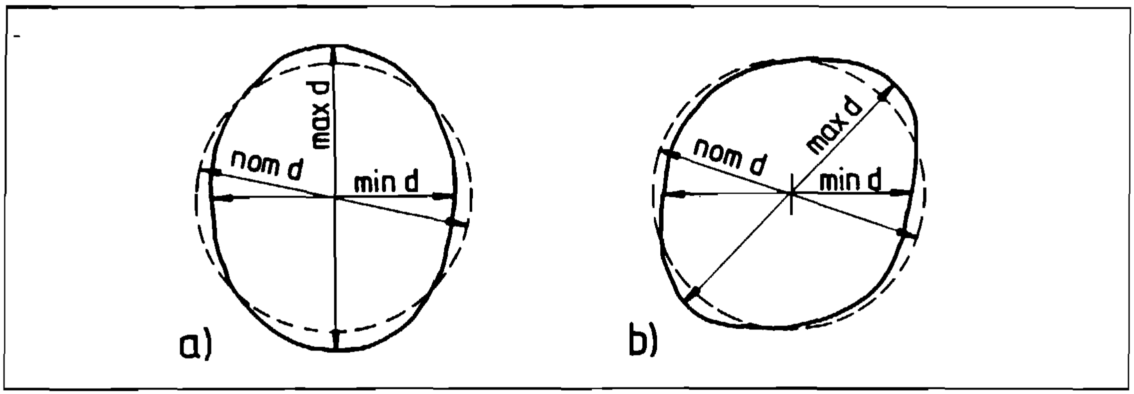

Figure 7.1: Measurement of out-of-roundness Δ d

(3) Accidental exxetricities e at joints in shell walls (see fig. 7.2) shall satisfy the following condition.
$e \le \boxed{0.2} \cdot t$ or $e \le \boxed{0.2} \cdot \min t$, but $e \le \boxed{3.0}$ mm

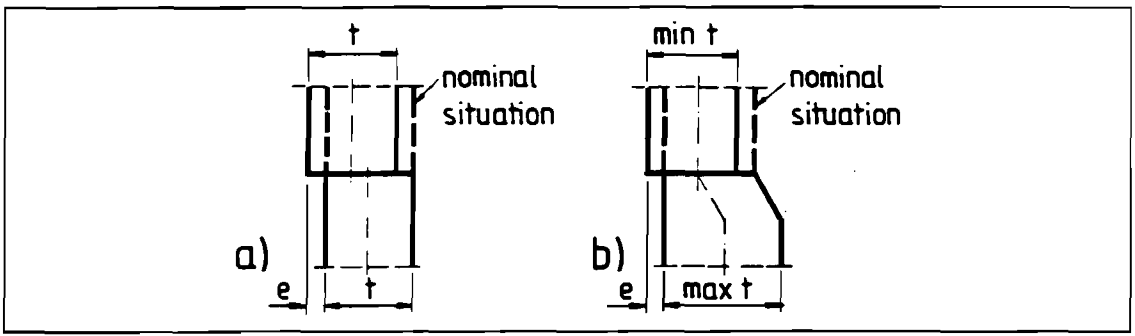

Figure 7.2: Accidental eccentricity e

(4) The depth Δ of initial dimples in the shell wall shall not be greater than 1% of the gauge length l_g, held anywhere against any meridian and against any parallel circle respectively (see ficure 7.3). The gauge length shall be taken as follows:

(a) In all cases, in meridional and in circum ferential direction:
$l_g = 4\sqrt{r \cdot t}$

(b) In case of circumferential compressive stresses and of shear stresses, alternatively in circumferential direction:
$l_{g\theta} = 2.3\, r / [\, (r/t)\, (r/t)^{0.5}\,]^{0.5}$, but not larger than r.

(c) In all cases, in addition across welds:
$l_g = 25\, t$ or $l_g = 25 \min t$, but not larger than 500mm.

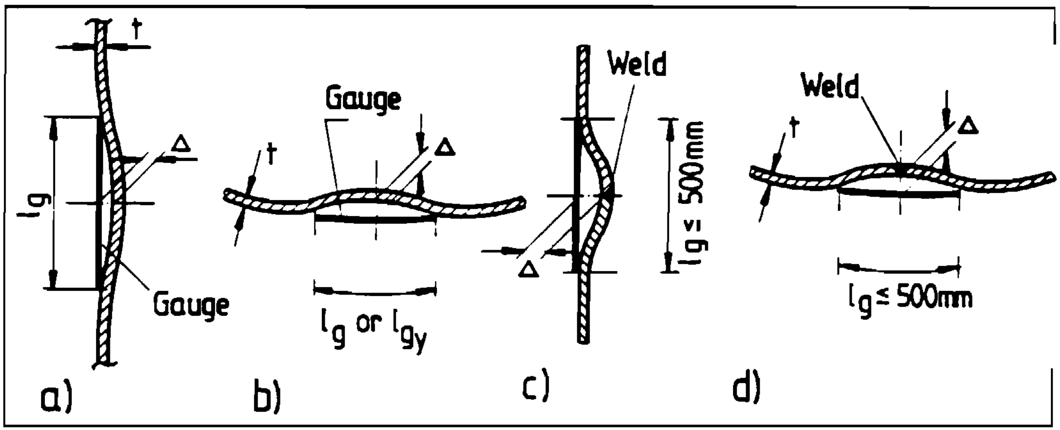

Figure 7.3: Measurement of depth Δ of initial dimples

(5) The deviations from the circular shape of a shell shall not be greater than 0.005 r.

(6) The deviations from the planeness of structures that supprot continouosly a shell (e.g. foundations) shall not have greater local slopes in circumferential direction than $\boxed{0.1}$ %.

(7) It shall be established by representative sample checks that the actual geometrical imperfections stay within the geometrical tolerances stated in the above items (2) to (6).

(8) If the actual geometrical imperfections do not comply with the geometrical tolerances stated in the above clauses (2) to (6), any correction steps, e.g. straightening, should be investigated and decided individually. Before a decision is made in favour of straightening, it should be kept in mind that this may, in fact, cause additional residual stresses. The degree to which the design buckling resistances are utilized should also be considered. In cases of doubt, the design engineer responsible for the structural analysis should be consulted.

7.4 Stress design

7.4.1 Design values of stresses

(1) The design values of stresses σ_{xSd}, $\sigma_{\theta Sd}$ and τ_{Sd} shall be taken as the keyvalues of compressive and shear membrane stresses as obtained by linear shell analysis (LA). If applicable, membrane theory may be used. This is generally the case under purely axisymmetric conditions (loading and supporting)

(2) If no specific rules are given for defining the key values of membrane stresses, the maximum values should be introduced.

(3) In some cases it is necessary to calculate the membrane stresses by geometrically nonlinear elastic analysis (GNA). These cases are defined in the relevant application rules in Parts 3.1, 3.2, 4.1, 4.2, 4.3.

(4) For basic loading cases the membrane stresses may be taken from Annex A or Annex C.

(5) In some case the key values of membrane stresses may be of fictituous type and larger than the real maximum values (see Annex D1.2).

7.4.2 Design values of resistance (buckling strength)

(1) The buckling resistance (buckling strength) is described by the buckling stresses as defined in section 7.2.1. The design buckling stresses shall be obtained from:

$$\sigma_{xRd} = \sigma_{xRk}/\gamma_M,\ \sigma_{\theta Rd} = \sigma_{\theta Rk}/\gamma_M,\ \tau_{Rd} = \tau_{Rk}/\gamma_M\ .$$

(2) The partial safety factor for resistance shall be taken from the relevant EC3 application parts.

(3) The characteristic buckling stresses shall be obtained by multiplying the characteristic yield stress by the stability reduction factors:

$$\sigma_{xRk} = \chi_x \cdot f_{y,k}, \sigma_{\theta Rk} = \chi_\theta \cdot f_{y,k}, \tau_{Rk} = \chi_\tau \cdot f_{y,k} / \sqrt{3}$$

(4) The stability reduction factors χ_x, χ_θ and χ_τ shall be determined as a function of the non-dimensional shell slenderness parameter $\bar{\lambda}$ from:

$$\chi = 1 \qquad \text{when} \quad \bar{\lambda} \le \bar{\lambda}_0$$

$$\chi = 1 - 0.6 \frac{\bar{\lambda} - \bar{\lambda}_0}{\bar{\lambda}_p - \bar{\lambda}_0} \qquad \text{when} \quad \bar{\lambda}_0 < \bar{\lambda} < \bar{\lambda}_p$$

$$\chi = \alpha / \bar{\lambda}^2 \qquad \text{when} \quad \bar{\lambda}_p \le \bar{\lambda}$$

where a is the elastic imperfection reduction factor and $\bar{\lambda}_0$ is the squash limit slenderness, both to be taken from the relevant sections in Annex D, and $\bar{\lambda}_p = (2.5\ \alpha)^{0.5}$.

(5) The non-dimensional shell slenderness parameters shall be determined from:

$$\bar{\lambda}_x = \sqrt{f_{y,k} / \sigma_{xRc}}, \qquad \bar{\lambda}_\theta = \sqrt{f_{y,k} / \sigma_{\theta Rc}} \qquad \bar{\lambda}_\tau = \sqrt{(f_{y,k} / \sqrt{3}) / \tau_{Rc}},$$

(6) The critical buckling stresses σ_{xRc}, $\sigma_{\theta Rc}$ and τ_{Rc} shall be obtained by means of the formulas in the relevant sections of Annex D.

7.4.3 Stress limitation (buckling strength verification)

(1) Although buckling is not a purely stress-initiated failure phenomenon, the buckling limit state is, within this section, defined by limiting the design values of membrane stresses.

(2) Depending on the loading and stressing situation, one or more of the following checks for the key values of single membrane stress components shall be carried out:

$$\sigma_{xSd} \le \sigma_{xRd}, \qquad \sigma_{\theta Sd} \le \sigma_{\theta Rd}, \qquad \tau_{Sd} \le \tau_{Rd}.$$

(3) If more than one of the three buckling-relevant membrane stress components are present under the actions under consideration, the following interaction check for the combined membrane stress state shall be carried out:

$$\left(\frac{\sigma_{xSd}}{\sigma_{xRd}}\right)^{k_x} + \left(\frac{\sigma_{\theta Sd}}{\sigma_{\theta Rd}}\right)^{k_\theta} + \left(\frac{\tau_{Sd}}{\tau_{Rd}}\right)^{k_\tau} \le 1$$

where σ_{xSd}, $\sigma_{\theta Sd}$ and τ_{Sd} are the interaction-relevant groups of the significant values of compressive and shear membrane stresses in the shell and $k_x = k_\theta = \boxed{1.25}$ and $k_\tau = \boxed{2.0}$.

(4) *If no specific rules are given for defining the interaction-relevant groups of membrane stress components, the maximum values should be introduced.*

7.5 Direct design

At present no formulas are provided for this limit state.

7.6 Design by global LA or GNA analysis

7.6.1 Design value of action

See 7.1 .

7.6.2 Design value of resistance

(1) The buckling resistance is described by those values of the action quantities (e.g. loads) that are, under the applied combination of actions, present at the buckling limit state. The design buckling resistance shall be obtained from:

$$R_d = R_k/\gamma_M$$

where R_k is the characteristic buckling resistance,
γ_M is the partial safety factor for resistance acc. to 7.4.2 (2).

(2) The characteristic buckling resistance shall be obtained from:

$$R_k = \chi_{ov} \cdot R_{pl}$$

where χ_{ov} is the overall shell stability reduction factor,
R_{pl} is the plastic reference resistance.

(3) The plastic reference resistance R_{pl} shall be obtained by linear shell analysis (LA) under the applied combination of actions. The action quantities F under which the buckling-relevant membrane stress resultants at any point of the shell firstly satisfy the yield criterion

$$n_x^2 - n_x n_\theta + n_\theta^2 + 3n_{x\theta}^2 = (t \cdot f_{y,k})^2,$$

shall be taken as plastic reference resistance R_{pl} (see fig. 7.4).

(4) The overall shell stability reduction factor χ_{ov} shall be obtained as a function of the overall shell slenderness parameter

$$\bar{\lambda}_{ov} = (R_{pl} / R_{cr})^{0.5}$$

where R_{cr} is the critical buckling resistance. The function $\chi_{ov} = f\ (\bar{\lambda}_{ov})$ shall be taken as in 7.4.2 (4).

(5) In order to determine the critical buckling resistance R_{cr}, an eigenvalue analysis shall be performed on the linearly calculated stress/deformation state of the geometrically perfect shell (LA) under the applied combination of actions. The lowest eigenvalue (bifurcation) defines the critical buckling resistance (see fig. 7.1).

(6) *It should be undoubtedly sure that the applied algorithm finds reliably the eigenmode which leads to the lowest eigenvalue. In cases of doubt it is recommended to calculate neighboured eigenvalues with their eigenmodes as well, in order to get a complete insight into the bifurcation behaviour of the shell. Also, the applied numerical tool should be authenticated against benchmark cases with physically similar buckling characteristics.*

(7) In some cases it is necessary to use geometrically nonlinear elastic analysis (GNA) to determine the resistance pair R_{pl} and R_{cr}. Such cases are:

- Conical and spherical caps if they are either shallow or supported displaceably,
- assemblies of cylindrical and conical partial shells with no ring stiffeners at the meridional breaks if loaded meridionally,

Drafting note: Further cases to be identified.

If in such cases no bifurcation happens before reaching the elastic maximum value (snap through load) the latter defines the critical buckling resistance (see fig. 7.4).

(8) The overall elastic imperfection factor α_{ov} and squash limit slenderness $\bar{\lambda}_{ov,0}$ which are to be introduced into the function $\chi_{ov} = f(\bar{\lambda}_{ov})$ according to 7.4.2 (4) shall consider the imperfection sensitivity of the particular shell buckling case.

(9) The values α_{ov} and $\bar{\lambda}_{ov,0}$ shall be conservatively determined by analogous conclusion from known shell buckling cases with physically similar buckling characteristics.

(10) If that is not possible without any reasonable doubt, specific tests have to be carried out.

(11) *If no specfic values α_{ov} and $\bar{\lambda}_{ov,0}$ are available according to (9) or (10), they may be taken from the case of an axially compressed cylinder (see Annex D, subsection D1.1.2.2, clause 1).*

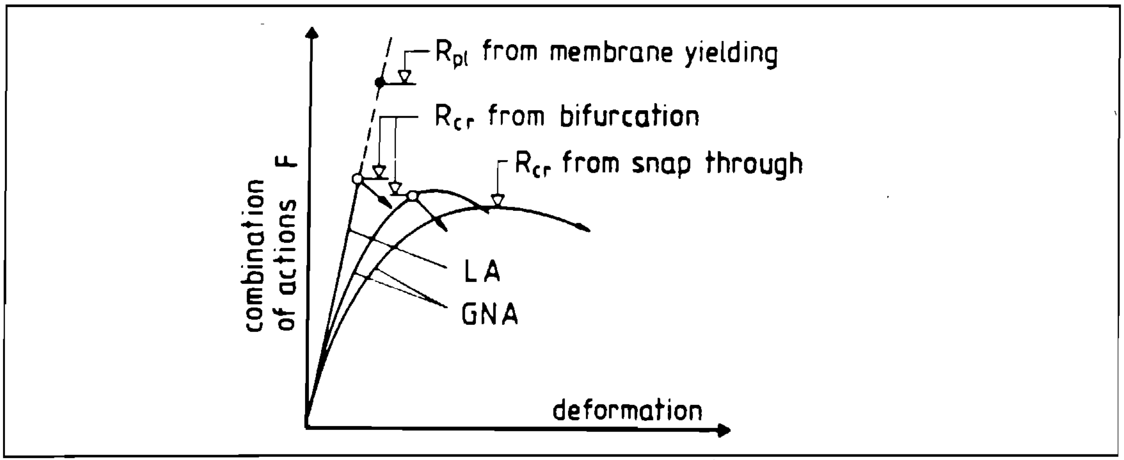

Figure 7.4: Definition of plastic reference resistance R_{pl} and critical buckling resistance R_{cr} from global LA or GNA analysis

7.6.3 Buckling strength verification

(1) It shall be verified that

$F_d \leq R_d$.

7.7 Design by global GMNA analysis

7.7.1 Design values of actions

See 7.1

7.7.2 Design value of resistance

(1) As in 7.6.2 (1).

(2) The characteristic buckling resistance shall be obtained from:

$$R_k = \alpha_{ov,GMNA} \cdot R_{cr,GMNA}$$

where $\alpha_{ov,GMNA}$ is the overall GMNA imperfection reduction factor,
$R_{cr,GMNA}$ is the critical GMNA buckling resistance.

(3) In order to determine the critical GMNA buckling resistance, a GMNA analysis of the geometrically perfect shell under the applied combination of actions, accompanied by an eigenvalue analysis, shall be performed. The critical buckling state is defined by either of the three following criterions (fig. 7.2):

(a) Maximum load of the load-deformation-curve (limit load),

(b) bifurcation load if occurring during the loading path before reaching the limit point of the load-deformation-curve,

(c) largest tolerable deformation if occurring during the loading path before reaching the bifurcation load or the limit load

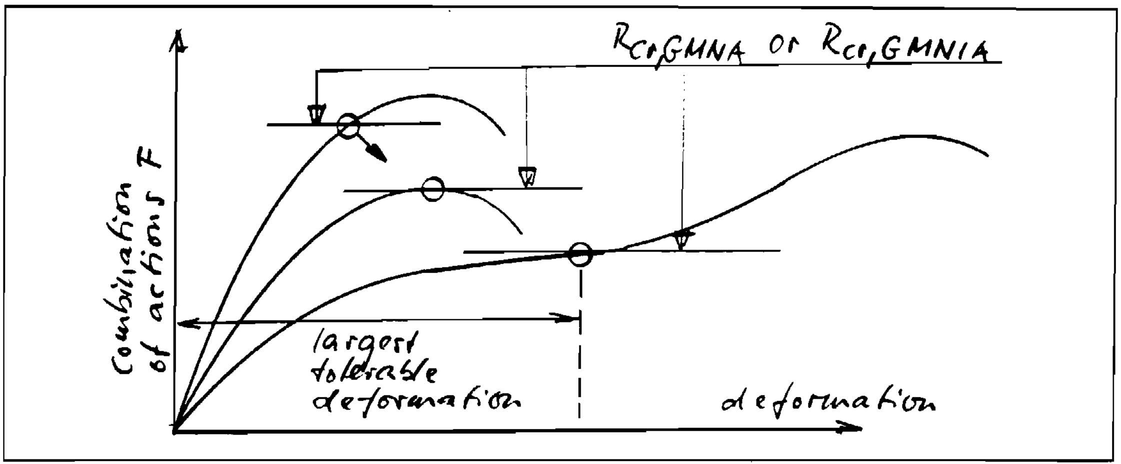

Figure 7.5: Definition of buckling resistance from global GMNA or GMNIA analysis

(4) The overall GMNA imperfection reduction factor $\alpha_{ov,GMNA}$ shall consider the imperfection sensitivity of the particular shell buckling case.

(5) *The determination of the value* $\alpha_{ov,GMNA}$ *shall be analogously to 7.6.2 (9)-(11)*

7.7.3 Buckling strength verification

(1) It shell be verified that

$$F_d \leq R_d.$$

7.8 Design by global GMNIA analysis

7.8.1 Design values of actions

See 7.1 .

7.8.2 Design value of resistance

(1) As in 7.6.2 (1).

(2) The characteristic buckling resistance shall be obtained from:

$$R_k = k_{GMNIA} \cdot R_{cr,GMNIA}$$

where $R_{cr,GMNIA}$ is the critical GMNIA buckling resistance,
k_{GMNIA} is a calibration factor.

(3) As in 7.7.2 (3), replacing GMNA by GMNIA.

(4) In the GMNIA analysis appropriate allowances shall be incorporated to cover the effects of practically unavoidable imperfections, including:

(a) geometric imperfections, such as:

- deviations from the nominal geometric shape of the midsurface (predeformations, out-of-roundness),
- irregularities at welds (minor excentricities, shrinkage kinks),

- deviations from nominal thickness,
- lack of evenness of supports;

(b) material imperfections, such as:
- residual stresses because of rolling, pressing, welding, straightening etc.,
- inhomogenities and anisotropies.

Further possibly negative influences on the critical GMNIA buckling resistance, such as flexibilities of connections or of supports and ground settlements, are no imperfections in the sense of these rules.

(5) Imperfections shall be allowed for in the GMNIA analysis by including appropriate additional quantities in the analytical shell model for the numerical computation.

(6) If no better method is known, the imperfections shall be allowed for by means of equivalent geometric imperfections in the form of initial shape deviations perpendicular to the midsurface of the perfect shell. Superposing the equivalent geometric imperfections to the perfect shape yields the midsurface of the equivalent-geometrically imperfect shell.

(7) The pattern of the equivalent geometric imperfections shall be chosen of such a kind that it influences the buckling behaviour most unfavourably. If the most unfavourable pattern cannot be identified without any doubt, the analysis shall be repeatedly carried out with varied imperfection patterns.

(8) *One unfavourable pattern is, as a rule, the critical buckling mode that yields the elastic critical buckling resistance* $R_{cr,GNA}$ *of the perfect shell (see 7.6.2 (7)). This is called the eigenmode-affine pattern.*

(9) The pattern of the equivalent geometric imperfections shall, if relevant, be adapted to the constructional detailing and to the boundary conditions in an unfavourable manner. However, patterns which may, because of fabricating or manufacturing reasons, be reliably excluded, need not to be allowed for.

(10) *Adapting to constructional detailing could, for example, mean that along a ring-weld an axisymmetric shrinkage necking is superimposed to the eigenmode-affine pattern.*

(11) The sign of the equivalent geometric imperfections shall be chosen in such a manner that the maximum initial shape deviations are unfavourably oriented towards the centre of the shell curvature.

(12) The maximum size of th equivalent geometric imperfections shall be taken as the larger of:

$w_{0,eff} = 1.5\ l_g/100$

where l_g is the gauge length for the limitation of imperfections according to 2.3.2 (3),

$w_{0,eff} = 1.5\ t$

where t is the shell wall thickness.

Drafting note: The factors are to be investigated further.

(13) The reliability of the numerically determined critical GMNIA buckling resistance shall be checked

(a) either by calculating other shell buckling cases, for which characteristic buckling resistance values are known, with the same program and basicly similar imperfection assumptions. The check cases should be similar in their buckling controlling parameters (e.g. non-dimensional shell slenderness, postbuckling behaviour, imperfection-sensitivity, material behaviour);

(b) or by comparison of calculated values with test resuslts. Regarding the similarity of the check cases, the same statements as made above are valid.

(14) Depending on the results of the reliability checks, a calibration factor shall be evaluated from:

$$k_{GMNIA} = \frac{R_{k,known,check}}{R_{cr,GMNIA,check}}$$

7.8.3 Buckling strength verification

(1) It shall be verified that

$F_d \leq R_d$.

ANNEX D (normative) of EC 3 Part 1.6
Strength and Stability of Shell Structures
Formulas for buckling design

D1 Unstiffened cylindrical shells

D1.1 Unstiffened cylindrical shells of constant wall thickness

D1.1.1 Notation and boundary conditions

(1) Geometrical quantities

l	cylinder length
r	radius of cylinder midsurface

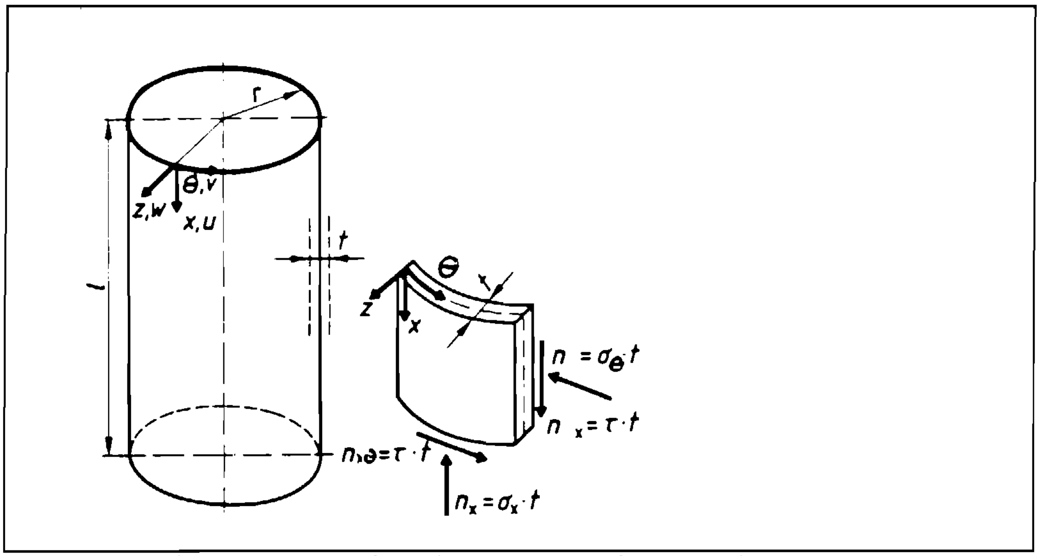

Figure D.1: Cylinder geometry and membrane stresses and stress resultants

(2) The boundary conditions are defined in chapter 4 of the main document.

D1.1.2 Meridional (axial) compression

D1.1.2.1 Critical meridional buckling stresses

(1) The following formulas are applicable only to shells with boundary conditions BC 1 or BC 2 at both edges.

(2) The critical meridional buckling stress shall be obtained from:

$$\sigma_{xRc} = 0.605\, C_x \cdot E \cdot t/r \qquad \text{(D.1)}$$

The factor C_x shall be determined as specified in the following.

(3) For medium-long and short cylinders with

$$l/r \leq 0.5\, \sqrt{r/t} \qquad \text{(D.2)}$$

the factor C_x shall be taken as:

$$C_x = 1 \qquad \text{(D.3)}$$

(4) For short cylinders with

$$l/r \leq 1.7\sqrt{r/t} \qquad (D.4)$$

the factor C_x may be taken as:

$$C_x = 1.36 + \frac{2.07}{(l/r)^2(r/t)} - \frac{1.83}{(l/r)(r/t)^{0.5}} \qquad (D.5)$$

(5) For long cylinders with

$$l/r > 0.5\sqrt{r/t} \qquad (D.6)$$

the factor C_x shall be obtained from:

$$C_x = 1 - [\,0.4\,(l/r)\,(t/r)^{0.5} - 0.2]/\eta \geq 0.6 \qquad (D.7)$$

where η is a parameter depending on the boundary conditions and being taken from table D.1.

Table D.1: Parameter η to determine the critical meridional buckling stress in long cylinders

Case	Combination of boundary conditions	η
1	BC 1 BC 1	6
2	BC 2 BC 1	3
3	BC 2 BC 2	1

D1.1.2.2 Meridional buckling parameters

(1) The meridional elastic imperfection factor shall be obtained from:

$$\alpha_x = \frac{0.83 \cdot 0.75}{\sqrt{1 + 0.01\,r/t}} \qquad \text{for } r/t \leq 212 \qquad (D.8)$$

$$\alpha_x = \frac{0.70 \cdot 0.75}{\sqrt{0.1 + 0.01\,r/t}} \qquad \text{for } r/t > 212$$

Drafting note: Formula (D.8) is to be investigated further.

(2) The meridional squash limit slenderness shall be taken as:

$$\bar{\lambda}_{x0} = 0.20 \qquad (D.9)$$

(3) Cylinders satisfying condition (D.10) need not be checked against meridional shell buckling.

$$\frac{r}{t} \leq \frac{E}{25 f_{y,k}} \qquad (D.10)$$

D1.1.3 Circumferential (hoop) compression

D1.1.3.1 Critical circumferential buckling stressses

(1) The following formulas are applicable to shells with all boundary conditions.

(2) For medium-long and short cylinders with

$$l/r \leq 1.63\ C_\theta \sqrt{r/t} \tag{D.11}$$

the critical circumferential buckling stress shall be otained from:

$$\sigma_{\theta cr} = 0.92\ C_\theta \cdot E \cdot (r/l) \cdot (t/r)^{1.5} \tag{D.12}$$

The factor C_θ depends on the boundary conditions and shall be taken from table D.2.

(3) For short cylinders with

$$l/r \leq \ldots \tag{D.13}$$

the factor C_θ in eqn. (D.12) may be replaced by C_θ^* taken from table D.3.

Table D.2: Factors C_θ

Case	Combination of boundary conditions	C_θ
1	BC 1 BC 1	1.5
2	BC 2 BC 1	1.25
3	BC 2 BC 2	1.0
4	BC 3 BC 1	0.6
5	BC 3 BC 3 BC 2 BC 3	0

Table D.3: Factors C_θ^*

	C_θ^*
1	$1.5 + 10.0 / \bar{l}^2 - 5.0 / \bar{l}^3$
2	$1.25 + 8.0 / \bar{l}l^2 - 4.0 / \bar{l}^3$
3	$1.0 + 3.0 / \bar{l}^{1.35}$
4	$0.6 + 1.0 / \bar{l}^2 - 0.3 / \bar{l}^3$

In table D.3 $\bar{l}$ means $\bar{l} = (l/r)\,(r/t)^{0.5}$

(4) For long cylinders with

$$l/r > 1.63\ C_\theta \cdot \sqrt{r/t} \tag{D.14}$$

the critical circumferential buckling stress shall be obtained from:

$$\sigma_{\theta Rc} = E\left(\frac{t}{r}\right)^2\left[0.275 + 2.03\left(\frac{C_\theta}{(l/r)\sqrt{t/r}}\right)^4\right] \tag{D.15}$$

D1.1.3.2 Circumferential buckling parameters

(1) The circumferential elastic imperfection factor shall be taken as:

$$\alpha_\theta = 0.65 \tag{D.16}$$

(2) The squash limit slenderness shall be taken as:

$$\bar{\lambda}_{\theta 0} = 0.4 \tag{D.17}$$

(3) Cylinders satisfying condition (D.18) need not be checked against circumferential shell buckling.

$$r/t = \sqrt{E/23 f_{y,k}} \tag{D.18}$$

(4) The non-uniform distribution of area load q_w resulting from external wind loading on cylinders (see figure D.2) may, for the purpose of shell buckling design, be substituted by an equivalent uniform external pressure

$$q_{eq} = \delta \cdot \max q_w \tag{D.19}$$

where max q_w is the maximum wind pressure, and δ as follows:

$$\delta = 0.46\left(1 + 0.1\sqrt{C_\theta \frac{r}{l}\sqrt{\frac{r}{t}}}\right) \le 1 \qquad \text{(D.20)}$$

with C_θ being taken from table D.2 as a function of the boundary conditions.
For long cylinders as defined by eqn. (D.14) $\delta = 0.5$ may be assumed conservatively.
The circumferential design stress to be introduced into section 7.4.3 of the main document follows from:

$$\sigma_{\theta Sd} = q_{eq} \cdot r/t \qquad \text{(D.21)}$$

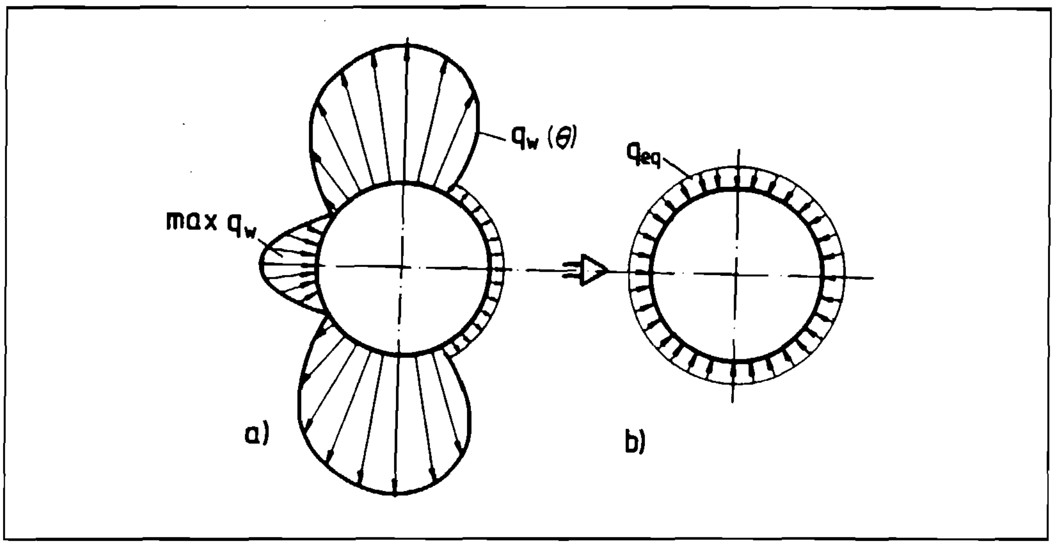

Figure D.2:Typical load distribution from external wind loading

Drafting note: The validity of item (4) for long cylinders is to be investigated further.

D1.1.4. Shear

D1.1.4.1 Critical shear buckling stresses

(1) The following equations are applicable only to shells with boundary conditions BC1 or BC2 at both edges. For medium-long and short cylinders with

$$l/r \le 8.7\sqrt{r/t} \qquad \text{(D.22)}$$

the critical shear buckling stress shall be obtained from:

$$\tau_{Rc} = 0.75\, C_\tau \cdot E \cdot (r/l)^{0.5} \cdot (t/r)^{1.25} \qquad \text{(D.23)}$$

The factor C_τ shall be taken as:

$$C_\tau = 1 \qquad \text{(D.24)}$$

(2) For short cylinders with

$$l/r \le \ldots \qquad \text{(D.25)}$$

the factor C_τ may be obtained from:

$$C_\tau = [1 + 42(r/l)^3 \cdot (t/r)^{1.5}]^{0.5} \qquad \text{(D.26)}$$

(3) For long cylinders with

$$l/r > 8.7\sqrt{r/t} \qquad \text{(D.27)}$$

the critical shear buckling stress shall be obtained from:

$$\tau_{cr} = 0.25\, E \cdot (t/r)^{1.5} \qquad \text{(D.28)}$$

D1.1.4.2 Shear buckling parameters

(1) The shear elastic imperfection factor shall be taken as:

$$\alpha_\tau = 0.65 \quad (D.29)$$

(2) The squash limit slenderness shall be taken as:

$$\bar{\lambda}_{\tau 0} = 0.4 \quad (D.30)$$

(3) Cylinders satisfying condition (D.31) need not be checked against shear shell buckling.

$$r/t \leq [E / (15\, f_{y,k})]^{0.67} \quad (D.31)$$

D1.2 Unstiffened cylindrical shells of stepwise variable wall thickness

D1.2.1 Notation and boundary conditions

(1) Geometry
It is assumed that the wall thickness of the cylinder increases progressively stepwise from top to bottom (see fig. D.4a).

l overall cylinder length
r radius of cylinder midsurface
j integer index denoting the individual cylinder sections with constant wall thickness (from $j = 1$ to $j = n$)
t_j constant wall thickness of the j-th cylinder section
l_j length of the j-th cylinder section

(2) Intended offsets e_0 between plates of adjacent sections (see fig. D.3) are covered by the following formulas if they are not greater than the permissible values perm e_0 acc. to table D.4.

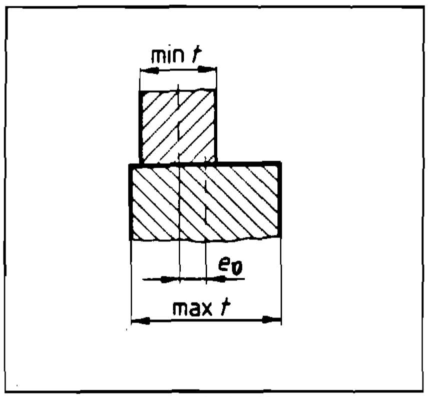

Figure D.3: Intended offset e_0

Table D.4: Permissible intendend offset values

type of bucklingrelevant membrane stress	perm e_0
circumferential compression, shear	1.1 min t
meridional compression	0.5 (max t -min t) or 0.5 min t whichever is smaller

(3) *For cylinders with permissible intended offsets between plates of adjacent sections, the radius r may be taken as the mean value of all sections.*

(4) The following formulas are applicable only to shells with boundary conditions BC 1 or BC 2 at both edges, with no distinction made between them.

D1.2.2 Meridional (axial) compression

(1) Each cylinder section j shall be treated as equivalent cylinder of overall length l and of uniform wall thickness $t = t_j$ according to D1.1.2.

(2) For long equivalent cylinders acc. to eqn. (D.6), the parameter η shall be given the value $\eta = 1$.

D1.2.3 Circumferential (hoop) compression

D1.2.3.1 Critical circumferential buckling stresses

(1) If the cylinder consists of more than three sections with different wall thicknesses, it shall first be replaced by an equivalent cylinder comprising three sections (see fig. D.4b). The length of its upper section, l_o, shall extend to the upper edge of the next section with a wall thickness greater than 1,5 times the smallest wall thickness t_1, but shall not comprise more than half the total length of the cylinder. The length of both the other sections shall be obtained as follows:

$$l_m = l_o,\ l_u = l - 2l_o, \qquad \text{if} \qquad l_o \leq l/3 \tag{D.40a}$$

$$l_m = l_u = 0.5\,(l - l_o), \qquad \text{if} \qquad l/3 < l_o \leq l/2 \tag{D.40b}$$

The fictitious wall thicknesses t_o, t_m and t_u shall be determined by avaraging the wall thickness over each of the three fictitious sections.

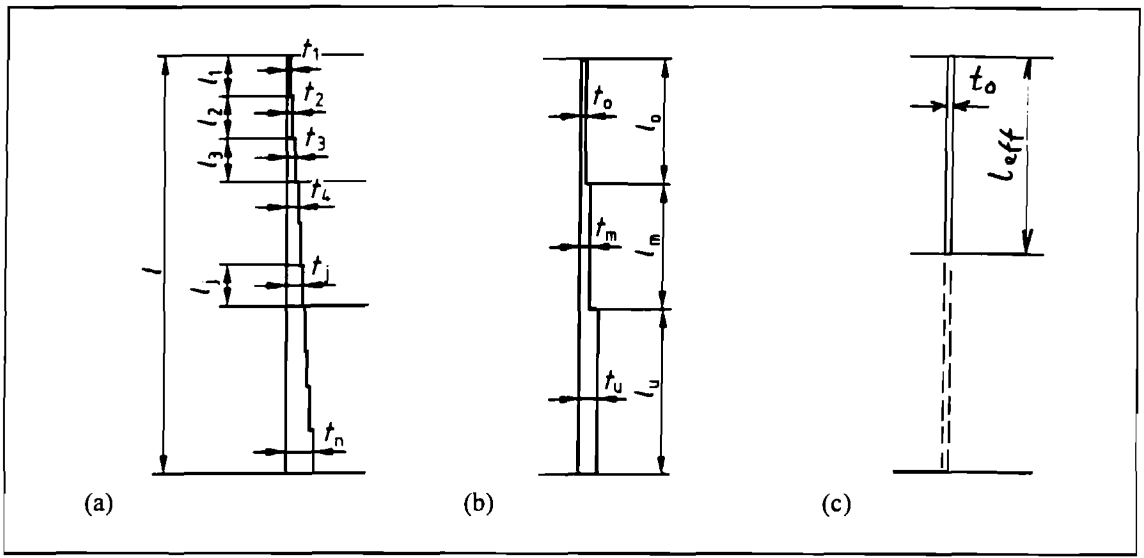

Figure D.4: (a) Cylinder of stepwise variable wall thickness
(b) Equivalent cylinder comprising three sections
(c) Equivalent single cylinder with uniform wall thickness

(2) The three-section-cylinder (i.e. the equivalent one or the real one respectively) shall be replaced by an equivalent single cylinder of effective length l_{eff} and of uniform wall thickness $t = t_o$ (see fig. D.4c). The effective length shall be determined from:

$$l_{eff} = l_o/\beta, \tag{D.41}$$

where b is a nondimensional factor to be taken from fig. D.5.

(3) The critical circumferential buckling stress of each cylinder section j of the original cylinder of stepwise variable wall thickness shall be determined from:

$$\sigma_{\theta Rc,j} = (t_o/t_j) \cdot \sigma_{\theta Rc,eff} \tag{D.42}$$

where $\sigma_{\theta Rc,eff}$ is the critical circumferential buckling stress of the equivalent single cylinder acc. to clause (2) derived from eqns. (D.12) or (D.15) respectively. The factor C_θ in these eqns. shall be given the value $C_\theta = 1$.

(4) For long cylinder sections with

$$l_j/r > 1.63\ \sqrt{r/t_j} \tag{D.43}$$

the critical circumferential buckling stress shall alternatively be determined from:

$$\sigma_{\theta Rc,j} = E\left(\frac{t_j}{r}\right)^2\left[0.275 + 2.03\left(\frac{1}{(l_j/r)\sqrt{(t_j/r)}}\right)^4\right] \qquad (D.44)$$

(5) The smaller one of the two values obtained from eqns. (D.42) and (D.44) shall be used for the buckling design check of the cylinder section j.

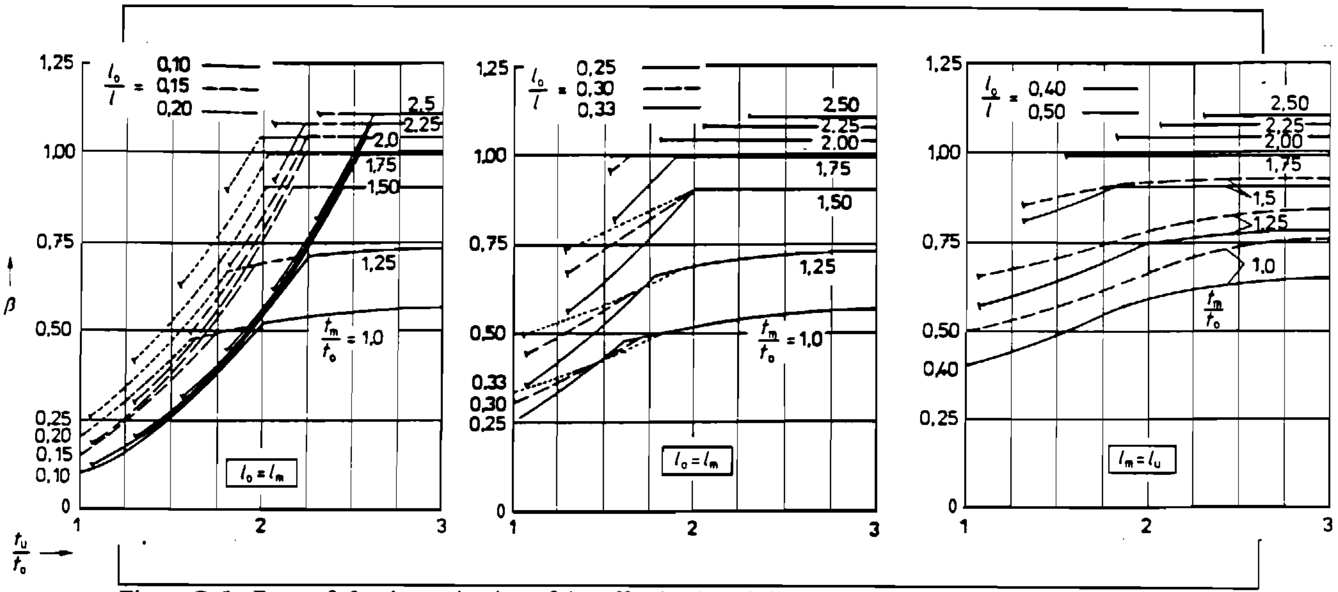

Figure D.5: Factor β for determination of the effective length l_{eff} acc. to eqn. (D.41)

D1.2.3.2 Buckling strength verification for circumferential compression

(1) For each cylinder section the basic check acc. to section 7.4.3 of the main document shall be carried out:

$$\sigma_{\theta Sd,j} \le \sigma_{\theta Rd,j} \qquad (D.45)$$

where $\sigma_{\theta Sd,j}$ is the key value of the circumferential compressive membrane stress, as defined in the following clauses,

$\sigma_{\theta Rd,j}$ is the design circumferential buckling stress, as derived from the critical circumferential buckling stress acc. to section D1.2.3.1.

(2) If $n_{\theta Sd}$ is constant along l, the elementary value

$$\sigma_{\theta Sd,j} = n_{\theta Sd}/t_j$$

shall be taken as key value of the circumferential compressive membrane stress.

(3) If $n_{\theta Sd}$ is variable along l, the fictitious value

$$\sigma_{\theta Sd,j}^{*} = \max n_{\theta Sd}/t_j$$

shall be taken (instead of $\sigma_{\theta Sd,j}$) as key value of the circumferential compressive membrane stress,
where max $n_{\theta Sd}$ is the maximum circumferential compressive membrane force anywhere along l (see fig. D.6).

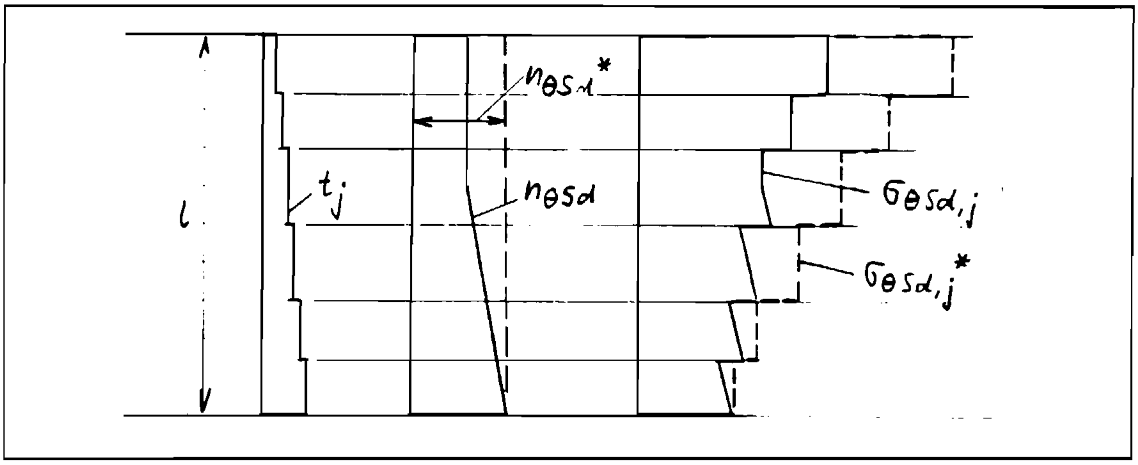

Figure D.6: Key values of the circumferential compressive membrane stress in cases where $n_{\theta Sd}$ is variable along l

D2 Unstiffened conical shells

D2.1 Notation and boundary conditions

(1) Only truncated cones of uniform wall thickness and with semi-vertex angle $\beta \leq 65°$ (see fig. D.7) are covered by the following rules.

(2) Geometry

h	axial length (height) of cone
l	meridional length of cone
r	radius of cone midsurface, perpendicular to axis of rotation, linearly variable along length
r_1, r_2	radii at small and large end of cone
β	semi-vertex angle of cone

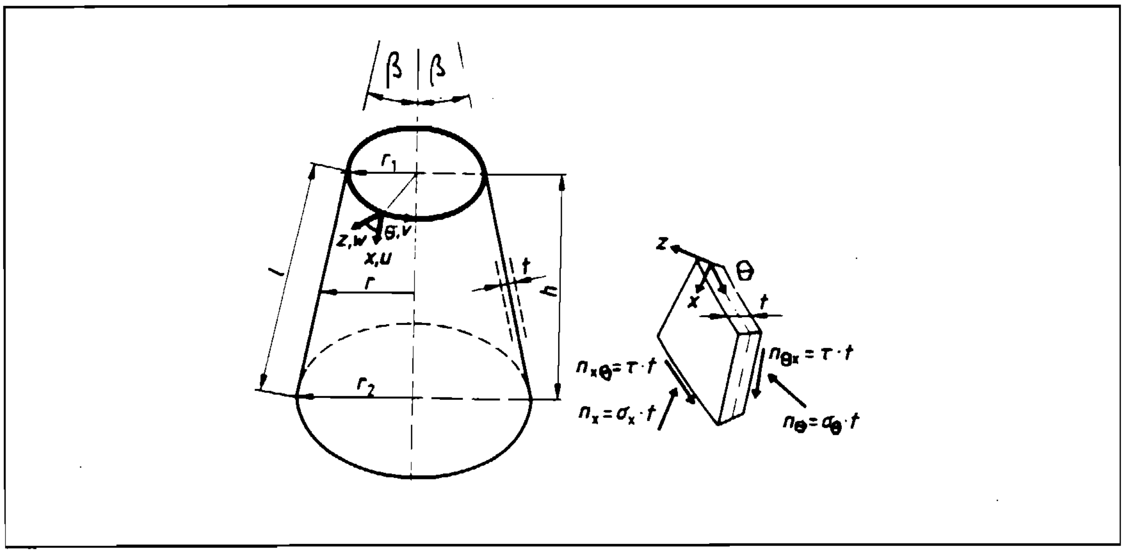

Figure D.7: Cone geometry, membrane stresses and stress resultants

(3) Only boundary conditions BC1 or BC2 acc. to section 4.2.2 of the main document are covered by the following rules, with no distinction made between them.

(4) In practical applications the radial displacement restraint condition may be

- *either* $w = 0$ *("cylinder condition")*
- *or* $u \cdot \sin\beta + w \cdot \cos\beta = 0$ *("ring condition").*

No distinction is made between them in the following rules.

D2.2 Design buckling stresses

D2.2.1 Equivalent cylinder

(1) The design buckling stresses which are needed for the buckling strength verification acc. to section 7.4.3 of the main document, may be derived from an equivalent cylinder of length l_e and of radius r_e. The defintion of l_e and r_e depends on the type of stresses and is given in the following subsections.

D2.2.2 Meridional compression

(1) The equivalent cylinder length shall be taken as:

$$l_e = l \qquad \text{(D.46)}$$

(2) The equivalent cylinder radius shall be taken as:

$$r_e = r / \cos\beta \qquad \text{(D.47)}$$

D2.2.3 Circumferential (hoop) compression

(1) The equivalent cylinder length shall be taken as:

$$l_e = l \qquad \text{(D.48)}$$

(2) The equivalent cylinder radius shall be taken as:

$$r_e = 0.5\,(r_1 + r_2)/\cos\beta \qquad \text{(D.49)}$$

D2.2.4 Shear

(1) The equivalent cylinder length shall be taken as:

$$l_e = h \qquad \text{(D.50)}$$

(2) The equivalent cylinder radius shall be taken as:

$$r_e = \left[1 + \left(\frac{r_1 + r_2}{2r_1}\right)^{0.5} - \left(\frac{2r_1}{r_1 + r_2}\right)^{0.5}\right] r_1 \cos\beta \qquad \text{(D.51)}$$

D2.3 Key values of design stresses

D2.3.1 Meridional compression

(1) The buckling design check shall be carried out at both the small radius and the large radius end of the cone, using the meridional design membrane stresses acting at these radii.

D2.3.2 Circumferential (hoop) compression

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D2.3.3 Shear

Drafting note: Rules will be formulated acc. to DIN 18800-4