Bauforschung

Koordination und Entwicklung eines probabilistischen Sicherheitskonzepts für neue und bestehende Tragwerke

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KOORDINATION UND ENTWICKLUNG EINES PROBABILISTISCHEN SICHERHEITSKONZEPTS FÜR NEUE UND BESTEHENDE TRAGWERKE

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ZUSAMMENFASSUNG

Seit einiger Zeit ist man bestrebt, die Normen auf internationale Ebene zu harmonisieren, um einheitliche Sicherheitsniveaus bei den Konstruktionen zu erreichen. Im Bauwesen ist die internationale Vereinigung Joint Committee on Structural Safety (JCSS) maßgebend an diesen Vorarbeiten beteiligt. Sie ist als Verbindungskomite mehrerer Internationalen Organisationen wie z.B. Organisationen für Beton - und Spannbetonkonstruktionen, für Stahlbauten, für Brückenbau und für Hochbau, oder auch für Bauforschung entstanden. Die Arbeitsgruppe des JCSS besteht aus etwa 30 Mitgliedern und wird seit 1996 von Prof. Dr.-Ing. D. Diamantidis geleitet. Das vorliegende Forschungsvorhaben ist mit der Arbeit des JCSS verknüpft und beinhaltet die Organisation der Arbeiten und die Zusammenstellung deren Ergebnisse als Empfehlungen für spätere Normenwerke.

In diesem Schlußbericht werden die im Zeitraum 1996-1998 erzielte Ergebnisse der Arbeiten dargestellt: Die Arbeiten umfassen die numerische Überprüfung des Sicherheitsniveaus in den Eurocodes, die Entwicklung eines Nachweisformats unter Zugrundelegung eines probabilistischen Konzepts und die Erweiterung des Sicherheitskonzepts zur Beurteilung der Sicherheit bestehender Konstruktionen.

SUMMARY

Since several years efforts are beeing undertaken to harmonize building codes at an international level in order to achieve uniform structural safety. A main contribution to such precodification efforts has the Joint Committee on Structural Safety, JCSS. It is a liaison committee between several international associations, i.e. CIB, FIB, RILEM, ECCS, IABSE. The working party of the JCSS consists of 30 members and is leaded since 1996 by Prof. D. Diamantidis.The current research project is related to the work of the JCSS and particularly to the organisation of the work and to the reporting of the results in terms of guidelines for future codes.

The results achieved during the period 1996 – 1998 are presented in this report. They include the numerical evaluation of the safety level inherent in the Eurocodes, the development of a probabilistic model code and its extension to assess the reliability of existing structures.

ZUSAMMENFASSUNG UND WERTUNG FÜR DIE PRAKTISCHE ANWENDUNG

Seit einiger Zeit ist man bestrebt, die Normen auf internationale Ebene zu harmonisieren, um einheitliche Sicherheitsniveaus bei den Konstruktionen zu erreichen. Im Bauwesen ist die internationale Vereinigung Joint Committee on Structural Safety (JCSS) maßgebend an diesen Vorarbeiten beteiligt. Sie ist als Verbindungskomitee mehrerer internationaler Organisationen, wie z.B. Organisationen für Beton - und Spannbetonkonstruktionen, für Stahlbauten, für Brückenbau und für Hochbau, oder auch für Bauforschung entstanden. Die Arbeitsgruppe des JCSS besteht aus etwa 30 Mitgliedern und wird seit 1996 von Prof. Dr.-Ing. D. Diamantidis geleitet. Das vorliegende Forschungsvorhaben ist mit der Arbeit des JCSS verknüpft. Das wesentliche Ziel ist die Organisation der Arbeit des JCSS und die Zusammenstellung der Arbeitsergebnisse als Empfehlungen für spätere Normenwerke.

In diesem Schlußbericht werden die im Zeitraum 1996-1998 erzielten Ergebnisse der Arbeiten des JCSS dargestellt: Die Arbeiten umfassen:

a) numerische Überprüfung des Sicherheitsniveaus in den Eurocodes und Entwicklung eines Nachweisformats unter Zugrundelegung eines probabilistischen Konzepts;
b) Erweiterung des Sicherheitskonzepts zur Beurteilung der Sicherheit bestehender Konstruktionen.

Die Ergebnisse der Arbeiten sind detailliert in den Anhängen des Schlußberichts enthalten. Es wurden große Fortschritte erreicht: das Sicherheitskonzept zur Beurteilung bestehender Konstruktionen wurde fertiggestellt, das Sicherheitsniveau in den Eurocodes wurde für charakteristische Fälle überprüft und wesentliche Teile eines probabilistischen Bemessungskonzepts mit entsprechenden Modellen für Lasten und Bauteilwiderstände wurden erarbeitet und in den Sitzungen diskutiert.

Die praktische Anwendung der erzielten Ergebnisse umfaßt ein breites Spektrum:

a) die Forschungsergebnisse dienen als Entscheidungshilfe für die Beurteilung bestehender Tragwerke und der damit verbundenen Probleme (Erweiterung der geplanten Nutzungsdauer, wirtschaftliche Sanierung, Inspektionen, usw.),

b) die Forschungsergebnisse sind direkt anwendbar bei der Planung spezieller Bauwerke, wie z.B. Bohrplattformen, Kraftwerke, Tunnel, Brücken, bei denen wirtschaftliche Lösungen mit annehmbaren Sicherheitsniveaus gefragt sind,

c) die Forschungsergebnisse können direkt bei der Aufbereitung der zukünftigen Normen bezüglich Sicherheitsanforderungen, Belastungen und Festigkeitseigenschaften der Tragwerke verwendet werden.

Es sind aber weitere Arbeiten auf dem Vornormenniveau notwendig, um besonders das probabilistische Konzept für die Tragwerkbemessung zu vervollständigen und im Hinblick auf dessen Anwendbarkeit zu prüfen.

1 EINLEITUNG

1.1 Hintergrund

Primäres Ziel des konstruktiven Ingenieurbaus ist die Sicherheit der Bauwerke. Sie wird mit Hilfe von in den Normen vorgeschriebenen Sicherheitsnachweisen geprüft. Seit einiger Zeit ist man bestrebt, die Normen auf internationale Ebene zu harmonisieren, um einheitliche Sicherheitsniveaus bei den Konstruktionen zu erreichen. Eine internationale Normung entwickelt sich schrittweise in internationalen Organisationen unterschiedlicher Struktur und Aufgabenstellung. Hier treffen sich Fachleute aus vielen Ländern, um wissenschaftliche Erkenntnisse und praktische Erfahrungen auszutauschen. Dabei werden Statusberichte erstellt und Empfehlungen für spätere Regeln erarbeitet. Ohne diese aufwendigen Vorarbeiten und ohne die fachliche Verständigung in diesen Organisationen wäre eine internationale Normung unmöglich.

Im Bauwesen ist die internationale Vereinigung Joint Committee on Structural Safety (JCSS) maßgebend an diesen Vorarbeiten beteiligt. Sie ist als Verbindungskomitee mehrerer internationaler Organisationen, wie z.B. Organisationen für Beton - und Spannbetonkonstruktionen, für Stahlbauten, für Brückenbau und für Hochbau, oder auch für Bauforschung, entstanden. Die Arbeitsgruppe des JCSS besteht aus etwa 30 Mitgliedern (s. Anhang I) und wird seit 1996 von Prof. Dr.- Ing. D. Diamantidis geleitet. Das vorliegende Forschungsvorhaben ist mit der Arbeit des JCSS verknüpft. Es hat folgende Ziele:

a) Organisation der Arbeit des JCSS,
b) Vorbereitung und Leitung der Sitzungen der Arbeitsgruppe,
c) Beteiligung an der technischen Arbeit und speziell an der Entwicklung eines Sicherheitskonzepts für bestehende Tragwerke,
d) Zusammenstellung der Ergebnisse und Empfehlungen für spätere Regeln

In diesem Schlußbericht werden die im Zeitraum 1996-1998 erzielten Ergebnisse zusammengefaßt.

1.2 Literaturauswertung

Für die Bearbeitung der Forschungsziele des Vorhabens wurden sehr viele Literaturstellen hauptsächlich aus dem Gebiet der Sicherheit und der Zuverlässigkeit im Bauwesen kritisch betrachtet. Einige dieser Literaturstellen sind hier angegeben. Aus der Literaturauswertung ergibt sich die Notwendigkeit von sicherheitstheoretischen Untersuchungen im Hinblick auf:

- Anforderungen an die Zuverlässigkeit der Bauwerke in Abhängigkeit von der Nutzungsart des Tragwerks und somit von der erwünschten Nutzungsdauer,
- Anforderungen an die Zuverlässigkeit in Abhängigkeit von den möglichen Schadenfolgen,
- Vergleichmäßigung des Sicherheitsniveaus.

Für den Einsteiger in die Thematik ist folgende ausgewählte Literatur angegeben. Hinweise auf die Literatur findet man im Text.

American Society of Civil Engineers (ASCE), 1995, Minimum Design Loads for Buildings and other Structures, ASCE 7-95.

Benjamin, J.R., Cornell, A.C., 1970, Probability, Statistics and Decision for Civil Engineers, Mac Graw-Hill Book Company, New York, 1970.

Beranger, M., Diamantidis, D. and G.M. Manfredini, 1993, Probabilistic Reanalysis of Existing Offshore Structures, IABSE Colloquium, Copenhagen.

Breitschaft, G. , 1994, Harmonisierung technischer Regel für das Bauwesen in Europa, Betonkalender, Teil II, 1-18.

Danish Technical Research Council "Safety and Reliability", 1997, Probabilistic Methods and Models for Reliability - Based Reassessment, Copenhagen, Denmark.

Ditlevsen, D., 1986, Reliability Updating of Existing Structures, Appendix A in Structural Reliability of Existing Bridges, Report No. 1, The Road Directorate, Ministry of Transport, Denmark.

Diamantidis, D., 1987, Reliability Assessment of Existing Structures, Engineering Structures, Vol. 9, pp. 177-182.

Eurocode 1, Basis of Design, CEN/TC250-N 105.

Goyet, J., Faber, M., Paygnard, J.C., and A. Maroini, 1994, Optimal Inspection and Repair Planning, ASME-OMAE Conference.

International Standards Organization (ISO), 1996, General Principles on Reliability for Structures, Revision of IS 2394, in Vorbereitung, Genf.

Kersken-Bradley, M., Diamantidis, D., 1985, Sicherheit von Baukonstruktionen, in Handbuch der Sicherheitstechnik, Teil 1, C. Hanser Verlag, S. 253-333.

König, G., Hosser, D., Schobbe, W., 1982, Sicherheitsanforderungen für die Bemessung von baulichen Anlagen nach den Empfehlungen des NABau - eine Erläuterung, der Bauingenieur 57.

Rackwitz, R. and Fießler, B., Structural Reliability under Combined Random Load Sequences, Computers and Structures, 9, 489-494.

Schneider, J., 1990, Beurteilung der Tragsicherheit bestehender Tragwerke, SIA-Heft Nr. 46.

Schneider, J., 1996, Sicherheit und Zuverlässigkeit im Bauwesen, Hochschulverlag an der ETH Zürich und B.G. Teubner Verlag Stuttgart, 1996.

Schweizerischer Ingenieur- und Architekten Verein (SIA), 1994, Beurteilung der Tragsicherheit bestehender Bauwerke, Richtlinie 462, Zürich.

TNO Building and Construction Research, 1995, Beoordeling Bestaande Bouwconstructies, Vornorm in Vorbereitung, Delft.

Vrouwenvelder, T., 1993, Codes of Practice for the Assessment of Existing Structures, Proc. Of IABSE Conference, Copenhagen, Denmark.

1.3 Ziele

Für den Arbeitsablauf wurden folgende drei Ziele gesetzt:

a) kurzfristiges Ziel (bis Ende 1997): Nachprüfen des Sicherheitsniveaus in den Eurocodes,
b) mittelfristiges Ziel (bis Ende 1998): Erarbeiten eines zuverlässigkeitstheoretischen Konzepts für den Nachweis der Sicherheit bestehender Bauwerke,

c) langfristiges Ziel (bis zum Jahr 2001): Erstellung eines probabilistischen Model Code (probabilistisches Sicherheitskonzept) für neue und bestehende Tragwerke.

Die Ergebnisse der bisherigen Arbeiten wurden in den Sitzungen präsentiert und diskutiert. Im Zeitraum 1996 - 1998 fanden folgende Sitzungen statt:

- April 1996 in Delft,
- Oktober 1996 in Kopenhagen,
- März 1997 in Regensburg,
- September 1997 in Innsbruck,
- April 1998 in Leipzig,
- Oktober 1998 in London.

Die Ergebnisse bezüglich des Nachprüfens des Sicherheitsniveaus in den Eurocodes sind hier im Kapitel 2 zusammengefaßt. Kapitel 3 stellt das Konzept zum Nachweis existierender Konstruktionen dar. Der bisher erarbeitete probabilistischer Model Code ist in Kapitel 4 beschrieben. Schlußfolgerungen sind im Kapitel 5 angegeben.

2 ÜBERPRÜFUNG DES SICHERHEITSNIVEAUS IN DEN EUROCODES

2.1 Grundlagen des Sicherheitskonzepts

Das für die Bemessung von Bauwerken maßgebende Sicherheitskonzept gemäß Eurocode 1 verlangt:

a) Die Gewährleistung der Tragfähigkeit, der Gebrauchstauglichkeit und der Dauerhaftigkeit der Tragwerke während ihrer *vorgesehenen Nutzungsdauer* mit *angemessener Zuverlässigkeit.*

b) Die Untersuchung von Grenzzuständen für den Verlust der Funktionsfähigkeit des Tragwerks in ausgewählten Bemessungssituationen; insbesondere werden folgende Grenzzustände untersucht:

- *Grenzzustand der Tragfähigkeit*
- *Grenzzustand der Gebrauchstauglichkeit*

c) Den Nachweis der Tragsicherheit und der Gebrauchstauglichkeit unter Verwendung von Teilsicherheits- und Kombinationsbeiwerten für Einwirkungen sowie von baustoffabhängigen Teilsicherheitsbeiwerten für Widerstände.

Den Sicherheitsanforderungen wird somit entsprochen durch den Nachweis, daß die Bemessungswerte der Beanspruchung kleiner sind als die Bemessungswerte der Widerstände. Die Bemessungswerte der Beanspruchung erhält man aus der Multiplikation der charakteristischen Werte der Einwirkungen mit den entsprechenden Teilsicherheitsbeiwerten, die Bemessungswerte der Widerstände aus der Division der charakteristischen Werte der Materialkenngrößen durch die entsprechenden Teilsicherheitsbeiwerte. Die Anwendung des erläuterten Sicherheitskonzepts soll zu einem annehmbaren Sicherheitsniveau führen. Das Sicherheitsniveau wird durch die Versagenswahrscheinlichkeit p_F oder durch den Sicherheitsindex β dargestellt.

2.2 Ergebnisse

Das Sicherheitsniveau in den Eurocodes wurde im Rahmen der Arbeit des JCSS für folgende charakteristische Bauteile überprüft:

- Stahlbetonstützen,
- Stahlstützen,
- Pfahlgründungen.

Für jedes Bauteil wurden mehrere repräsentative Bemessungsfälle gewählt. Die Zuverlässigkeitsanalysen haben gezeigt, daß die berechneten Sicherheitsniveaus von Fall zu Fall unterschiedlich sind. Die Ergebnisse für die Stahlbetonstützen wurden veröffentlicht und sind im Anhang II enthalten. Sie zeigen, daß der Sicherheitsindex Werte zwischen 2.9 und 6.1 annimmt. Dies bedeutet, daß in einigen Fällen das Sicherheitsniveau unterhalb der Annehmbarkeitskriterien liegt (vgl. Tabellen 5a und 5b)

Ähnliche Veröffentlichungen sind für die Untersuchungen des Sicherheitsniveaus der Stahlstützen und der Pfahlgründungen in Vorbereitung.

3 SICHERHEITSKONZEPT FÜR BESTEHENDE TRAGWERKE

3.1 Notwendigkeit

Die Notwendigkeit, das Tragwerk eines bestehenden Bauwerks einer Beurteilung zu unterziehen, ergibt sich

- aufgrund der Ergebnisse einer periodischen Zustandsuntersuchung,
- aufgrund des Ablaufs der anläßlich einer früheren Beurteilung der Tragsicherheit zugestandenen Restnutzungsdauer,
- bei Bekanntwerden von Bemessungs- oder Ausführungsmängeln,
- anläßlich einer geplanten Nutzungsänderung des Bauwerks,
- bei Zweifeln an der Tragsicherheit, hervorgerufen durch sichtbare Schäden,
- bei offensichtlich mangelhafter Gebrauchstauglichkeit,
- durch außerordentliche Vorkommnisse während der Nutzung (wie z.B. Anprall von Fahrzeugen, Lawinen, Brand im Gebäude, Erdbeben etc.), die das Tragwerk möglicherweise geschädigt haben könnten,
- bei baustoff-, bauweisen- oder systembedingtem Verdacht auf mögliche Beeinträchtigung der Tragsicherheit.

3.2 Übersicht über Normen bezüglich des Sicherheitsnachweises bestehender Tragwerke

3.2.1 Allgemein

Gegenwärtig haben nur wenige Länder ein allgemein anwendbares Normenwerk zur Beurteilung von bestehenden Baukonstruktionen. Zu diesen Ländern zählen die ehemalige Tschechoslowakei, die Schweiz und die Niederlande. In den USA und in Kanada sind solche Normen in Vorbereitung. Empfehlungen gibt es bereits in mehreren Ländern, wie z.B. in Großbritannien und Dänemark.

Es muß aber zwischen bereits anwendbaren Normen und nur richtungsweisenden Empfehlungen unterschieden werden. Normen fordern für die Konstruktionen Vorschriften mit Mindestanforderungen, die eingehalten werden müssen. Währenddessen stellen Empfehlungen hauptsächlich Hinweise dar, wie die Planung zu erstellen ist und wie die

Beurteilung von existierenden Bauwerken in einem systematischen und zugleich wirtschaftlichen Weg durchzuführen ist.

Im Anschluß werden die Normenwerke der Schweiz, der Niederlande und Dänemarks kurz erörtert. Eine zusammenfassende Darstellung der Normenwerke zur Beurteilung existierender Bauwerke findet man in Vrouwenvelder (1992).

3.2.2 Schweiz (SIA, 1994)

In der Schweiz ist der Schweizer Ingenieur- und Architektenverein (SIA) für die Beurteilung existierender Bauwerke und für die Erstellung der dafür notwendigen Normen zuständig.

Die Normen des SIA fordern für jedes Bauwerk in den frühen Phasen der Planung die Aufstellung eines Nutzungsplans und eines Sicherheitsplans. Diese Pläne sind die beiden zentralen Dokumente für die Vorbereitung und Durchführung von Bauprojekten.

Auf der Basis der Normen des SIA stellt der Bauherr nach seinen Anforderungen mit der Sachkenntnis des Architekten oder des Ingenieurs gemeinsam den Nutzungsplan auf. Dieser dient als Grundlage für den von Architekten/Ingenieuren ausgearbeiteten Sicherheitsplan. Im weiteren Ablauf folgt ein Kontrollplan, eine Dokumentation der akzeptierten Risiken, ein Überwachungsplan, ein Unterhaltsplan und Nutzungsanweisungen.

Der Sicherheitsplan stellt in einer Reihe von Dokumenten die als maßgebend erkannten Gefährdungsbilder (Gefahren-Kombinationen; Hazard Scenarios), den zu ihrer Abwehr als geeignet befundenen Maßnahmen, gegenüber. Es geht hier darum, sowohl den Bau- als auch den Nutzungsprozeß in allen erkennbaren Details auf Gefahren im voraus durchzudenken. Dieser Teil des Ablaufs kann auch als „Gefahrenerkennung" bezeichnet werden. Allerdings sind nicht alle Gefährdungsbilder relevant. Man wird versuchen, die Fülle des Erkannten auf das für den betrachteten Fall Wesentliche und Notwendige zu reduzieren, indem man eine Bewertung der Gefahren bzw. der Gefährdungsbilder anschließt. Alle Gefahren, die als akzeptierbare Risiken ohne Gegenmaßnahmen stehengelassen werden können oder müssen, werden in eine Liste der akzeptierten Risiken eingetragen. Es werden sich auch Gefährdungsbilder zeigen, die man im nachhinein als vernachlässigbar einstufen kann und gar nicht mehr in die Liste der akzeptierten Risiken aufnimmt.

Allerdings werden bei vielen bestehenden Bauwerken sowohl der von den SIA - Normen geforderte Nutzungsplan als auch der darauf aufbauende Sicherheitsplan fehlen. Diese Pläne sind aufgrund der angestrebten Restnutzungsdauer neu zu erstellen bzw. zu berichtigen und bilden in der Folge eine wichtige Basis für die Beurteilung der Frage, unter welchen Bedingungen und Vorkehrungen ein bestehendes Bauwerk weiterhin in Betrieb bleiben darf.

Als weiteres wichtiges Mittel zur Beurteilung von existierenden Bauwerken kann die „Beurteilung in drei Phasen" (SIA 462, 1994) angesehen werden. Diese Phasen können wie folgt zusammengefaßt werden:

- Phase I: Grobe Erstbeurteilung,
- Phase II: Detaillierte Untersuchung,
- Phase III: Beratung im Experten - Kollegium.

Es ist allerdings nicht nur von Bedeutung, die Größe oder die Ausdehnung des Schadens so schnell und so gut wie möglich festzustellen und durch sofort eingeleitete geeignete Maßnahmen beseitigen zu lassen, sondern auch die Schadensursache zu ermitteln. Es werden dabei einige Methoden vorgeschlagen, der Ursache auf den Grund zu gehen. Die wichtigsten Verfahren, die in der Literatur auch als „Gefährdungsanalysen" bezeichnet werden, sind:

- Fehlerbaum (Fault Tree),
- Ereignisbaum (Event Tree),
- Ursachen/Folgen-Diagramm (Cause/Consequence Chart),
- Entscheidungsbaum (Decision Tree).

Die oben erwähnte Richtlinie SIA 462 befaßt sich mit der Beurteilung der Tragsicherheit von bestehenden Bauwerken. Der erste Teil dieser Richtlinie steckt den Geltungsbereich ab und definiert wichtige Begriffe. Weiterhin werden

- mögliche Gründe bzw. Anlaß für die Beurteilung,
- Phasen der Beurteilung,
- mögliche Darstellungen der Untersuchungsergebnisse

gezeigt. Anschließend wird darauf hingewiesen, wie vorzugehen ist und was zu beachten ist bei:

- der Bestandsaufnahme der Bauwerksakten,
- der Zustandsaufnahme und Aktualisierung von Informationen,
- der Aktualisierung des Sicherheitsplans,
- dem Verhalten gegenüber Risiken.

Im weiteren wird aufgezeigt, wie die Tragsicherheit eines bestimmten Systems nachgewiesen werden kann. Hierbei werden sowohl Gründe für die Notwendigkeit eines Nachweises erbracht als auch eine Möglichkeit zur Eingliederung der Restnutzungsdauer in eine entsprechende Kategorie gegeben. Hierbei ist die Restnutzungsdauer wie folgt eingeteilt:

- Nachweis für eine unbestimmte Restnutzungsdauer,
- Nachweis für eine lange Restnutzungsdauer → maximal 5 Jahre,
- Nachweis für eine kurze Restnutzungsdauer → maximal 6 Monate.

Außerdem werden einige Berechnungshinweise für die drei oben genannten Kategorien gegeben. Abschließend werden noch einige Sicherheitsmaßnahmen genannt, die je nach Ergebnis der Beurteilung eingeleitet werden können bzw. müssen.

3.2.3 Niederlande (TNO, 1995)

In der Niederlande ist eine Vornorm zur Beurteilung der Sicherheit existierender Bauwerke in Vorbereitung. Die wichtigsten Punkte dieser Vornorm sind hier zusammengefaßt:

- Für die Beurteilung der Sicherheit ist eine Tabelle mit Mindestwerten für den Zuverlässigkeitsindex angegeben. Auf dieser Grundlage sind auch Teilsicherheitsbeiwerte für die Belastungen abgeleitet,
- Es gibt keine Forderungen eingehender Gebrauchsfähigkeit in Bezug auf Durchbiegungen,
- Es werden keine außergewöhnliche Belastungen mit Ausnahme von Brand betrachtet,
- Für die Bestimmung von Abmessungen und Materialeigenschaften muß von Tatsachen ausgegangen werden, die in der Baukonstruktion vorhanden sind,
- Die Berechnung geschieht auf der Grundlage von Normen für Neubauten.

Die akzeptablen Sicherheitsniveaus sind in der Tabelle 1 dargestellt.

	Neubau β_n		Bestehendes Bauwerk β_b	
Bezugszeitraum t	t = 50 Jahre		t = 1 Jahr	
	Normal	Wind dominant	Normal	Wind dominant
Klasse 1	3,2	2,3	1,7	1,3
Klasse 2	3,4	2,4	3,1	3,1
Klasse 3	3,6	2,6	3,6	3,6

Tabelle 1: Zulässige Werte für den Sicherheitsindex β bei Neubauten und bei bestehenden Bauwerken.

Hierbei wird davon ausgegangen, daß:

- Neubauten für einen Zeitraum von 50 Jahren geplant und gebaut werden (β_n),
- die Untersuchung eines bestehenden Gebäudes für einen Bezugszeitraum von einem Jahr durchgeführt wird (β_b).

Für diese beiden Bezugszeiträume sind die Zielsicherheitsindizes in der Tabelle 1 dargestellt. Für die angestrebte erwünschte Restlebensdauer des zu untersuchenden Bauwerks kann das entsprechende β interpoliert werden. Wie in der Tabelle 1 weiter zu sehen ist, werden die β - Werte für drei Klassen ermittelt, nach dem Prinzip der *safety class differentiation*, die wie folgt unterteilt sind:

- Klasse 1: vernachlässigbare Gefährdung von Menschenleben,
- Klasse 2: geringe Gefährdung von Menschenleben,
- Klasse 3: große Gefährdung von Menschenleben.

Teilsicherheitsbeiwerte für Lasten sind ebenfalls in der Vornorm enthalten.

Es wird nach folgenden Lastarten unterschieden:

- Ständige Lasten (z.B. Eigengewicht),
- Veränderliche Lasten (z.B. Wind, Schnee),
- Außergewöhnliche Lasten (z.B. Brand).

Die außergewöhnlichen Lasten werden, wie in Tabelle 2 zu sehen ist, nur in einer speziellen Lastkombination berücksichtigt. Diese Tatsache entspricht auch der Bemessungsphilosophie in den Eurocodes.

Lastkombinationen	Ständige Lasten		Veränderliche Lasten	Außergewöhnliche Lasten
	Ungünstig	günstig	Wind dominant ja / nein	Brand allein
LK: 1 (fundamental)				
- Klasse 1	1,0	0,9	1,00 / 1,00	-
- Klasse 2	1,2	0,9	1,60 / 1,20	-
- Klasse 3	1,2	0,9	2,30 /1,35	-
LK: 2 (Ständige Lasten allein)				
- Klasse 1	1,0	0,9	-	-
- Klasse 2	1,2	0,9	-	-
- Klasse 3	1,2	0,9	-	-
LK: 3 (nur Brandlast)				
- alle Klassen	1,0	1,0	1,0	1,0

Tabelle 2: Teilsicherheitsbeiwerte für Traglast - Grenzzustände bei bestehenden Bauwerken.

3.2.4 Dänemark (Danish Technical Research Council, 1995)

Richtlinien für den nachträglichen Nachweis der Sicherheit existierender Stahlbetonkonstruktionen wurden in Dänemark von der Danish Technical Research Council entwickelt (1997). Es werden dabei folgende Aspekte behandelt:

- Grundlagen der sicherheitstheoretischen Beurteilung,
- Materialeigenschaften von Beton,
- Materialeigenschaften von Bewehrungsstahl,
- Materialeigenschaften von Spannstahl,
- Risikoakzeptanzkriterien für existierende Tragwerke.

3.3 JCSS Vorschlag für ein Nachweiskonzept

3.3.1 Zusammenfassende Darstellung

Tabelle 3 zeigt in übersichtlicher Darstellung den Inhalt des von der JCSS erarbeiteten Konzepts zum Sicherheitsnachweis bestehender Konstruktionen. Der für jeden Abschnitt verantwortliche Wissenschaftler ist in der Tabelle ebenfalls aufgeführt.

Kapitel	Titel	Verantwortlicher
1	Einleitung	Diamantidis
2	Richtlinien	Diamantidis
3	Normung	Diamantidis
A	Methoden	Chryssanthopoulos
B	Updating	Faber
C	Zielsicherheit	Ambjerg-Nielsen
D1	Holzbalken	Vrouwenvelder
D2	Ermüdung	Vrouwenvelder/Faber
D3	Überstehen einer Last	Diamantidis
D4	Prüflast	Rackwitz
D5	Optimale Inspektionsstrategie	Vrouwenvelder
E1	Betonfestigkeit	Vrouwenvelder
E2	Offshore Pfähle	Nadim
E3	Brückenpfähle	Faber
E4	Ermüdungsrisse	Goyet
E5	Landwirtschaftliche Gebäude	Hergenröder

Tabelle 3: Übersicht über das JCSS Nachweiskonzept für bestehende Tragwerke.

Kapitel 1, 2 und 3 enthalten allgemeine Entscheidungsgrundlagen zur Überprüfung der Sicherheit bestehender Bauwerke. Sie sind im wesentlichen abgeschlossen. Anhang A faßt die Grundlagen der Zuverlässigkeitstheorie zusammen. Im Falle eines bestehenden Tragwerks verfügt der Ingenieur über zusätzliche Informationen (z.B. durch Beobachtungen, durch Messungen, usw.). Deswegen ist die Aktualisierung des Informationszustandes eine wesentliche Aufgabe und bezieht sich auf mehrere Berechnungs- und Bemessungsverfahren. Zuverlässigkeitstheoretische Modelle für die

Aktualisierung des Informationszustandes sind im Anhang B enthalten. Das Zielsicherheitsniveau wird im Anhang C diskutiert. Das gesamte Konzept wird im Abschnitt D und E in ausgewählten Beispielen dargestellt. Während Abschnitt D einfache Lehrbeispiele enthält, werden charakteristische Fälle aus der Praxis im Abschnitt E erläutert.

Die beigefügten Berichtsentwürfe (Anhang III) zeigen, daß in den letzten drei Jahren ein großer Fortschritt bei der Fertigstellung der Arbeiten erreicht wurde. Die Arbeiten sollen im nächsten Jahr (1999) veröffentlicht werden.

Im nächsten Abschnitt ist das von JCSS erarbeitete allgemeine Konzept zur Beurteilung der Sicherheit existierender Konstruktionen erörtert.

3.3.2 Allgemeines Konzept

3.3.2.1 Bereiche der Beurteilung

Es zeigt sich, daß bei bestehenden Bauwerken vor allem die Beurteilung der *Tragsicherheit* das wesentliche Problem ist, und zwar deshalb, da Aussagen über das Verhalten des Tragwerks in Extremsituationen gemacht werden müssen, die in der Regel außerhalb des Erfahrungsbereichs liegen. Dazu gehört oft auch die schwierige und für die Beurteilung der Tragsicherheit entscheidende Frage, in welchem Zustand sich gewisse schlecht oder gar nicht untersuchbare Tragelemente befinden, z.B. in Bezug auf Korrosion.

Auch die Beurteilung des Zustands eines bestehenden, dynamisch beanspruchten Tragwerks im Hinblick auf *Ermüdung* ist eine komplexe Angelegenheit. Eine einigermaßen zuverlässige Entdeckung von Ermüdungsrissen im frühen Zustand ist nur mit großem Aufwand möglich und setzt eine sachkundige, auf besonders gefährdete Tragwerksteile ausgerichtete intensive Suche voraus.

Zweifel an der *Gebrauchstauglichkeit* bestehen hingegen selten, denn entweder hat sich das Tragwerk als gebrauchstauglich erwiesen, oder die entsprechenden Mängel sind aus der vorhergehenden Nutzung bekannt. Durchbiegungen, die Rißbildung, die Charakteristik des Schwingungsverhaltens etc. zeigen sich jedenfalls unter den Umständen des normalen Gebrauchs und lassen schlüssige Aussagen ohne weiteres zu. Allenfalls stellen sich Fragen der Zumutbarkeit gewisser Erscheinungen für die Benützer von Bauwerken, wie z.B. von Schwingungen, Erschütterungen usw.

Auch die Frage der *Dauerhaftigkeit* zeigt sich bei bestehenden Bauwerken in anderem Licht. Wo es bei der Projektierung von Bauwerken vielfach an Erfahrung mangelt, läßt sich bei bestehenden Bauten anhand des angetroffenen Zustands leichter auf die zu erwartende weitere Entwicklung schließen. Die Festlegung der zum weiteren Erhalt der Bausubstanz notwendigen Vorkehrungen ist vergleichsweise einfacher.

Bei der Projektierung und Bemessung *neuer* Tragwerke liegen die Probleme im übrigen genau umgekehrt: Die normgemäße Tragsicherheit ist mit vergleichsweise einfachen Modellen nachweisbar, während die Gebrauchstauglichkeit wegen der großen Voraussage-Unschärfen und wegen der mangelnden Kenntnis über die Einzelheiten der späteren Nutzung auch mit „genauen“ Modellen lediglich abgeschätzt werden kann. Noch schwieriger ist die Voraussage der Dauerhaftigkeit von Bauwerken im Planungsstadium.

3.3.2.2 Phasen der Beurteilung

Es zeigt sich in Anlehnung an SIA (1994), daß eine Gliederung der Beurteilung eines bestehenden Bauwerks in drei Phasen sinnvoll ist. Jede dieser drei Phasen soll in sich abgeschlossen sein und dem Eigentümer nach Abschluß der Phase die Entscheidungsfreiheit zurückgeben. Bild 1 zeigt schematisch, wie man sich den Entscheidungsprozeß vorstellen kann. Die Entscheidungsfreiheit des Eigentümers ist durch die Bestimmungen des Strafgesetzbuches eingeengt, sowie durch die Empfehlungen des Ingenieurs bzw. des Experten - Kollegiums, denen er kaum zuwider handeln kann.

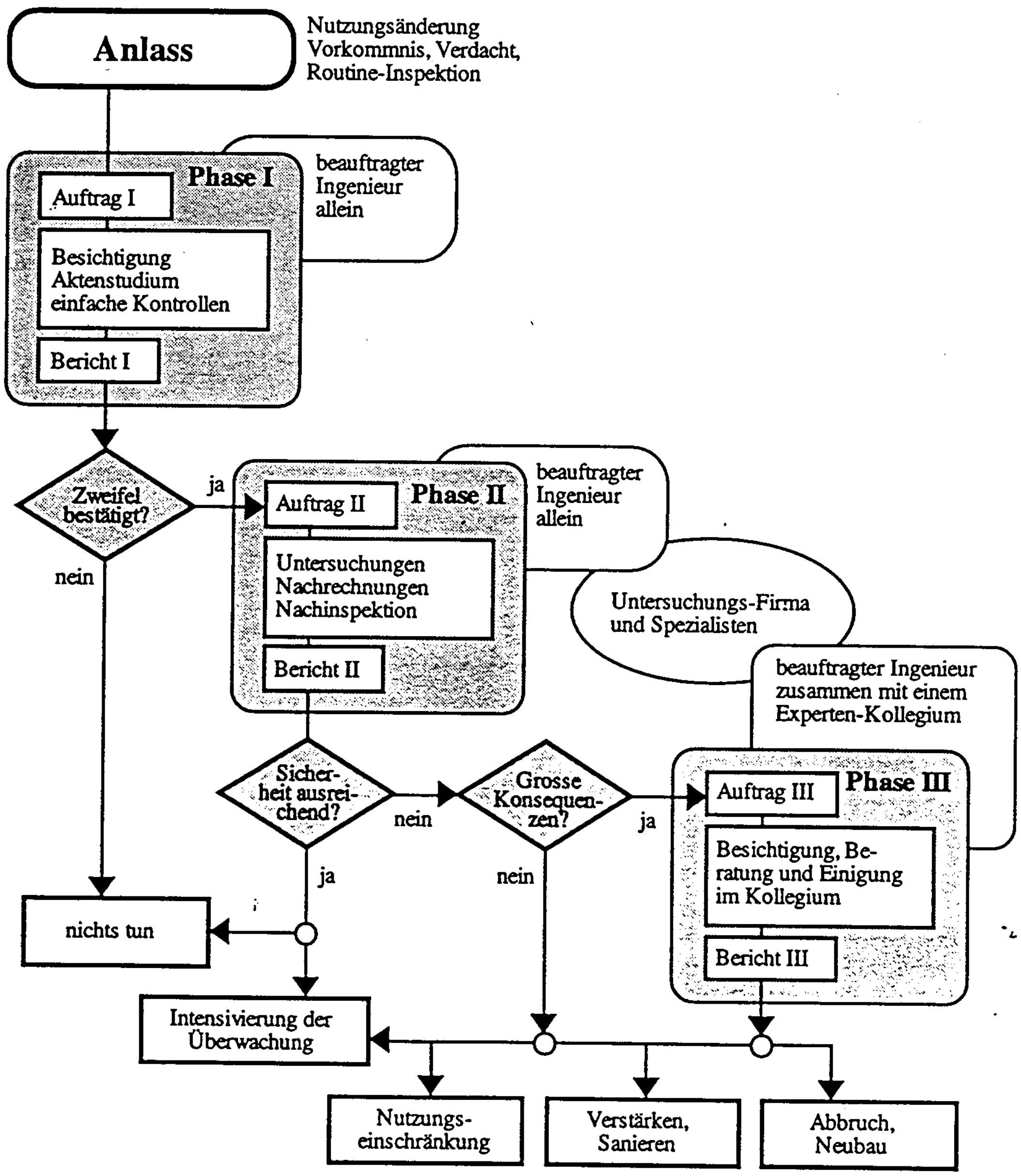

Bild 1: Phasen der Beurteilung bestehender Tragwerke.

3.3.2.2.1 Phase I: Grobe Erstbeurteilung

Die grobe Erstbeurteilung bezweckt, mit grundsätzlich bescheidenen, dem Problem aber angemessenen Mitteln, die vorhandenen Zweifel auszuräumen oder aber - falls dies nicht gelingt - vernünftige Vorschläge für das weitere Vorgehen zu machen. Die grobe Erstbeurteilung besteht aus einer Besichtigung, einem begleitendem Aktenstudium, einer überschlägigen Kontrolle der Tragsicherheit und einem abschließenden Bericht.

a) Besichtigung

Eine Besichtigung des fraglichen Objekts ist außerordentlich wichtig. Ziel ist unter anderem das Erkennen von ortstypischen Gefährdungsbildern, die das Tragwerk in Zukunft bedrohen könnten. Es geht weiter darum, Mängel, Schäden und alle Zeichen von Überlastung zu erkennen. Auch soll bei dieser Besichtigung eine erste, intuitive Beurteilung der Tragsicherheit erfolgen. Falls notwendig, sind auch erste sichernde Sofortmaßnahmen anzuordnen.

b) Aktenstudium

Falls überhaupt auffindbar, müssen in einem zweiten Teil der Phase I die vorhandenen Bauakten studiert werden: Pläne, Statische Berechnung, Bautagebuch, Rechnungen, Abnahme - Protokolle, Umbau - Pläne etc. Beim Studium dieser Akten muß versucht werden, die damalige Situation zu erfassen: Welche Ziele wurden angestrebt, welche Bauverfahren, welche Baustoffe angewendet? Welche ökonomische und organisatorische Struktur herrschte? Standen die Arbeiten unter Zeitdruck? Das sind sogenannte Qualitätsindikatoren.

Auch die Durchsicht der statischen Berechnung bringt eine Fülle von Informationen über Normen, Berechnungs- und Bemessungsmethoden, die zugrunde gelegten statischen Systeme und die zur Verfügung stehenden Rechenhilfsmittel. Sie zeigt gleichzeitig auf, wo aufgrund des heutigen Standes der Technik rechnerische Reserven liegen und, falls nötig, herangezogen werden könnten. Wie man diese Informationen bearbeitet, wird in 3.3.2.3 erläutert.

c) Ergänzung und Berichtigung der Bauakten

Dabei gilt festzuhalten, welche *Risiken akzeptiert* werden müssen. Man tut gut daran, diese aufzulisten und mit dem Eigentümer abzusprechen, damit rechtzeitig klar ist, wer im Schadensfall die finanziellen Konsequenzen trägt.

Es ist durch eine sorgfältige Zusammenstellung aller wesentlichen *Bauakten* dafür zu sorgen, daß für eine erneute Beurteilung die wichtigen Grundlagen unmißverständlich und lückenlos zur Verfügung stehen.

d) Grobe statische Beurteilung

Aufgrund der Einsicht in die Pläne und in die statische Berechnung kann in der Regel eine erste Abschätzung der vorhandenen Tragsicherheit vorgenommen werden.

Im Hinblick auf Tragwerke, die in dynamischer Hinsicht ein unbefriedigendes Verhalten zeigen, ist neben dem Tragwerkswiderstand auch die Tragwerkssteifigkeit zu beachten, die das Schwingungsverhalten entscheidend beeinflußt. Verstärkungen der Konstruktion, die in der Regel mit einer Erhöhung der Steifigkeit einhergehen, sind in solchen Fällen nicht immer das richtige Konzept.

Besonders sorgfältig muß man die dynamischen Beanspruchungen unter Erdbeben oder bei Anprallstößen angehen, denn hier besteht zwischen dem Tragwiderstand und der Duktilität des Tragwerks ein empfindliches Wechselspiel. Das Verstärken einer Konstruktion kann als Folge der damit einhergehenden Versteifung in Bezug auf Erdbeben durchaus zu einer Schwächung führen.

Schließlich sind auch ermüdungsbeanspruchte Bauteile sachgerecht zu beurteilen.

e) Bericht

Alle in der Phase I gewonnen Erkenntnisse werden in einem Bericht zu Händen des Auftraggebers zusammengefaßt. Falls die Zweifel, die zum Auftrag geführt hatten, im Verlauf der Phase I nicht aus dem Weg geräumt werden konnten, müssen mit der Phase II weitere Schritte eingeleitet werden.

3.3.2.2.2 Phase II: Detaillierte Untersuchung

Es ist eventuell sinnvoll, den gleichen Ingenieur mit den Arbeiten der Phase II zu betreuen, um so das gewonnene Vorwissen zu nutzen.

a) Untersuchungen am Bauwerk

Typisch für die Phase II sind die Untersuchungen am Bauwerk. Man nennt das *Aktualisieren* der Informationen über das Bauwerk. Hierzu müssen in der Regel spezialisierte Fachinstanzen hinzugezogen werden.
Oft werden standardmäßig alle möglichen Einflußparameter geprüft: Menge und Zustand der Bewehrung, Karbonatisierungstiefen, Chloridgehalte, Risse und Rißweiten, Anrisse, Ermüdungsrisse, Festigkeiten, Verhalten unter Last, Durchbiegungen, Verankerungen,

Injektion bei Spannkabeln etc. Ein solches undifferenziertes Vorgehen ist selten vernünftig. Auch können allzu intensive Untersuchungen, die selten völlig zerstörungsfrei sind, das Tragwerk unnötig schädigen.

Es ist vielmehr vernünftig und kostengünstig, auf der Basis der in Phase I gewonnenen Einsichten und Fragen gezielt ein geeignetes Untersuchungsprogramm aufzustellen und darin festzulegen, was zu prüfen ist. Die gründlich vorbereitete Untersuchung sollte vom beauftragten Ingenieur verantwortlich begleitet werden. Auch soll er soweit wie möglich Einfluß nehmen auf die Formulierung des Untersuchungsberichts

b) Nachrechnung

Die aus den Untersuchungen gewonnenen Zusatzinfomationen werden in die Nachrechnung einbezogen mit dem Ziel, die am Ende der Phase I noch immer bestehenden Zweifel nun endlich auszuräumen und eine ausreichende Tragsicherheit nachzuweisen. Man wird für die konventionelle Statik die aktualisierten Werte und modifizierte Rechenmodelle verwenden. Hier sei auf Abschnitt 3.3.2.3 verwiesen.

c) Bericht

Alle Ergebnisse der Phase II werden in einem Bericht zusammengefaßt, der wiederum an den Auftraggeber geht. Der Bericht gibt insbesondere über die Frage der Tragsicherheit Auskunft. Wenn die Tragsicherheit als ungenügend eingeschätzt wird, müssen eine intensivierte Überwachung, Nutzungseinschränkungen, eine Verstärkung und eventuell ein Abbruch und Neubau in Betracht gezogen werden.

Bei *geringer Tragweite* der Entscheidung zu einer dieser Maßnahmen ist es durchaus vertretbar, die Untersuchung mit der Phase II abzuschließen. Das ist z.B. der Fall, wenn keine Menschen gefährdet sind und erhöhte Sachschaden- oder Vermögensschadenrisiken in Kauf genommen werden können. Wenn keine Menschenleben gefährdet sind, sind auch Kosten-Nutzen-Überlegungen angebracht. Ein solcher Abschluß des Beurteilungsprozesses ist auch dann vertretbar, wenn man sich zu einer Verstärkung bzw. Sanierung der Konstruktion oder für Abbruch und Neubau entscheidet, sofern dies keine unverhältnismäßig großen finanziellen Konsequenzen zur Folge hat.
Treffen diese Kriterien jedoch nicht zu, hat man es mit einem komplexen Problem zu tun, bei dem die Entscheidung weitreichende Folgen hat. Der verantwortliche Ingenieur muß dann in seinem die Phase II abschließenden Bericht die Einleitung der Phase III

vorschlagen.

3.3.2.2.3 Phase III: Beratung im Experten Kollegium

Bei Problemen von großer Tragweite wird ein Experten - Kollegium einberufen, das die Vorschläge für den bevorstehenden Entscheid sorgfältig prüft. Der Eigentümer oder der Betreiber ist nicht Mitglied des Kollegiums, steht diesem aber für Auskünfte zur Verfügung.

Ein solches Experten - Kollegium tritt bei der Beurteilung bestehender Bauwerke gewissermaßen an die Stelle der Normen, die bei der Projektierung von Neubauten ein ausgewogenes Sicherheitsniveau gewährleisten. Insbesondere das Akzeptieren erhöhter Risiken sollte im Prinzip einem solchen Experten - Kollegium vorbehalten bleiben.

Der mit den Phasen I und II beauftragte Ingenieur wird dem Kollegium alle verfügbaren Unterlagen weitergeben und seine Vorschläge für das weitere Vorgehen begründen. Das Kollegium wird gut daran tun, das Bauwerk gemeinsam zu besichtigen und dann gemeinsam zu beraten.

Das Kollegium kann den Entscheid vertagen, um Zeit für weitere Untersuchungen am Bauwerk zu gewinnen. Es kann vom beauftragten Ingenieur auch weitere Untersuchungen fordern, bevor es zu einem Entscheid kommt. Dieser Entscheid sollte einstimmig sein und gemeinsam vor dem Eigentümer - gegebenenfalls auch vor der Öffentlichkeit - vertreten werden. Für den Entscheid tragen die Mitglieder des Kollegiums dem Eigentümer gegenüber gemeinsam die Verantwortung.

Das Experten - Kollegium berät den Bauherrn oder Betreiber demnach in letzter Instanz über die zu treffenden Maßnahmen. Es ist auch der Öffentlichkeit gegenüber verpflichtet und muß bei Gefahr für Leib und Leben alles Nötige in die Wege leiten, d. h. auch für den Eigentümer oder Betreiber eventuell unangenehme Maßnahmen durchzusetzen.

3.3.2.3 Aktualisierung von Informationen

Hauptaufgabe der Phase II ist das Aktualisieren der Kenntnisse über das Tragwerk. Diese Aktualisierung bezieht sich auf mehrere Problemkreise und sollte mit aller möglichen Objektivität vorgenommen werden. Es geht im wesentlichen um die Aktualisierung

- der Einwirkungen,
- der Festigkeiten,
- der Abmessungen,
- des statischen Systems,
- der Berechnungsverfahren,
- der Bemessungsmethoden
- sowie die Untersuchung auf Mängel und Schäden.

Auf die genannten Bereiche soll in der Folge eingegangen werden. Nicht behandelt wird allerdings, wie man Bauwerke untersucht und was man wie messen kann.

3.3.2.3.1 Einwirkungen

Bei der Aktualisierung der Lastannahmen muß beachtet werden, daß seit der Erstellung des fraglichen Bauwerks manche Anforderungen erheblich erhöht wurden.

Es liegt oft z.B. nahe, bei zeitabhängigen Einwirkungen (Schnee, Wind, Erdbeben usw.) die Wiederkehrperiode angemessen zu reduzieren. Diese Tatsache zeigt das nächste Beispiel.

Beispiel:

Die Bestimmung von Bemessungswerten für zeitabhängige Einwirkungen wird durch eine Modellierung dieser Zeitabhängigkeit durchgeführt. Ausgehend von der Verteilung der Grundgesamtheit oder einer Extremwertverteilung können statistische Parameter wie Auftretenswahrscheinlichkeit oder Wiederkehrperiode nur mit Bezug zu gegebenen Zeitdauern, bzw. der Anzahl von Realisationen der betreffenden Einwirkung angegeben werden.

In der Praxis des konstruktiven Ingenieurbaus bedeutet dies, daß die Bauwerke mit Bezug auf zeitabhängige Lasten für eine gegebene Nutzungsdauer berechnet werden. Im Rahmen des Sicherheitskonzepts werden charakteristische Werte für die Lasten berücksichtigt, d.h. Werte, die einer Wiederkehrperiode T entsprechen. Zum Beispiel ein Jahrhundertereignis, d.h. eine der 100-Jahre Wiederkehrperiode entsprechende Lastintensität entspricht der 99%-Fraktile der Verteilung der jährlich maximalen Lastintensitätswerte.

Da nun Bauwerke für eine endliche Nutzungsdauer T bemessen bzw. ausgelegt werden, ist für den entwerfenden Ingenieur auch der Zusammenhang zwischen der geplanten

Nutzungsdauer T und der Wiederkehrperiode P von Interesse. Die Wahrscheinlichkeit p, daß die der Wiederkehrperiode P entsprechende Lastintensität während der Nutzungsdauer T auftritt, ergibt sich z.B. zu:

$$p=1-(1-1/P)^T$$

Ist man nun vor die Frage gestellt, wie groß die Wahrscheinlichkeit ist, daß ein Jahrhundertereignis während der Nutzungsdauer eines Bauwerks von 50 Jahren auftritt, ergibt sich:

$$p = 1 - (1 - 1 / 100)^{50} = 0.395$$

Beträgt die Nutzungsdauer dagegen 5 Jahre:

$$p = 1 - (1 - 1 / 100)^{5} = 0.049$$

Man stellt somit fest, daß bei einer abnehmenden Nutzungsdauer die Wahrscheinlichkeit des Auftretens des Ereignisses abnimmt. Daraus kann man schließen, daß bei kleineren Restnutzungsdauern eine Abminderung der Bemessungswerte von zeitlich veränderlichen Einwirkungen vorgesehen werden kann (vgl. Tab. 4).

	T=1	T=5	T=20	T=50
P=10	0.1	0.41	0.88	0.99
P=100	0.01	0.05	0.18	0.39
P=1000	0.001	0.005	0.02	0.05

Tabelle 4: Auftretenswahrscheinlichkeit p als Funktion der Wiederkehrperiode P und der Nutzungsdauer T (P, T in Jahren).

Auch in Bezug auf Eigenlasten und ständige Lasten ist eine Aktualisierung nötig und sinnvoll. Gerade letztere geben oft zu Überraschungen Anlaß (zusätzliche Beläge, unberücksichtigt Zwischenwände etc.). Die Aktualisierung mag angesichts ausgeräumter Unsicherheiten allerdings eine gewisse Reduktion der entsprechende Teilsicherheitsbeiwerte rechtfertigen.

Auch Lasten in Lagern, Fabrikgebäuden usw. müssen aktualisiert werden. Dabei ist in der Regel durch unmißverständliche Nutzungsanweisungen und ausreichende Überwachung dafür zu sorgen, daß die berücksichtigten Lasten nicht überschritten werden. Eine Reduktion der Teilsicherheitsbeiwerte für die Lasten ist hier jedoch nicht angebracht.

Auch Lasten auf Bahn- und Straßenbrücken dürfen reduziert werden, wenn zuverlässig dafür gesorgt ist, daß festgelegte Maximalwerte nicht überschritten und Fahrvorschriften eingehalten werden.

3.3.2.3.2 Festigkeiten und andere Baustoffeigenschaften

Bei der damaligen Bemessung des zu beurteilenden Bauwerks wurden gewisse Anforderungen an die Eigenschaften von Baustoffen gestellt. Angesichts der Möglichkeit, daß solche Anforderungen bei der Erstellung nicht erfüllt werden, hat die entsprechende Norm Sicherheitsvorgaben geschaffen.

Im Beurteilungszeitpunkt besitzt man weit bessere Kenntnis über die maßgebenden Baustoffeigenschaften aufgrund von aus dem Bauwerk entnommenen Proben. Es ist gerechtfertigt, diese bessere Kenntnis in die Beurteilung einzubringen und außerdem gewisse Sicherheitsvorhalte abzubauen. Andererseits sind aber auch die Beobachtungen bezüglich Korrosion, Ermüdung, Abnützung, Versprödung etc. in die Beurteilung einzubeziehen. Auch im Hinblick auf Verstärkungen sind die Beobachtungen sorgfältig zu dokumentieren, z.B. bezüglich Schweißneigung bestimmter Stähle, usw.

Es ist zu beachten, daß die Aussagekraft der wenigen aus dem Bauwerk entnommenen Proben nicht sehr groß ist. Die daraus gewonnenen Zahlenwerte sind lediglich als Anhaltspunkte aufzufassen, die aufgrund von Erfahrungswerten zu ergänzen sind. So ist z.B. die Streuung der Eigenschaften von Baustoffen älterer Bauwerke sicher größer als diejenige in neueren Konstruktionen.

Es ist zweckmäßig, die Prüfresultate der Stichproben auf statistischer Basis zu bearbeiten, um sich bei der Interpretation von den Zufälligkeiten der Einzelwerte zu lösen. Bei der Extrapolation auf Fraktilwerte muß der Umfang der Stichprobe beachtet werden (vgl. 4.2.3 Modell für die Betonfestigkeit).

3.3.2.3.3 Abmessungen

Planmäßige Werte sind - sofern das von Belang ist - durch gemessene Werte zu ersetzen, und zwar im günstigen wie auch im ungünstigen Fall.

3.3.2.3.4 Statische Systeme

Die für das Tragverhalten des Bauwerks wichtigen statischen und kinematischen Randbedingungen (Einspannungen, Lagerungsbedingungen, freie Beweglichkeit von Lagern und Fugen, u.a.) sind zu überprüfen. Sie legen die bei der Beurteilung der Tragsicherheit anzunehmenden statischen Systeme fest.

Auch die statischen Systeme waren früher angesichts der zur Verfügung stehenden Rechenhilfsmittel einfacher und damit gröber. Durch detailliertere Modelle, z.B. durch das Erfassen räumlicher Tragwirkung, lassen sich oft Reserven ausnützen. Das ist jedoch nicht immer möglich, denn oft ist das, was z.B. als kreuzweise tragende Platte erscheint, rechnerisch nur in einer Richtung tragend und dementsprechend bewehrt. Auch können die anschließenden, die Auflagerkräfte weiterleitenden Konstruktionselemente zu schwach sein, um einen alternativen Kraftfluß zu ermöglichen. Es sind unter Umständen gezielte Nachinspektionen zweckmäßig oder notwendig, um sich in diesem Bereich Sicherheit zu verschaffen.

Auf der anderen Seite enthalten Bauwerke oft sogenannte nichttragende Teile, die man in einer Nachrechnung zum Tragen heranziehen kann. Ein typisches Beispiel sind die sog. nichttragenden Wände, die oft entscheidend zur Stabilisierung bestehender Bauten beitragen. Man kann sie rechnerisch heranziehen, muß dann natürlich deren Funktion auch über die Restnutzungsdauer sicherstellen.

Ein anderes Beispiel sind durchlaufende Stabtragwerke, die man jedoch seinerzeit als gelenkig verbundene Teilsysteme angesehen hat. Man kann die Durchlaufwirkung gegebenfalls zumindest teilweise in Rechnung stellen und damit Reserven mobilisieren.

3.3.2.3.5 Bemessungsmethoden

Die Berechnungsverfahren haben sich im Laufe der Zeit geändert. So stützt man sich heute oft nicht mehr auf elastische Verfahren sondern auf die statischen Methoden des Traglastverfahrens; dies gestattet, gewisse Reserven bei der Nachrechnung aufzulösen. Ähnliches gilt für die dynamische Auslegung von Gebäuden gegen Erdbeben.

3.3.2.3.6 Bemessungskonzepte

Hier unterscheidet man zwei unterschiedliche Aspekte:

a) In einigen früheren Normen wurden Tragelemente auf der Basis der zulässigen Spannungen unter Berücksichtigung eines sogenannten globalen Sicherheitsbeiwerts bemessen. Heute stellen wir in der Regel die als Schnittkräfte gegebenen Beanspruchungen dem entsprechenden Querschnittswiderstand gegenüber. Dabei wird ein semiprobabilistisches Nachweisformat unter Berücksichtigung von Teilsicherheitsbeiwerten zugrunde gelegt. Dadurch lassen sich oft Reserven mobilisieren.

b) Auf der anderen Seite ist zu beobachten, daß bezüglich einigen Grenzzuständen in den heutigen Normen strengere Anforderungen gestellt werden. Beispiele hierfür sind die Stabilitäts- und Ermüdungsgrenzzustände im Stahlbau oder das Versagen infolge Durchstanzen im Stahlbetonbau.

3.3.2.3.7 Mängel und Schäden

Das Tragwerk muß auf Mängel, Schäden und Alterserscheinungen sorgfältig geprüft werden.

3.3.2.3.8 Bauwerksgeschichte

Weitere interessante und wertvolle Hinweise bringt das Studium der Bauwerksgeschichte: Was ist während der Lebensdauer des Bauwerks alles vorgefallen? Es lohnt sich, Erkundigungen einzuziehen. Man erfährt so z.B., daß eine ganze Reihe von mehr oder weniger starken Erdbeben über das Tragwerk hinweggegangen sind, ohne Schäden anzurichten. Solche Auskünfte sind für die Beurteilung der Erdbebensicherheit eines Bauwerks von Belang.

3.3.2.4 Auswertung der zusätzlichen Informationen und Nachweis der Sicherheit

Zusätzliche Informationen können auf statistischer Basis ausgewertet und entsprechend aktualisiert werden. Diese Prozedur wird als Updating bezeichnet. Man unterscheidet zwischen:

a) Updating von einzelnen Variablen (z.B. Betonfestigkeit, Verkehrslast, usw.),
b) Updating von Grenzzuständen (z.B. Überstehen einer extremen Last, Messungen von Verformungen, usw.).

Die dazugehörigen Verfahren sind im Anhang C des von JCSS vorbereiteten Konzepts beschrieben. Beispiele sind hier im Abschnitt 3.4 gegeben. Mit Hilfe dieser Methoden kann der Zuverlässigkeitsindex ß für den betrachteten Grenzzustand neu bestimmt werden und an Hand von Zuverlässigkeitsanforderungen entsprechend beurteilt werden.

Die Definition von Zuverlässigkeitsanforderungen muß sich vor allem an der gegenwärtigen Praxis orientieren. Als Arbeitshypothese für eine wahrscheinlichkeitstheoretische Formulierung solcher Anforderungen muß gelten, daß die derzeitige Praxis auf bestimmten Gebieten bereits von der Wirtschaftlichkeit her optimal ist und das Sicherheitsbedürfnis der Gesellschaft befriedigt.

Wenn davon ausgegangen wird, daß bei jedem Bauwerksversagen, zumindest gedanklich, systematischer Wiederaufbau erfolgt, so ist über die Versagensrate (Wahrscheinlichkeit des Versagens pro Zeiteinheit) zu optimieren. Daraus können Zielsicherheitsindizes in Abhängigkeit von der Sicherheitsklasse hergeleitet werden. Tabelle 5a und 5b zeigen die von JCSS vorgeschlagenen Sicherheitsindizes. Sie gelten für den Bezugszeitraum von einem Jahr.

Relative Kosten der Sicherheitsmaßnahmen	Geringe Versagens-konsequenzen	Mittlere Versagens-konsequenzen	Große Versagens-konsequenzen
groß	3,2	3,7	4,2
mittel	3,7	4,2	4,7
gering	4,2	4,7	5,2

Tabelle 5a: Erforderliche Sicherheitsindizes, gültig für ein Jahr - Grenzzustände der Tragfähigkeit.

Relative Kosten der Sicherheitsmaßnahmen	Zielsicherheitsindex (irreversible Grenzzustände der Gebrauchsfähigkeit)
groß	1,3
mitttel	1,7
gering	2,3

Tabelle 5b: Erforderliche Sicherheitsindizes, gültig für ein Jahr - irreversible Grenzzustände der Gebrauchsfähigkeit.

3.3.2.5 Maßnahmen

3.3.2.5.1 Sichernde Sofortmaßnahmen

Sobald es der Augenschein oder andere Umstände als nötig erscheinen lassen, müssen zum Schutz von Menschen und Umwelt unverzüglich sichernde Maßnahmen angeordnet werden. Dies ist spätesten dann der Fall, wenn der dringende Verdacht besteht, daß die Tragsicherheit nicht gewährleistet ist. Als sichernde Sofortmaßnahmen sind in Betracht zu ziehen:

- ausreichende Beschränkung der Nutzung
- Abstützung von Bauteilen, die Menschen und Umwelt gefährden
- Absperrung von Teilen des Bauwerks
- Außerbetriebnahme und Absperrung des Bauwerks.

3.3.2.5.2 Administrative Maßnahmen

Sinnvoll ist oft eine *Intensivierung der Überwachung* von Bauwerken, sofern diese eine graduelle Verschlechterung der Tragsicherheit einer Konstruktion rechtzeitig aufdecken kann. Dies ist der Fall, wenn sich ein Versagen, z.B. durch wachsende Verformungen und Risse vorzeitig ankündigt. Bei Konstruktionen, bei denen ein unangekündigtes Versagen möglich scheint, ist hingegen eine Intensivierung der Überwachung in der Regel kein zuverlässiges Mittel.

Oft sind auch die Verfügung von Nutzungs- bzw. Nutzlastbeschränkungen denkbar. Es ist wichtig, solche Maßnahmen auf ihre Wirksamkeit und Durchsetzbarkeit hin zu untersuchen.

3.3.2.5.3 Verstärken der Konstruktion

Verstärken von Teilen eines Bauwerks oder Verstärken des gesamten Bauwerks.

3.3.2.5.4 Abbruch und Neubau

D.h. Ersetzen des beschädigten Tragwerks.

3.3.2.6 Entscheidungskriterien

Die Entscheidung, welche Maßnahmen getroffen werden sollen, ist mit folgenden Kriterien verknüpft:

Akzeptables Sicherheitsniveau: Das akzeptable Zuverlässigkeitsniveau ist als Funktion der Versagensfolgen (Personenschäden, Sachschäden, Umweltschäden) zu betrachten (s. Tab. 5a und 5b).

Wirtschaftlichtkeitsüberlegungen: Folgende Parameter müssen gegenübergestellt werden:

- wirtschaftliche Vorteile aus dem weiteren Nutzen des Bauwerks,
- Kosten für die notwendigen Maßnahmen (Inspektion und Instandsetzung).

Restnutzungsdauer: Vom besonderen Einfluß auf den Entscheid ist die vom Eigentümer angestrebte oder die vom Ingenieur auf Grund seiner Beurteilung zugestandene Restnutzungsdauer des Bauwerks. Darunter wird diejenige Zeitdauer verstanden, während der das Bauwerk noch in Betrieb bleiben soll bzw. darf. Sie legt gegebenenfalls auch den Zeitpunkt fest, zu dem im Hinblick auf eine weitere Nutzung eine erneute Beurteilung der Tragsicherheit erforderlich ist.

Soziale und politische Verhältnisse: Die allgemeine soziale Lage und die politische Situation und das aktuelle Umweltbewußtsein der Menschen kann besonders bei größeren Projekten (Abbruch einer Bohrplattform , Instandsetzung eines Kernkraftwerks) eine maßgebende

Rolle beim Entscheidungsprozeß spielen.

3.4 Beispiele

Die Beispiele beziehen sich auf den nachträglichen Sicherheitsnachweis von Offshore-Konstruktionen. Die umfangreichen Offshore-Aktivitäten in der Nordsee haben auch zur Intensivierung der Erforschung der Seegangsbedingungen geführt. Die Bemessung wird mit der wahrscheinlich größten Wellenhöhe in 100 Jahren , H_{100}, durchgeführt. Durch neue Messungen bzw. Beobachtungen können die Bemessungswerte erheblich verbessert werden.

3.4.1 Beispiel 1: Air Gap

Ein wichtiger Grenzzustand bei extremen Wetterbedingungen entsteht, wenn der Wellenkamm den untersten Deckträger der Plattform berührt; das bedeutet, es muß immer ein Luftspalt (air gap) *a* zwischen dem Wellenkamm und dem untersten Deckträger existieren (s. Bild 2).

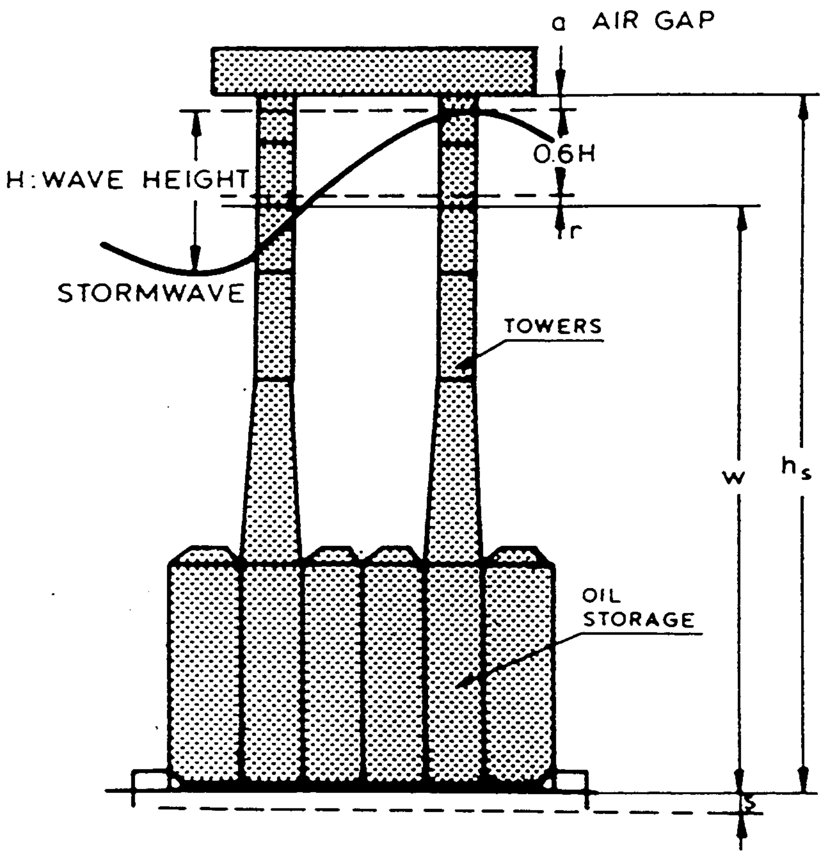

Bild 2: Technische Beschreibung - air gap Beispiel.

Der Grenzzustand kann folgenderweise formuliert werden.

$a \leq 0 \quad \Leftrightarrow \quad H \geq (h_s - r - w - s)/0,6$

Der Faktor 0,6 entsteht durch die Nichtlinearität der Wellenbewegung. Die zur Zeit gültigen Bemessungsrichtlinien verlangen einen minimalen Luftspalt von a = 1,5 m. Dabei wird die Jahrhundertwelle H_{100} zugrunde gelegt.

Eine Gewichtsplattform wurde in diesem Beispiel anhand von vorhandenen Wellendaten für eine erwünschte Lebensdauer von 20 Jahren bemessen. Die Wellendaten wurden einer Verteilungsfunktion angepaßt. Nach 15 Jahren ist die Möglichkeit einer Restlebensdauer von 10 Jahren gefragt; das entspricht einer Verlängerung der ursprünglichen Lebensdauer von 5 Jahren. Während der 15 Jahre sind neue Wellendaten registriert worden und eine neue realistischere Verteilungsfunktion konnte angepaßt werden. Die Sicherheit wird dadurch nachgewiesen, daß der Luftspalt-Grenzzustand mit einer ausreichenden Wahrscheinlichkeit p_F über den Bezugszeitraum T nicht erreicht wird:

$p_F = P\,[a \leq 0] = P\,[H \geq (h_s - r - w - s)/0,6]$

Tabelle 6 zeigt Eingabe- und Ausgabeparameter für drei charakteristische Zustände:

a) wie wurde bemessen (ursprüngliche Wellendaten, angenommene Abmessungen),
b) wie wurde installiert (neue Abmessungen),
c) nach 15 Jahren unter Berücksichtigung der zusätzlichen Daten.

Der Vergleich der drei Zustände wird durch die über den betrachteten Zeitraum bezogene Wahrscheinlichkeit p_F gegeben. Aus den Ergebnissen folgt, daß eine 5jährige Verlängerung der ursprünglichen Lebensdauer keine Gefährdung im Vergleich zum ursprünglichen Sicherheitsniveau darstellt.

Parameter	Wie bemessen	Wie installiert	Nach 15 Jahren
Gesamthöhe der Konstruktion h_s	128,5 m	130,0 m	130,0 m
Wassertiefe w	110,0 m	110,0 m	110,0 m
Flutbereich r	2,0 m	2,0 m	2,0 m
Senkung s	0,0 m	0,0 m	0,5 m
Luftspalt (air gap) a	1,5 m	3,0 m	1,3 m
Jahrhundert - Welle	25,0 m	25,0 m	27,0 m
Erwartete Lebensdauer	20 Jahre	20 Jahre	10 Jahre
Wahrscheinlichkeit $p_F = P[a \leq 0]$	0,063	0,018	0,045

Tabelle 6: Eingabe und Ausgabewerte des air–gap Beispiels.

3.4.2 Beispiel 2: Pfahlgründung

Das Beispiel stellt den Vergleich einer deterministischen und einer probabilistischen Nachrechnung der Pfahlgründung einer Plattform in der Nordsee dar (s. Bild 3).

Die Plattform wurde im Jahr 1975 mittels weniger Baugrunddaten (s. Bild 4) geplant und gebaut. Im Jahr 1993 wurde die Sicherheit der Bohrplattform und speziell deren Pfahlgründung anhand von neuen Baugrundinformationen (s. Bild 5) untersucht. Durch neue Wellendaten konnte die Verteilung der extremen Wellenlast neu ermittelt werden (s. Bild 6).

Die Ergebnisse der Untersuchungen sind in Bild 7 dargestellt. Zunächst wurde für beide Zustände:

a) 1975 Planungszustand (geringe Informationen),
b) 1993 Inspektionszustand (zusätzliche Informationen),

der globale Sicherheitsbeiwerts (deterministischer Ansatz) der Pfähle ermittelt. Anschließend wurde unter Berücksichtigung der Streuungen der Einflußparameter für beide Zustände die Versagenswahrscheinlichkeit (probabilistischer Ansatz) berechnet. Obwohl der globale Sicherheitsbeiwert im Zustand b) kleiner als im Zustand a) ist, ist die dazugehörige Versagenswahrscheinlichkeit aufgrund der geringeren Streuung ebenfalls kleiner. Das zeigt, daß die Sicherheit der Pfähle in Wirklichkeit größer ist, als im Planungszustand angenommen.

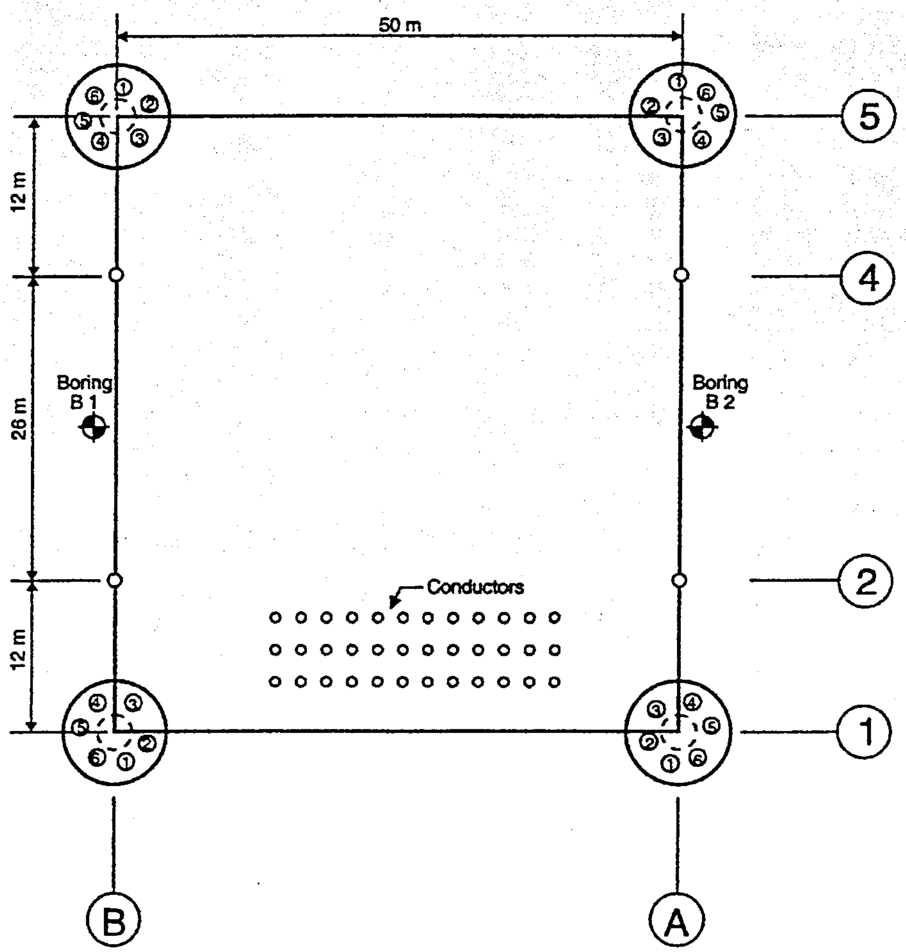

Bild 3: Layout der Plattformgründung

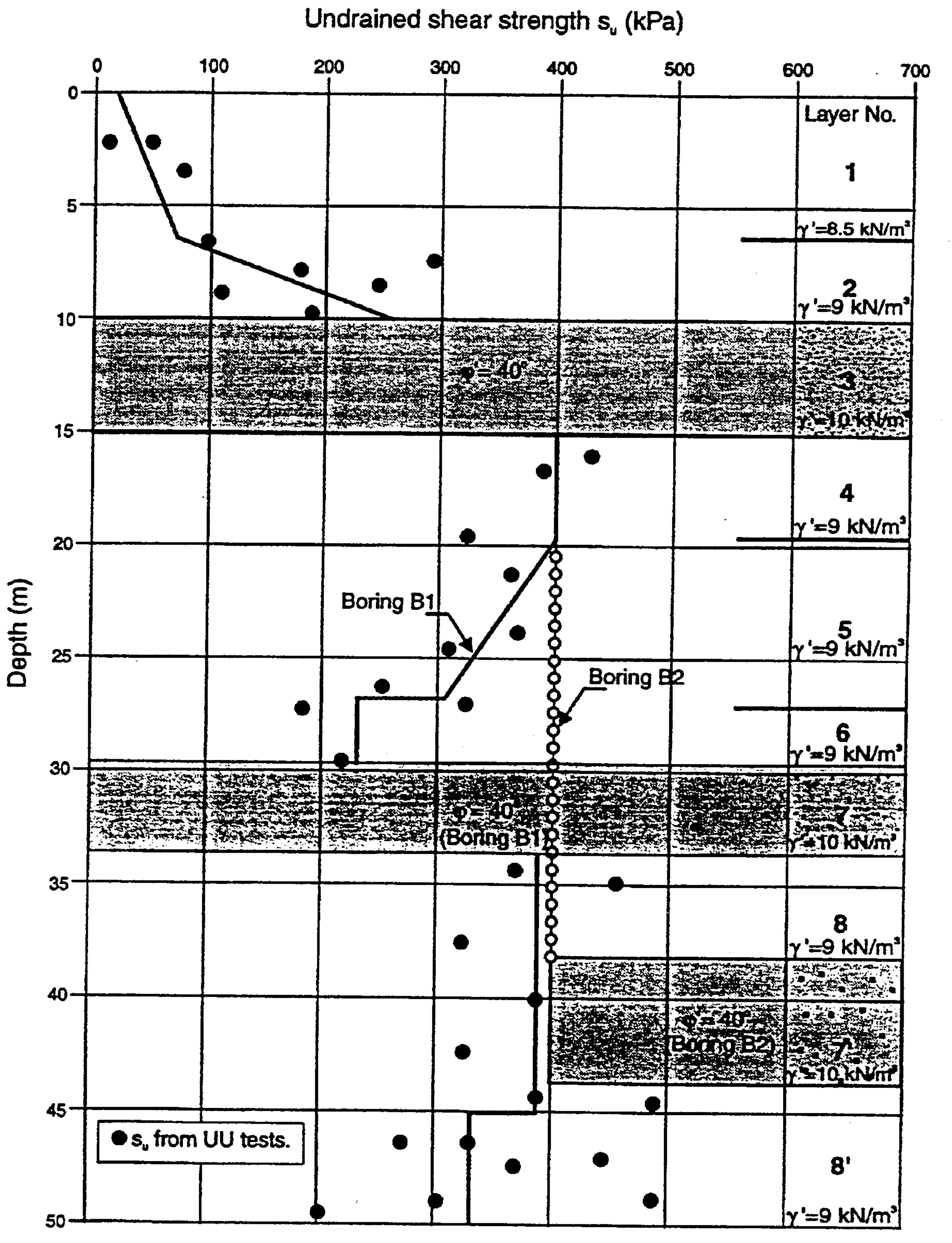

Bild 4: Bodenprofile für den axialen Bohrpfahlwiderstand, Daten von 1975.

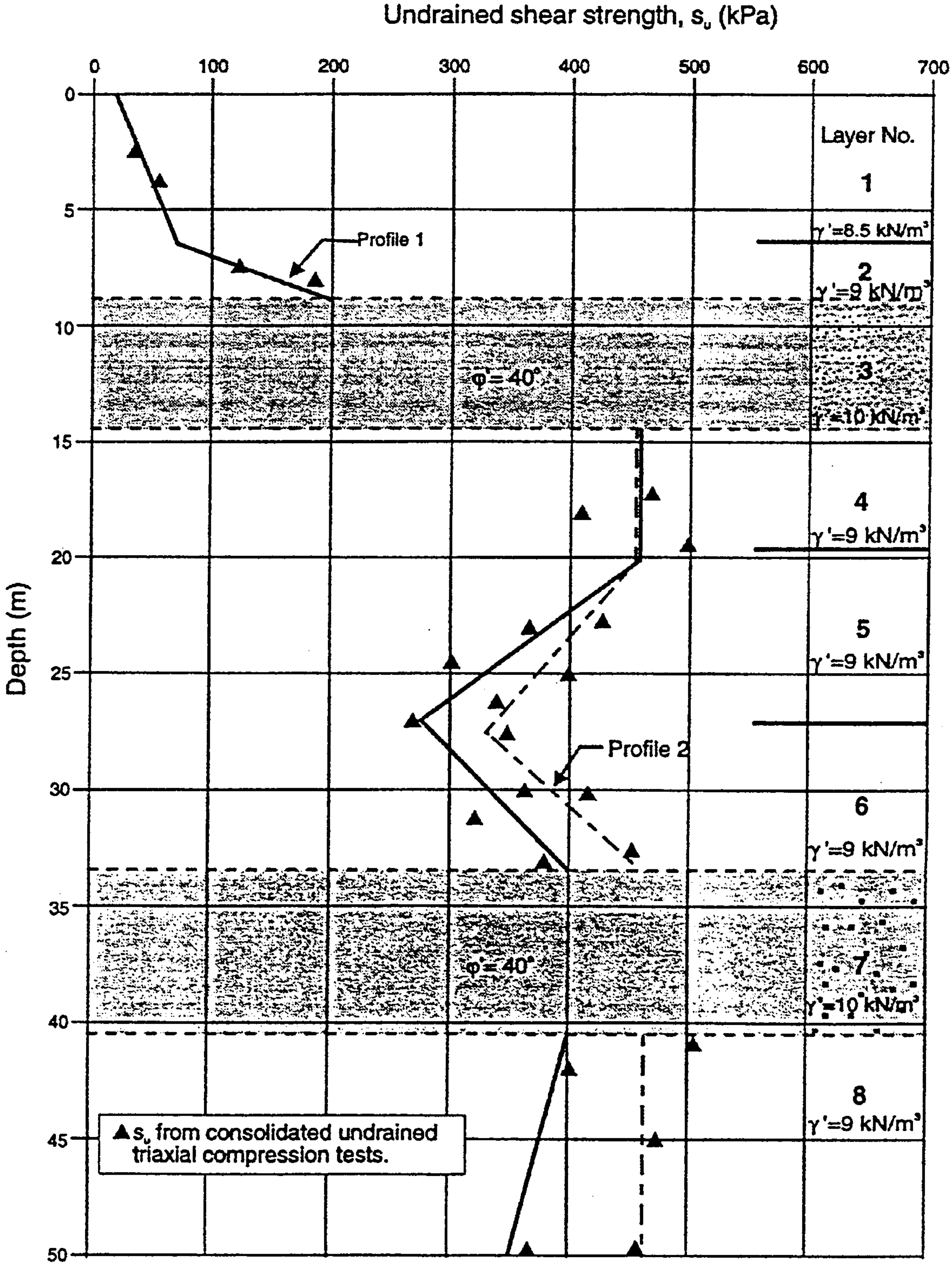

Bild 5: Bodenprofile für den axialen Bohrpfahlwiderstand, Daten von 1993.

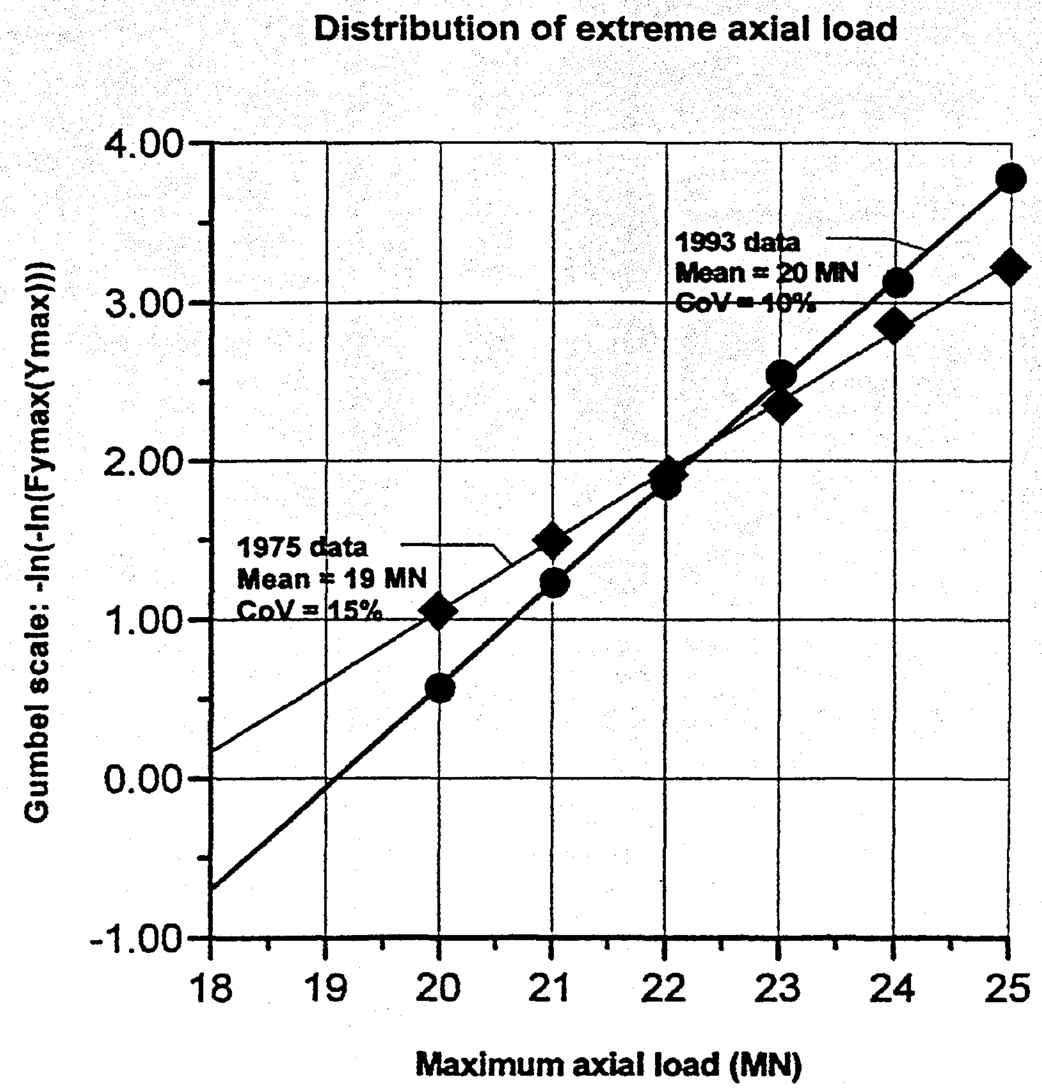

Bild 6: Verteilung der Jahrhundert-Wellenlast (Gegenüberstellung der Daten von 1975 und 1993).

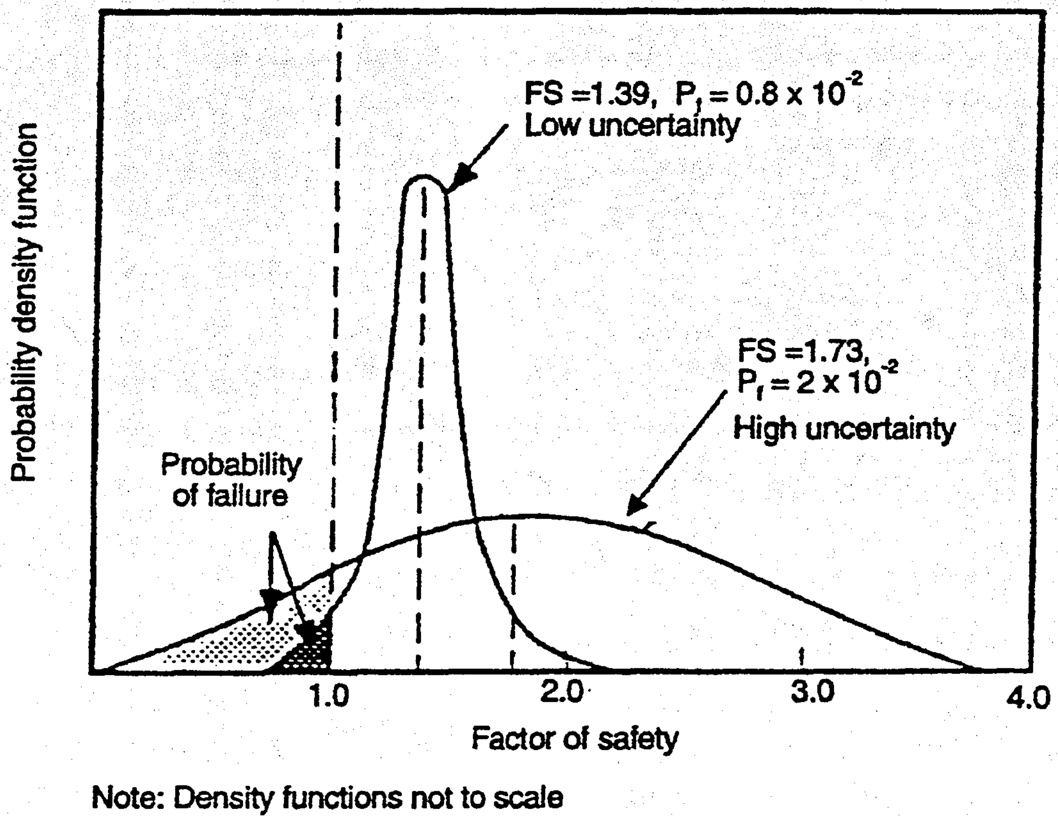

Bild 7: Sicherheitsfaktor und Versagenswahrscheinlichkeit des am meisten beanspruchten Bohrpfahls.

4 Probabilistischer Model Code

4.1 Übersicht

Die wesentlichen Aspekte für ein probabilistisches Sicherheitskonzept, d. h. ein Konzept dessen Elemente aus Sicherheitsbetrachtungen direkt ableitbar sind, wurden in den Sitzungen des JCSS ausführlich diskutiert. Sie sind in dem probabilistischen Model Code enthalten, der sich aus drei Teilen zusammensetzt:

Teil 1 Bemessungsgrundlagen: Hier werden grundlegende Aspekte, wie Bemessungssituationen, Grenzzustände, Lasten, Bauteilwiderstände, Berechnung der Versagenswahrscheinlichkeit, Zielsicherheit usw., erläutert.

Teil 2 Lasten: Für jede Last wird das Basismodel kurz beschrieben und ein operatives stochastisches Model angegeben.

Teil 3 Baustoffeigenschaften: Ein baupraktisches stochastisches Model wird für jede festgelegte Baustoffeigenschaft beschrieben.

Durch die Anwendung des Model Code kann die Bemessung und der Sicherheitsnachweis direkt auf der Grundlage der Zuverlässigkeitstheorie in folgenden Schritten durchgeführt werden:

- Festlegung der Sicherheitsanforderungen (Festlegung der Sicherheitsklasse und Anwendung der Tabellen 5a und 5b);
- Definition der Grenzzustände (Grenzzustände der Tragfähigkeit und der Gebrauchsfähigkeit);
- Vordimensionierung und Berechnung der Wahrscheinlichkeit des Überschreitens der jeweiligen Grenzzustände mit Hilfe von Software - Programmen zufriedenstellender Genauigkeit (10%) und unter Anwendung der in den Teilen 2 und 3 angegebenen stochastischen Modelle und der entsprechenden Software;
- Sicherheitsnachweis (der berechnete Sicherheitsnindex muß größer oder gleich dem zulässigen Sicherheitsindex nach Tabelle 5a und 5b sein) und endgültige Dimensionierung.

Somit erfolgt die Bemessung direkt auf einer zuverlässigkeitstheoretischen (probabilistischen) Grundlage. Dadurch ist im Vergleich zur Bemessung mit starr vorgeschriebenen Sicherheitselementen eine Flexibilität bezüglich des Sicherheitsniveaus und der damit verbundenen Wirtschaftlichkeit der Konstruktionen möglich.

Das Inhaltsverzeichnis für jeden Teil des Model Code und die verantwortlichen Personen für die schon erarbeiteten Notes sind in Tabelle 7 dargestellt. Ausgewählte Notes aus dem Probabilistischen Model Code werden im Abschnitt 4.2 beschrieben.

TEIL 1: BEMESSUNGSGRUNDLAGEN

Kapitel	*Titel*	*Verantwortliche*
1-8	Bemessungsgrundlagen	Diamantidis et. al.

TEIL 2: LASTEN

Kapitel	*Titel*	*Verantwortliche*
2.0	Allgemeines	Vrouwenvelder
2.1	Eigengewicht	Vrouwenvelder
2.2	Verkehrslast	Petschacher
2.3	Industrial Storage	
2.4	Kräne	
2.5	Bewegliche Verkehrslasten	Östlund
2.6	Parkhauslasten	Rackwitz
2.7	Silos	Rackwitz
2.8	Flüssigkeiten/Gase	
2.9	Temperatur	
2.10	Erddruck	Denver/Calle/Ditlevsen
2.11	Wasser/Grundwasser	
2.12	Schnee	Vrouwenvelder/Rackwitz
2.13	Wind	Lungu/Rackwitz
2.14	Temperatur	
2.15	Wellen	Hagen/Guedes -Soares
2.16	Lawinen	
2.17	Erdbeben	Lungu/Diamantidis
2.18	An/Aufprall	Vrouwenvelder
2.19	Explosion	Vrouwenvelder
2.20	Brand	Vrouwenvelder

TEIL 3: BAUSTOFFE

Kapitel	*Titel*	*Verantwortliche*
3.1	Beton	Vrouwenvelder/Rackwitz
3.2	Betonstahl	Rackwitz/Chryssanthopoulos
3.3	Spannstahl	
3.4	Baustahl	Chryssanthopoulos/Goyet
3.5	Holz	Ditlevsen/Larsson
3.6	Aluminium	Rackwitz/Costeas
3.7	Boden	Calle/Denver
3.8	Mauerwerk	
3.9	Modellunsicherheiten	
3.10	Dimensionen	Holicky/Vrouwenvelder
3.11	Imperfektionen	Holicky/Vrouwenvelder

Tabelle 7: Inhaltsverzeichnis für jeden Teil des Model Code und verantwortliche Personen.

4.2 Ausgewählte Notes

4.2.1 Eigenlast

Eigenlasten werden in zwei Gruppen eingeteilt:

- Eigenlast der tragenden Konstruktion, welche in der Regel über die Zeit unveränderlich ist,
- Eigenlast der nicht-tragenden Bauteile und Einbauten, bei der zeitliche Änderungen möglich sind.

Die Eigenlast G ist im allgemeinen gegeben durch $\gamma * V$, mit γ als spezifischer Wichte und V als Volumen. Sowohl γ als auch V sind in der Regel als streuende Größen anzunehmen. Die Streuung von V ist von der Streuung der jeweiligen Abmessungen abhängig. Bei Betonbauteilen beobachtet man eine von der absoluten Größe der Abmessungen d_i im wesentlichen unabhängige Streuung von σ_{di} = 0,5 cm - 1,5 cm, bei Stahlbauteilen ist σ_{di} = 0,1 cm - 0,5 cm. Bei dicken Bauteilen sind Abmessungsstreuungen daher vernachlässigbar. Für γ und $V = f(d_i)$ können Lognormalverteilungen angenommen werden. Dann ist G ebenfalls lognormalverteilt.

Die Eigenlast nicht-tragender Bauteile wird häufig ebenfalls als zeitunabhängig angesetzt. Bei genaueren Betrachtungen ist die Modellierung als *Poisson'scher* Rechteckwellenprozeß möglich. Die Lastwechselrate liegt in der Größenordnung 0,02 - 0,1 pro Jahr. Die in DIN 1055, Bl. 1 und 2 angegebenen Werte können als 50% bis 75% der Fraktile der zugehörigen Verteilung interpretiert werden.

Wasserdruck, insbesondere infolge natürlichen Grundwassers, läßt sich oft durch einen *Gaußschen* Prozeß mit einer Autokovariazfunktion vom Typ (σ^2 e^{-at}cos bt) erfassen. Die Parameter m, σ, a und b sind jeweils aus einer ausreichenden Anzahl von Pegelstandsmessungen zu ermitteln. Für die Austrittsrate über gegebene Pegel bzw. die Extremwertverteilung gelten die Angaben für *Gaußsche* Prozesse. Im gegebenen Fall kann eine Stutzung der Verteilung bei einem oberen bzw. unteren Grenzwert zweckmäßig sein.

Bei Lasten aus Boden- oder Gesteinsdruck gelten für das Eigengewicht analoge Betrachtungen. Bei der Bestimmung des Druckes ist zu beachten, daß die gängigen boden- und felsmechanischen Theorien z.T. mit erheblichen Modellunsicherheiten behaftet sind, sowohl in Bezug auf die Größe als auch in Bezug auf die räumlich Verteilung des Druckes.

4.2.2 Verkehrslast

Die statistische Analyse von Verkehrslasten ist schwierig, da diese in ihrer Größe und räumlichen Zuordnung über die Zeit veränderlich sind. Für die praktische Anwendung werden gleichförmig verteilte Ersatzlasten gesucht, welche die Wirkung der vorhandenen Verkehrslasten näherungsweise wiedergeben.

Die gleichförmig verteilten Verkehrslasten vernachlässigen die räumliche Abhängigkeit und erfassen deswegen nur unzureichend die Wirkung der tatsächlichen Lasten im Bauwerk. Zur Erfassung der räumlichen Abhängigkeit stehen verschiedene Lastmodelle zur Verfügung.

Die zeitliche Abhängigkeit der Verkehrslast in Hochhäusern wird durch zwei Lastprozesse gekennzeichnet. Die quasi ständig vorhandene Verkehrslast wird durch einen stationären Prozeß aus rechteckförmigen Amplituden dargestellt. Die Last resultiert aus Einrichtungs- oder Betriebsgegenständen sowie aus Personenlasten, die den Raum im Regelfall über einen längeren Zeitraum konstant belasten. Die vorübergehende Verkehrslast wird auch durch einen stationären Prozeß mit rechteckförmigen Impulsen von kurzer Dauer dargestellt und erfaßt außergewöhnliche Menschenansammlungen oder Anhäufung von Einrichtungsgegenständen, bedingt z.B. durch Renovierung. Lastmessungen bezüglich beider Verkehrslasttypen sind in vielen Ländern durchgeführt worden. Das stochastische Modell für beide Verkehrslasttypen wird durch die Extremwertverteilung, Typ I (*Gumbel*verteilung), beschrieben.

$$F_{Qmax}(x) = \exp\left[-\lambda T(1-F_Q(x))\right]$$

mit

λ : Erneuerungsrate

T : Bezugszeitraum

F_Q : Augenblicksverteilung der Verkehrslast mit Mittelwert (wird als Gumberlverteilung angenommen).

Die statistischen Parameter von $F_Q(x)$ werden ermittelt aus:

$$E[Q] = m$$

$$Var[Q] = \sigma_V^2 + \sigma_U^2 \, (A_0/A) \, \kappa(A)$$

A_0 : tatsächliche Fläche

A : Bezugsfläche

$\kappa(A)$: 0,5 ÷ 1,5 (geometrischer Faktor)

In der Tabelle 8 sind die statistischen Parameter je nach Nutzungsart dargestellt. Die angegebenen Werte sind nur Anhaltswerte und können durch zusätzliche Information erheblich verbessert werden. Die statistischen Werte für Hotel-, Schul- und Krankenhauslasten sind anhand von wenigen Messungen ausgerechnet und deswegen mit Vorsicht anzuwenden.

		Dauernd vorhandene Verkehrslast				vorübergehende Verkehrslast			
Gebäudetyp:	Fläche [m²]	m_Q [KN/m²]	σ_Q [KN/m²]	$1/\lambda$ [a]	σ_U [KN/m²]	m_P [KN/m²]	σ_U [KN/m²]	$1/\lambda$ [a]	d_P [d]
Büro	20	0,50	0,30	5	0,59	0,20	0,39	0,30	1-3
Wohnraum	20	0,30	0,15	7	0,29	0,30	0,59	1,00	1-3
Warenhaus	100	0,90	0,60	1-5	1,60	0,40	1,10	1,00	1-14
Schulzimmer	100	0,60	0,15	>10	0,41	0,50	1,40	0,30	1-5
Hotelzimmer	20	0,30	0,05	10	0,10	0,20	0,39	0,10	1-3
Krankenhaus-zimmer	20	0,40	0,30	5-10	0,59	0,20	0,39	1,00	1-3

Tabelle 8: Statistische Parameter der Verkehrslasten.

4.2.3 Betonkenngrößen

Die wesentlichen Betonkenngrößen sind folgende:

Betondruckfestigkeit X_1,
Betonzugfestigkeit X_2,
Elastizitätsmodell des Betons X_3,
Bruchdehnung X_4.

Alle Betonkenngrößen können in Abhängigkeit von der 28-Tage-Zylinderdruckfestigkeit X_c ausgedrückt werden, z.B. als:

$X_1 = X_c$ [MN/m²]
$X_2 = 0{,}3X_1^{2/3}$ [MN/m²]
$X_3 = 10{,}5X_1^{1/3}$ [MN/m² * 10³]
$X_4 = 6{,}0 * 10^3 X_1^{-1/6}$ [m/m]

Die geringere Betondruckfestigkeit im Bauwerk gegenüber der Zylinderfestigkeit wird durch einen Faktor λ = 0,96 erfaßt. Zeitliche Abhängigkeiten, z.B. Alterungseffekte, Kriechen, können durch entsprechende Funktionen $\alpha(t, \tau)$ mit τ als Belastungsalter näherungsweise nachgebildet werden.

Für das statistische Modell der Betondruckfestigkeit kann folgende Beziehung angenommen werden:

$$X_1 = X_c^{\lambda} Y_1,$$

wobei

X_c : Zylinderdruckfestigkeit als lognormalverteilte Variable, deren Mittelwert und Standardabweichung selbst Zufallsgrößen sind

Y_1 : lognormalverteilte Variable, welche zusätzliche Unsicherheiten auf Grund unterschiedlicher Sorgfalt beim Einbringen des Betons auf der Baustelle und etwaiger Nachbehandlung berücksichtigt.

Für die übrigen Betonkenngrößen erhält man dementsprechend:

$$X_2 = 0{,}3 X_c^{2/3} Y_2$$
$$X_3 = 10{,}5 X_1^{1/3} Y_3$$
$$X_4 = 6{,}0 * 10^3 X_c^{-1/6}\, Y_4.$$

Die Variablen Y_2, Y_3, Y_4 sind analog zu Y_1 definiert. In Tabelle 9 sind die statistischen Parameter für Y_1 Y_2, Y_3 und Y_4 zusammengestellt.

Variable	Mittelwert m_y	Variationskoeffizient
Y_1	1,0	0,06
Y_2	1,0	0,3
Y_3	1,0	0,15
Y_4	1,0	0,152

Tabelle 9: Statistische Parameter für Y_1 Y_2, Y_3 und Y_4 zur Beschreibung der Betonkenngrößen.

Die verschiedenen Streumaße reflektieren die unterschiedliche Empfindlichkeit der Kenngrößen gegenüber der jeweiligen Ausführungssorgfalt.

Statistische Parameter für X_c sind in Tabelle 10 aufgeführt.

		Betonklasse				
		B15	B25	B35	B45	B55
Ortbeton	m_{lnX}	3,40	3,65	3,85	-	-
	s_{lnX}	0,15	0,12	0,09	-	-
	n	1,0	2,0	3,0	-	-
	ν	3,0	4,0	4,5	-	-
Transportbeton	m_{lnX}	3,40	3,65	3,85	3,98	-
	s_{lnX}	0,14	0,12	0,09	0,07	-
	n	1,5	1,5	1,5	1,5	-
	ν	6,0	6,0	6,0	6,0	-
Beton für Fertigteile	m_{lnX}	-	3,80	3,95	4,08	4,15
	s_{lnX}	-	0,09	0,08	0,07	0,05
	n	-	2,0	2,5	3,0	3,5
	ν	-	4,5	4,5	5,0	5,5

Tabelle 10: Statistische Parameter für die Zylinderdruckfestigkeit X_c.

Die Verteilung der Betondruckfestigkeit X_c im so definierten Makromodell ist (als Prediktorverteilung) zentral-t-verteilt

$$F_X(x) = T_\nu[((\ln x - m_{lnX})/ s_{lnXc}) \cdot (n/(n+1))^{1/2}]$$

und kann durch eine logarithmische Normalverteilung approximiert werden. Stehen zusätzliche (z.B. projektspezifische) Stichprobenergebnisse zu Verfügung, dann können Parameter von Tabelle 10 als Parameter der Priori-Verteilung verwendet werden. Die resultierende Prediktorverteilung für X_c unter Einbeziehung der aktuellen Information ist wieder zentral-t-verteilt.

Die Korrelation von Betongrößen eines Bauteiles (Korrelationsabstand) kann näherungsweise durch die Einführung einer Korrelationskoeffizienten berücksichtigt werden.

$$\rho(r) = \exp(-(r/b)^2)$$

mit b ~ 1 m für gedrungene Bauteile oder b ~ 5 m für schlanke, stabförmige Bauteile.

4.2.4 Baustahlkenngrößen

Ausgewählte Baustahlkenngrößen sind:

- Fließgrenze X_1 [N/mm²]
- Zugfestigkeit X_2 [N/mm²]
- E-Modul X_3 [N/mm²]
- Bruchdehnung X_4 [m/m]
- *Poisson*-Zahl X_5 –

Von diesen Kenngrößen läßt sich eine Reihe anderer Kenngrößen ableiten, wie z.B. die Schubfestigkeit $X_6 = X_1/3$ oder der Schubmodul $X_7 = X_3/2(1+X_5)$.

Die Kenngrößen $\underline{X}$ können als lognormalverteilt, näherungsweise auch als normalverteilt angenommen werden mit dem Mittelwert und der Variationskoeffizient nach Tabelle 11.

Variable	Mittelwert (m_x)	Variationskoeffizient (V_x)
X_1	$X_{1,N}\alpha(t)\exp(-uV_x)$-C*	0,07
X_2	μ_1 B**	0,04
X_3	2,1 10^5	0,03
X_4	0,3	0,06
X_5	0,3	0,03

Tabelle 11: Statistische Parameter von Baustahlkenngrößen.

* C = 20 N/mm²: Abminderungsfaktor zur Berücksichtigung der statischen Streckgrenze. Der Parameter gibt den Abstand zwischen Nennfestigkeit $X_{1,N}$ und Mittelwert der Festigkeit an (u = (-1,5) / (-2,0)). Für $\alpha(t)$ gilt:

$$\alpha(t) = \begin{cases} 1{,}0 & \text{für Flanschteile} \\ 1{,}5 & \text{für Stegteile} \\ -1{,}0 & \text{für geschweißte Elemente} \end{cases}$$

**

$$B = \begin{cases} 1{,}5 & \text{für normalen Baustahl} \\ 1{,}4 & \text{für Legierungsstahl} \\ 1{,}1 & \text{für vergüteten Stahl} \end{cases}$$

Die angegebenen Variationskoeffizienten gelten für die gesamte Stahlproduktion. Die Variationskoeffizienten für Stahl aus einem Stahlwerk können niedriger angesetzt werden.

Die Kreuzkorrelationsmatrix der Kenngrößen wird folgenderweise angegeben:

$$\underline{R}_x = \begin{bmatrix} 1 & 0{,}75 & 0 & 0 & -0{,}45 \\ 0{,}75 & 1 & 0 & 0 & -0{,}60 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -0{,}45 & -0{,}60 & 0 & 0 & 1 \end{bmatrix}$$

4.2.5 Betonstahlkenngrößen

Die wichtigsten Materialkenngrößen von Betonstahl sind:

- Streckgrenze X_1
- Zugfestigkeit X_2
- Elastizitätsmodul X_3.

Die Streckgrenze X_1 kann als lognormalverteilt oder, wegen der geringen Streuung, als näherungsweise normalverteilt angesetzt werden. Der Mittelwert kann mit Bezug auf den Nennwert bestimmt werden, welcher etwa einer 5%-Fraktile der Grundgesamtheit entspricht. Die Streuung der Streckgrenze von Bewehrungsstäben ist vom Stabdurchmesser und von der Kontrolle der chemischen Stahlzusammensetzung abhängig. Überschlägig kann die Standardabweichung unabhängig von der Betonstahlklasse für die gesamte Produktion mit 28 N/mm², für Bewehrungsstahl aus dem gleichen Stahlwerk mit 15 N/mm² angenommen werden.

Für die Zugfestigkeit X_2 empfiehlt sich das gleiche Modell wie bei der Streckgrenze. Beide Größen sind straff miteinander korreliert ($\varrho = 0{,}8 \div 1{,}0$).

Der E-Modul X_3 kann als normalverteilt mit einem Variationskoeffizienten von 0,05 angenommen werden oder auch als deterministische Größe.

5 Schlußbemerkungen

Das aktuelle Sicherheitskonzept der Europäischen Normen basiert auf einem semiprobabilistisches Nachweisformat. Die Unsicherheiten der Einflußparameter werden durch Teilsicherheitsbeiwerte in der Bemessung der Bauteile berücksichtigt. Bei starr vorgeschriebenen Sicherheitselementen ist eine Flexibilität bezüglich des Sicherheitsniveaus und der damit verbundenen Wirtschaftlichkeit der Konstruktionen nur bedingt möglich.

Seit einiger Zeit ist man bestrebt, die Normen auf internationaler Ebene zu harmonisieren, um einheitliche Sicherheitsniveaus bei den Konstruktionen zu erreichen. Im Bauwesen ist die internationale Vereinigung Joint Committee on Structural Safety (JCSS) maßgebend an diesen Vorarbeiten beteiligt. Sie ist als Verbindungskomitee mehrerer internationaler Organisationen, wie z.B. Organisationen für Beton - und Spannbetonkonstruktionen, für Stahlbauten, für Brückenbau und für Hochbau, oder auch für Bauforschung, entstanden. Die Arbeitsgruppe des JCSS besteht aus etwa 30 Mitgliedern und wird seit 1996 von Prof. Dr.- Ing. D. Diamantidis geleitet. Das vorliegende Forschungsvorhaben ist mit der Arbeit des JCSS verknüpft. Es hat als wesentliches Ziel die Organisation der Arbeit des JCSS und die Zusammenstellung der Arbeitsergebnisse als Empfehlungen für spätere Normenwerke.

In diesem Schlußbericht werden die im Zeitraum 1996-1998 erzielte Ergebnisse der Arbeiten dargestellt: Die Arbeiten umfassen:

a) Numerische Überprüfung des Sicherheitsniveaus in den Eurocodes und Entwicklung eines Nachweisformats unter Zugrundelegung eines probabilistischen Konzepts;
b) Erweiterung des Sicherheitskonzepts zur Beurteilung der Sicherheit bestehender Konstruktionen.

Die Ergebnisse der Arbeiten sind detailliert in den Anhängen des Schlußberichts enthalten. Es wurden große Fortschritte erreicht: das Sicherheitskonzept zur Beurteilung bestehender Konstruktionen wurde fertiggestellt, das Sicherheitsniveau in den Eurocodes wurde für charakteristische Fälle überprüft und wesentliche Teile eines probabilistischen Bemessungskonzepts mit entsprechenden Modellen für Lasten und Bauteilwiderstände wurden erarbeitet und in den Sitzungen diskutiert.

Es sind weitere Arbeiten auf dem Vornormenniveau notwendig, um besonders das probabilistische Konzept für die Tragwerksbemessung zu vervollständigen und im Hinblick auf dessen Anwendbarkeit zu prüfen.

ANHANG I:

LISTE DER MITGLIEDER DER JCSS ARBEITSGRUPPE

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ANHANG II:

ÜBERPRÜFUNG DES SICHERHEITSNIVEAUS IN DEN EUROCODES

Reliability analysis of a reinforced concrete column designed according to the Eurocodes

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Summary

Reliability analysis of a built in reinforced concrete column designed according to Eurocodes 1 and 2 is a part of an extended research activity on Eurocode Random Variable Models supervised by JCSS. Presented results indicate that the reliability level of reinforced concrete columns designed according to the present generation Eurocodes may considerably vary depending on actual arrangement of the structure. To harmonise reliability levels provided by the Eurocodes for various structural members further research and calibration is required.

1. Introduction

Reliability analysis of reinforced concrete columns is part of an extensive research activity on Eurocode Random Variable Models supervised by the Joint Committee for Structural Safety JCSS [1]. The whole project covers reliability analysis of different structural members of a model multi-storey frame structure made of concrete or steel. The JCSS aims at providing a standardised set of statistical models for loads and structural properties which would reflect the present state of knowledge. Where necessary, the models should be adjusted in the future. It is expected that these models will be used as a practical design tool in conjunction with a probabilistic design criterion.

In a probabilistic design procedure a decision theoretical approach seems to be the most natural. However, as the models are only partly based on the experimental data, the calculated failure probabilities should not be identified directly with actual failure frequencies. That is why reliability criteria are usually defined through calibration to existing practice. In such a calibration procedure a set of structural elements are designed according to current design practice. For each of these elements the failure probability or reliability index is calculated, using the set of standardised statistical models. The resulting reliability indices may be then used as target reliability for the subsequent probabilistic design procedure. In such a way a combination of mechanical models, statistical models and corresponding target reliability which renders on the average the same design as current practice procedures may be derived.

This contribution presents preliminary results of reliability analysis of a built in reinforced concrete column designed according to newly developing Eurocode 1 [2, 3 and 4] and

Eurocode 2 [5]. The reliability analysis has been carried out using software product COMREL [6] developed by RCP Műnchen. It is expected that submitted investigation will contribute to desired calibration and possible future improvement of present generation of Eurocodes.

2. Structural characteristics

A model multi-storey structure considered in the this study is schematically shown in Fig. 1. It is assumed that each plenary frame in the transversal direction of the structure may be considered as unbraced sway frame. These transversal sway frames consist of four columns at a constant distance a_1; in the longitudinal direction of the structure they are located within a constant distance a_2 (see Fig. 1). The columns are considered as fully clamped in booth ends, at the top and at the bottom.

In the following reliability analysis of the edge column of an internal transversal frame having the height L and rectangular cross section $b \times h$ is considered. The cross section dimensions are chosen in such a way that the height h is two times (in one study case three times) the width b, thus h/b = 2 or 3. Considering different structural arrangements the total of 12 study cases indicated in Table 1 are analysed.

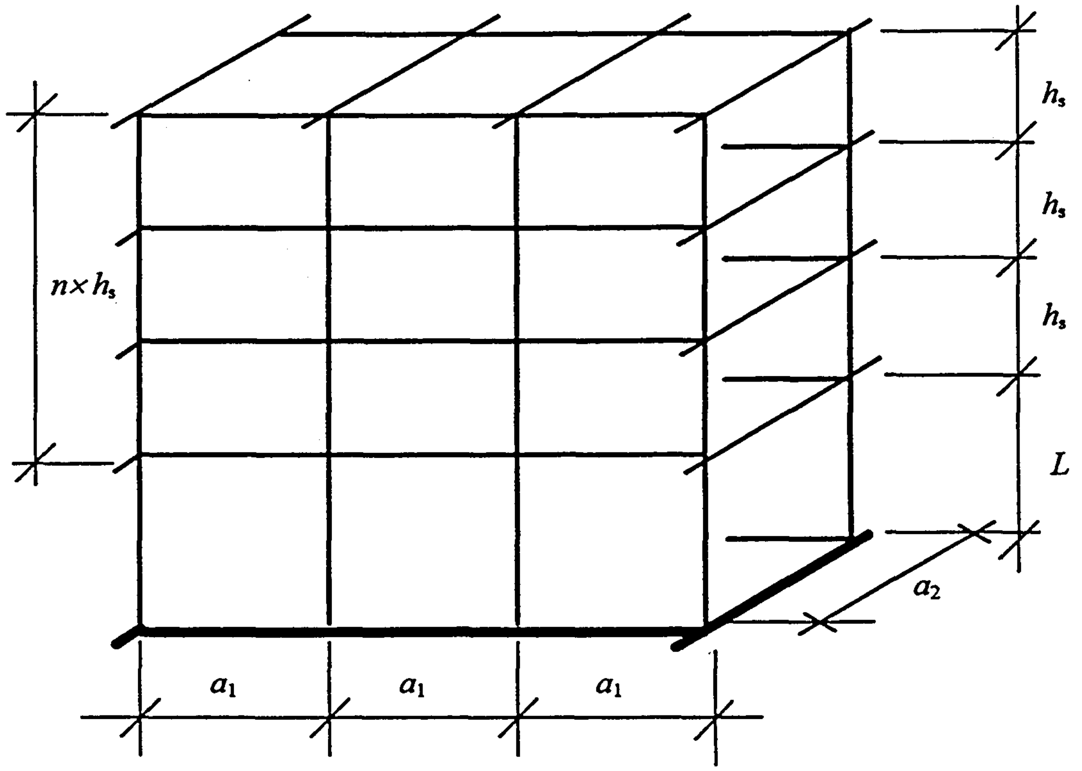


Fig. 1. Transversal frame of a multi-storey structure.

Study case	Number of storeys above the column n	Height of the analysed column L [m]	Transversal distance of columns a_1 [m]	Longitudinal distance of columns a_2 [m]	Cross section dimensions: width × height $b \times h$ [m×m]
1	10	6	5	5	0,35 × 0,70
2	10	3	5	5	0,25 × 0,50
3	10	9	5	5	0,35 × 0,70
4	10	12	5	5	0,45 × 0,90
5	10	6	4	5	0,35 × 0,70
6	10	6	7	5	0,35 × 0,70
7	10	6	5	4	0,30 × 0,60
8	10	6	5	7	0,40 × 0,80
9	1	6	5	5	0,25 × 0,50
10	3	6	5	5	0,25 × 0,50
11	20	6	5	5	0,40 × 0,80
12	10	6	5	5	0,25 × 0,75

Table 1. Study cases of a built in column.

Further it is assumed that the story height above the considered column is $h_s = 3$ m, permanent load is determined assuming reinforced concrete floor of a uniform equivalent thickness of 0.30 m (representing weight due to slab, columns, beams, floor and cladding).

3. Effect of actions

Effects of actions considered in the analysis of built in column consist of the axial force and bending moment, denoted again by N and M with appropriate subscripts. In the design calculation, the axial force and bending moment are represented by the design values N_d and M_d respectively. The maximum design axial force $N_{d,max}$ is given as

$$N_{d,max} = \gamma_G\ N_{W,k} + \gamma_Q \max \{N_{imp,k} + \psi_0\, N_{wind,k}\, ;\, N_{wind,k} + \psi_0 N_{imp,k}\} \quad (1)$$

where $\gamma_G = 1{,}35$ is the partial factor for permanent actions, $\gamma_Q = 1{,}50$ is the partial factor for the variable actions, ψ_0 is the factor for combination value, $N_{W,k}$ is the characteristic value of the axial force due to self weight, $N_{imp,k}$ is the characteristic value due to imposed load and $N_{wind,k}$ is the characteristic value due to wind action (positive values are accepted for compressive forces). The minimum design axial force $N_{d,min}$ is given as

$$N_{d,min} = \gamma_G\ N_{W,k} - \gamma_Q\, N_{wind,k} \quad (2)$$

where $\gamma_G = 1{,}00$ is the partial factor for favourable permanent actions, $\gamma_Q = 1{,}50$ is the partial factor for the variable actions.

Taking into account arrangement of the structure indicated in Fig. 1 the characteristic value due to self weight of n floors and one roof is given as

$$N_{W,k} = (n+1) a_1\, a_2\, t\, \rho_c\, / \, 2 \quad (3)$$

where ρ_c is the weight of concrete per unit volume considered as 0,024 MN/m^3. $N_{imp,k}$ is the characteristic value of imposed load from n floors given as

$$N_{imp,k} = n\, a_1 a_2\, p_{imp} / 2 \qquad (4)$$

Choosing a category B (Public Building) the characteristic value of floor imposed load $p_{imp\,k}$ equals 3 kN/m^2. For $n > 1$ the load reduction according to Eurocode 1 [3] should be included. $N_{wind,k}$ is the wind resulting from a pressure $C_p\, G\, p_{wind\,k}$ on a vertical area equal to $(L + nh_s)\, a_2$; multiplication by the height $(L + nh_s)/2$ gives the overturning moment. This moment is assumed to be balanced by the normal forces in the two outer columns, so:

$$N_{wind,k} = (1/2)(L + nh_s)^2 a_2 C_p G\, p_{wind,k} / (3\, a_1) = 0.271(L + nh_s)^2 a_2 / a_1 \qquad (5)$$

where the characteristic value of the wind action is taken for the return period of 50 years as $p_{wind,k} = 0.5$ kN/m^2; further for the gust (exposure) factor the value $G = 2.5$ and for the shape factor the value $C_p = 0.8 + 0.5 = 1.3$ is chosen [4].

The design value M_d of the bending moment M is given as

$$M_d = M_{d0} + N_d(e_a + e_2) = N_d(e_0 + e_a + e_2) \qquad (6)$$

where M_{d0} is the first order bending moment, $e_0 = M_{d0} / N_d$ is the first order eccentricity, e_a is the additional eccentricity taking into account geometric imperfections and e_2 is the second order eccentricity taking into account deformations of the column.

It is assumed that the first order moment M_{d0} is caused only by wind action, which is transmitted in each frame section of the width a_2 (see Fig.1) equally by the four columns fully clamped in and, therefore, the maximum first order bending moment M_{d0} due to wind load about the centroid of a column cross section is determined from the formula

$$M_{d0} = L[\gamma_Q\, C_p\, G\, p_{wind,k}\, (L+nh_s)\, a_2]/8 = 0{,}305\;\; L(L + nh_s)\, a_2 \qquad (7)$$

where L denotes the column height.

The eccentricities e_a and e_2 are determined in accordance with Chapter 2 and 4 of Eurocode 2 [5]. The additional eccentricity e_a is given as $e_a = \nu_a\, l_0/2$, where l_0 denotes the effective length of the column considered here by the lowest recommended value 1,12 L (for the case of a column of a sway frame), ν_a inclination from the vertical given by the minimum value 1/200 which is valid for all structures higher than 4 m when the second order effects are taken into account. Thus

$$e_a = 1{,}12\, L/(2 \times 200) = 0{,}0028\, L \qquad (8)$$

The second order eccentricity e_2 is dependent on the characteristics of the column cross section and should be generally determined by an iteration process. In accordance with equation (4.69) in [5] the second order eccentricity is given as

$$e_2 = 0{,}1 K_1\, l_0^2\, (1/r) \qquad (9)$$

where the coefficient K_1 depends on the slenderness ratio $\lambda = l_0 / i$ (i being radius of gyration) and is given by equations (4.70) and (4.71) in Eurocode 2 [5]. As in the all study cases here $\lambda \geq 35$ the value $K_1 = 1$ is considered. The curvature $1/r$ is given by equation (4.72) in [5] as

$$1/r = 2\, K_2\, \varepsilon_{yd} / (0{,}9\;\; (h - d_1)) \qquad (10)$$

where the coefficient K_2 is defined by equation (4.73) in [5] as follows

$$K_2 = (N_{ud} - N_d) / (N_{ud} - N_{bal,d}) \leq 1 \qquad (11)$$

where N_{ud} is the design capacity of the cross section, N_d is the design axial force and $N_{bal,d}$ is the force which maximises the ultimate moment of the cross section; in this study for symmetrical reinforcement $N_{bal,d} = 0,5\ \alpha f_{cd} A_c$, where α is a coefficient taking account of long term effects on the compressive strength.

The remaining variables entering equation (10), the design yield strength $\varepsilon_{yd} = f_{yd} / E_s$ and the effective depth of cross section $h - d_1$, are specified bellow (see also Fig. 2). Table 2 and 3 shows the resulting values of the effects of actions for all 12 study cases considered here.

Study case	$N_{d,max}$ [MN]	M_{d0} [MNm]	e_0 [m]	L [m]	e_a [m]	$A_s \times 10^4$ [m²]	A_s/bh [%]	e_2 [m]	M_d [MNm]
1	2,162	0,329	0,1522	6	0,0168	28,7	1,17	0,0245	0,418
2	2,078	0,151	0,0726	3	0,0084	22,1	1,23	0,0047	0,178
3	2,054	0,535	0,2373	9	0,0252	34,1	1,07	0,0591	0,725
4	2,353	0,768	0,3263	12	0,0336	38,2	0,94	0,1062	1,098
5	1,967	0,329	0,1673	6	0,0168	24,6	1,00	0,0265	0,415
6	2,736	0,329	0,1201	6	0,0168	41,4	1,69	0,0200	0,431
7	1,729	0,263	0,1523	6	0,0168	31,9	1,77	0,0285	0,343
8	3,028	0,461	0.1522	6	0,0168	37,4	1,17	0,0196	0,572
9	0,340	0,082	0,2422	6	0,0168	4,6	0,37	0,0485	0,105
10	0,702	0,137	0,1954	6	0,0168	10,9	0,87	0,0485	0,183
11	4,895	0,603	0,1232	6	0,0168	90,7	2,83	0,0141	0,755
12	2,162	0,329	0,1522	6	0,0168	37,5	2,00	0,0191	0,407

Table 2. Effects of actions for the maximum axial force $N_{d,max}$.

Study case	$N_{d,max}$ [kN]	M_{d0} [MNm]	e_0 [m]	L [m]	e_a [m]	$A_s \times 10^4$ [m²]	A_s/bh [%]	e_2 [m]	M_d [MNm]
1	0,464	0,329	0,7100	6	0,0168	17,9	0,73	0,0346	0,353
2	0,548	0,151	0,2755	3	0,0084	4,0	0,22	0,0101	0,161
3	0,372	0,535	1,4374	9	0,0252	31,4	0,98	0,0682	0,589
4	0,273	0,768	2,8125	12	0,0336	44,2	1,09	0,1078	0,806
5	0,134	0,329	2,4649	6	0,0168	24,0	0,98	0,0346	0,336
6	1,001	0,329	0,3289	6	0,0168	12,9	0,53	0,0346	0,381
7	0,372	0,263	0,7077	6	0,0168	18,6	1,03	0,0404	0,285
8	0,650	0,461	0.7093	6	0,0168	20,0	0,63	0,0303	0,491
9	0,147	0,082	0,5596	6	0,0168	6,8	0,54	0,0485	0,092
10	0,269	0,137	0,5106	6	0,0168	11,6	0,93	0,0485	0,155
11	0,120	0,603	5,0273	6	0,0168	40,5	1,27	0,0303	0,609
12	0,464	0,329	0,7100	6	0,0168	16,6	0,89	0,0323	0,352

Table 3. Effects of actions for the minimum axial force $N_{d,min}$.

4. Material characteristics

The following materials characteristics for concrete and reinforcing steel are considered in the deterministic design of reinforced concrete columns. Concrete class C 20/25 having the characteristics

$$f_{ck} = 20 \text{ MPa}, \gamma_c = 1{,}5, \ f_{cd} = 13{,}33 \text{ MPa}, \ \alpha = 0{,}85 \tag{12}$$

is considered here. It should be noted that the coefficient α equal to one is considered in some countries. Reinforcing steel S 500 having the strength values

$$f_{yk} = 500 \text{ MPa}, \gamma_s = 1{,}15, f_{yd} = 435 \text{ MPa} \tag{13}$$

is considered. Assuming further the modulus of elasticity $E_s = 200$ GPa, the design yield strain $\varepsilon_{yd} = 2{,}17$ ‰ corresponds to the yield strength f_{yd} given above.

5. Deterministic design

The following simplifications are accepted for design of column cross sections (see figure 2):

- symmetrical reinforcement ($A_{s1} = A_{s2} = A_s / 2$) is considered only,
- the square shape of the column cross section having dimensions h and b rounded to 5×10^{-2} m are chosen such that $h/b = 2$ (in the last study case $h/b = 3$).
- distance of reinforcing bars from the edge is chosen as $d_{1(2)} = 0.1\ h$.

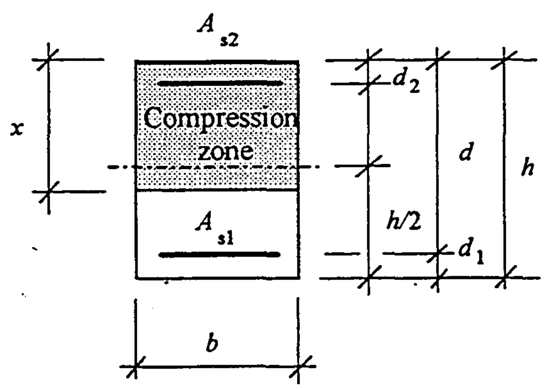


Fig. 2. Column cross section.

For given design values of the normal forces N_d and bending moments M_d, the column cross sections are designed using simplified interaction diagram described by the following formula:
for $N_d < \alpha\, b\, h f_{cd} / 2$

$$[A_s f_{yd} (h - 2d_1) + h N_d (1 - N_d / (\alpha\, b\, h f_{cd})] / 2 \ - \ M_d > 0 \tag{14}$$

for $N_d > \alpha\, b\, h f_{cd} / 2$

$$K_2 [A_s f_{yd} (h - 2d_1) / 2 + \alpha\, b\, h^2 f_{cd} / 8] \ - \ M_d > 0 \tag{15}$$

$$K_2 = (N_{ud} - N_d) / (N_{ud} - N_{bal,d}) \tag{16}$$

$$N_{ud} = \alpha\ b\,h f_{cd} + A_s f_{yd} \tag{17}$$

$$N_{bal,d} = \alpha\ b\,h f_{cd} / 2 \tag{18}$$

These relationships approximate well interaction diagrams derived from appropriate rules of Eurocode 2 [5] and, because of their simplicity, shall be used in the following reliability analysis. Moreover, detail analysis show that in common cases the ultimate bending moment given by these relationships is mostly on the safe side and differs insignificantly (by less than few percent) from that obtained by more accurate procedure based on Eurocode 2 [5]. The total reinforcement area A_s should satisfy the conditions of clause 5.4 in [5]:

$$0{,}15\ |N_d|\ / f_{yd} < A_s,\quad 0{,}003\ b\,h < A_s < 0{,}08\ b\,h \tag{19}$$

which specifies the minimum and maximum reinforcement ratio.

Using relationships (14) to (18.), material properties given by equations (12) and (13) and the design values of effects of actions described by equations (1) to (11), the resulting reinforcement areas A_s and rations A_s / bh shown in Table 2 and 3 have been obtained for the maximum axial force $N_{d,max}$ and the minimum axial forces $N_{d,min}$ respectively. Note that the reinforcement areas A_s given in Table 2 and 3 satisfy the conditions (19) required by Eurocode 2 [5]. Theoretical values of reinforcement area A_s rounded upward to the last digit indicated in Table 2 and 3, which do not correspond to any specific bar size, shall be considered in the following reliability analysis.

It follows from Tables 2 and 3 that in the study cases 4, 9 and 10 the greater reinforcement areas follow from the design situation corresponding to the minimum axial force $N_{d,min}$; this reinforcement should be used. However, to show the effect of the design procedure considering the maximum axial force $N_{d,max}$ only, both reinforcement areas (the greater due to the minimum axial force and smaller due to the maximum axial force) are considered in the following reliability analysis of the study cases 4, 9 and 10.

6. Limit state function

In the time variant reliability analysis the actual axial force N is considered as a simple sum of actual axial forces due to all the considered actions:

$$N = N_W + N_{imp} + N_{wind} \tag{20}$$

where N_W is the axial force due to self weight, N_{imp} is the axial force due to imposed load and N_{wind} is the axial force due to wind action (positive values are again accepted for compressive forces). Thus, the time variant reliability analysis presented here concerns only the permanent design situation with the maximum axial force (corresponding to $N_{d,max}$ given by (1)).

The bending moment M is given by equation (6) used in the design calculation in which actual values are applied instead of the design values and a new additional eccentricity e_a are considered, thus

$$M = M_0 + N(e_a + e_2) = N(e_0 + e_a + e_2) \tag{21}$$

where the first order eccentricity $e_0 = M_0 / N$, where M_0 is given as

$$M_0 = L[C_p\, G\, p_{wind}\, (L+nh_s)\, a_2]/8 \tag{22}$$

The additional eccentricity e_a is given in terms of the initial sway ζ as

$$e_a = \zeta L/2 \tag{23}$$

where ζ is given in Table 4. The second order eccentricity e_2 is given by modified equations (9) in which $l_0 = L$ (the minimum value $l_0 = 1{,}12\, L$ required by Eurocode 2 [5] is neglected in the reliability analysis), thus

$$e_2 = 0{,}1\, K_1 L^2 (1 / r) \tag{24}$$

where $K_1 = 1$ and r is given by equation (10), in which, again, actual values of basic variables shall be used instead of the design values.

The limit state function g may be expressed as the difference of resistance bending moment and the actual bending moment about the centroid.

$$g = \xi_R M_R - \xi_E M \tag{25}$$

Two coefficients of model uncertainties ξ_R and ξ_E are considered as random variables to cover imprecision and incompleteness of the relevant theoretical models. Taking into account (15) to (18) the limit state function (25) becomes
for $N < \alpha\, b\, h\, f_c / 2$

$$\xi_R [A_s f_y (h - 2d_1) + h N (1 - N / (\alpha\, b\, h f_c)] / 2 - \xi_E M > 0 \tag{26}$$

for $N > \alpha\, b\, h\, f_c / 2$

$$\xi_R\, \kappa [A_s f_y (h - 2d_1) / 2 + \alpha\, b\, h^2 f_c / 8] - \xi_E M > 0 \tag{27}$$

$$\kappa = (N_u - N) / (N_u - N_{bal}) \tag{28}$$

$$N_u = \alpha\, b\, h f_c + A_s f_y \tag{29}$$

$$N_{bal} = \alpha\, b\, h f_c / 2 \tag{30}$$

The limit state function given by equations (26) to (30) is applied in the reliability analysis of the column in conjunction with appropriate probabilistic models for basic random variables described bellow.

7. Statistical properties of basic variables

Basic variables applied in the reliability analysis are listed in Table 4. Note that the initial overall sway ζ_0 (which is not used in the design - see note (1) below Table 4) is applied now in the reliability analysis of the column. Some of the basic variables are assumed to be deterministic values - denoted "DET" (A_s, E_s, a_1, a_2, L, and n), the others are considered as random variables having the normal distribution - "N", lognormal distribution - "LN", Gumbel distribution - "GUM" and Gamma distribution - "GAM". Statistical properties of the random variables are further described by the moment characteristics, the mean and standard deviation, partly taken from CIB Reports [7] and [8].

Category of basic var.	Symbol	Name of basic variable	Distrib type	Dimen.	Mean	Standard deviation
Material	α	reduction factor	N	-	0,85	0,085
properties	A_s	reinforcement area	DET	m^2	nom	0
	f_c	concrete strength	LN	Mpa	30	5
	f_y	yield strength	LN	Mpa	560	30
	E	modulus of elasticity	DET	GPa	200	0
Geometric	a_1	column distance in plane	DET	m	nom	0
data	a_2	perpend. dist. of column	DET	m	nom	0
	b	width of cross section	N	m	nom	0,005
	$d_{1(2)}$	distance of bars from edge	N	m	0.1h+0.00	0,005
	h	height of cross section	N	m	nom	0,005
	L	height of column	DET	m	nom	0
	n	number of floors	DET	-	nom	0
	ζ	initial overall sway(1)	N	rad	0	0,0015(1)
Model	ξ_E	uncertainty of load	N	-	1,0	0,1
uncertainty	ξ_R	uncertainty of column	N	-	1,1	0,11
Actions	ρ	weight of reinf. concrete	N	MNm^{-2}	0,0240	0,00192
	C_p	shape coefficient	LN	-	1,0	0,15
	G	gust factor	GUM	-	2,5	0,25
	p_{wind}	wind pressure	GUM	MNm^{-2}	0,00035	0,00006(2)
	p_{impl}	imposed long term load	GAM	MNm^{-2}	0,0006	ean × ν(3)
	p_{imps}	imposed short term load	GAM	MNm^{-2}	0,0002	ean × ν(4)

Notes:

(1) The initial overall sway ζ is used to calculate the additional eccentricity e_a of the built in column according to equation (23).

(2) The mean and standard deviation correspond to the distribution of one year maximum.

(3) The mean and standard deviation correspond to the distribution of 7 years maximum; $v^2=(0,16+8/(a_1\ a_2))(1/n+\rho\ (1-1/n))$ (see CIB report [8]), where the coefficient of correlation of the long term loads in two floors is considered as $\rho = 0.5$ (see also table 5).

(4) The mean and standard deviation correspond to the distribution of the 12 hours (one day) maximum, $v^2= 50/(a_1\ a_2)$ (see also table 5).

Table 4. Statistical properties of basic variables for built in column.

Study case	$A_s \times 10^4$ [m²]	a_1 [m]	a_2 [m]	n	$\sigma_{p,\text{impl}}$ [MN/m²]	$\sigma_{p,\text{imps}}$ [MN/m²]
1	24,3	5	5	10	0,00031	0,00028
2	28,2	5	5	10	0,00031	0,00028
3	46,4	5	5	10	0,00031	0,00028
4	28,5	5	5	10	0,00031	0,00028
5	23,2	4	5	10	0,00033	0,00032
6	30,1	7	5	10	0,00028	0,00024
7	26,1	5	4	10	0,00033	0,00032
8	31,1	5	7	10	0,00028	0,00024
9	5,3	5	5	1	0,00042	0,00028
10	9,4	5	5	3	0,00034	0,00028
11	73,8	5	5	20	0,00030	0,00028
12	29,8	5	5	10	0,00031	0,00028

Table 5. Standard deviation $\sigma_{p,impl}$ and $\sigma_{p,imps}$ of the imposed loads.

8. Reliability analysis

Time variant reliability analysis is based on the Borges - Castanheta model for wind action, long term and short term imposed loads indicated in Fig. 3 (see also [1]). Program COMREL-JP [6] have been applied for time variant reliability analysis (jump process) of the columns assuming life time of 50 years and the probabilistic models given in Table 4 and 5.

The wind load is modelled as a sequence of independent rectangular pulses, each pulls having a duration of approximately 1 day. The statistical properties of the pulls intensity is tuned in such a way that the maximum pressure in a year has a distribution specified in Table 4. The long term imposed load is defined for the interval of 7 years. It is assumed to be changed simultaneously on all floors of a building. The short term load is present during one interval of 1 day in each year; the simultaneous occurrence of short term imposed loads on more than 1 floor at the same time may be neglected; so an independent short term single floor load imposed on the column occurs n times a year, n being the number of floors. Note that long term loads are considered as being correlated over various floors.

In the first type of the time variant analysis the short term action was assumed to be absent, $p_{\text{imps}} = 0$, and only wind action p_{wind} and long term imposed load p_{impl}, were considered as time dependent ergodic and stationary random variables. As the statistical properties of the wind action p_{wind} given in Table 4 refer to the distribution of one year maximum values and properties of the long term imposed load p_{impl} refer to 7 years maximum, the "jump rates" (number of jumps within one year) $\lambda_{p,\text{wind}}$ and $\lambda_{p,\text{impl}}$ of the rectangular wave renewal jump process were considered as follows:

$$\lambda_{p,\text{wind}} = 1{,}0/\text{year}\ ;\ \lambda_{p,\text{impl}} = 0{,}143/\text{year} \tag{31}$$

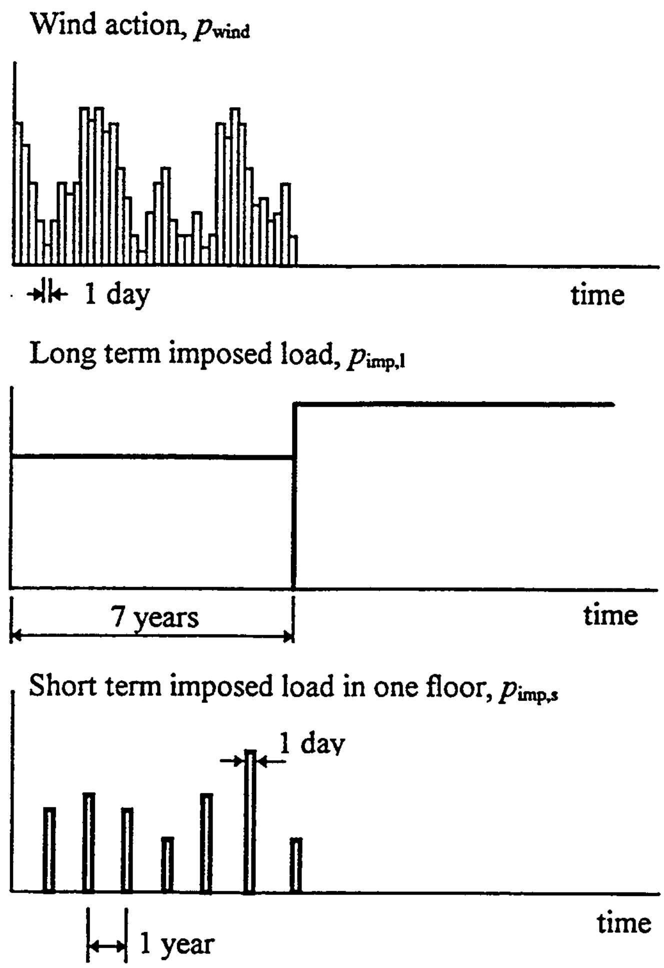


Fig. 3. Models of actions for time variant reliability analysis.

The second type of the time variant analysis concerns the period of time when the short term imposed load p_{imps} is present. As already mentioned above it is assumed that in each floor the short time imposed load may independently occur once a year. Thus, in every year there is n days, where n is the number of floors, when the short time load is active. The total number of 'active' days during the assumed life time of 50 years is therefore 50 n. This period is considered now as the total time of the time variant reliability analysis. One day is considered now as a unit of time. Jump rate of the short term imposed load p_{imps} is thus $\lambda_{p,imps}$ = 1,0/day.

Taking into account properties of the Gumbel distribution, statistical properties of the wind action p_{wind} were adjusted to one day period as follows

$$\mu_{day} = \mu_{year} - 0{,}78\ \sigma_{year} \ln(365) = 0{,}00035\text{-}0{,}00028\text{=}0{,}00007\ \text{MN/m}^2\ ,\ \sigma_{day} = \sigma_{year} \quad (32)$$

Jump rate of the wind action p_{wind} is thus $\lambda_{p,wind}$ = 1,0/day.

Statistical parameters of the long term imposed load p_{impl} given in Table 4 for 7 years correspond now to the period of *7n* "active" days (one year is "compressed" to *n* "active days"). Appropriate jump rate $\lambda_{p,impl}$ (number of jumps within one active day) is therefore

$$\lambda_{p,impl} = 1/\,(7\,n)\;/\;\text{day} \tag{33}$$

Using the FORM methods of probability integration [6], resulting values of the reliability index β_1 and β_2 of the first and second type of reliability analysis respectively for the 12 study cases are given in Table 6.

Study case	Reinforcement area	Reinforcement ratio	Cross section dimensions	Column height	Time variant analysis, short term load not present	Time variant analysis, short term load present
	$A_s \times 10^4$ [m^2]	A_s/bh [%]	$b \times h$ [m]	L [m]	β_1	β_2
1	28,7	1,17	0,35×0,70	6	5,6	6,1
2	22,1	1,23	0,25×0,50	3	4,7	5,3
3	34,1	1,07	0,35×0,70	9	4,0	4,6
4$^{(1)}$	44,2 (38,2)	1,09 (0,94)	0,45×0,90	12	4,5 (4,2)	5,1 (4,8)
5	~~24,0~~ 24,6	1,00	0,35×0,70	6	5,3	5,8
6	41,4	1,69	0,35×0,70	6	6,1	6,5
7	~~31,9~~ 31,8	1,77	0,30×0,60	6	5,5	6,0
8	37,4	1,17	0,40×0,80	6	5,7	6,2
9$^{(1)}$	6,8 (4,6)	0,54 (0,37)	0,25×0,50	6	3,7 (2,9)	4,9 (4,2)
10$^{(1)}$	11,6 (10,9)	0,93 (0,87)	0,25×0,50	6	3,9 (3,8)	4,8 (4,7)
11	90,7	2,83	0,40×0,80	6	5,6	6,0
12	37,5	2,00	0,25×0,75	6	5,6	6,2

Note: (1) In the study cases 4, 9 and 10 the reinforcement area is designed considering the minimum axial force $N_{d,min}$ due to permanent load and wind action only (imposed load being absent); values given in brackets ( ) correspond to the design considering the maximum axial force $N_{d,max}$.

Table 6. Reliability indices β_1, and β_2 of time variant analysis for built in column.

It follows from Table 6 that obtained values of the reliability indices are within a broad ranges from 3,7 (2,9 when the 'the maximum axial force design' is considered only) to 6,5. Such a broad range for reliability indices has been, however, reported also in previous probabilistic analyses (see for example [9]). Values of the reliability index β_1 are within a range from 3,7 (2,9) up to 6,1, values of β_2 within a range from 4,6 (4,2) up to 6,5. In the study cases 9 the reliability index β_1 = 3,7 (2,9) is less than recommended value 3,8 [1], relatively low value of β_1 are obtained also for the study cases 3, 4 and 10 (see Table 6). In all these cases the reinforcement ratio is relatively low (around or less than 1%), though still above the required minimum 0,3 %. In the study case 9 and 10 there may be also an unfavourable effect of relatively small cross section dimensions (0,25 × 0,50 m). Higher and perhaps uneconomical values of the reliabilty indices (around 6) seem to correspond to relatively great reinforcement ratios (study cases 7, 11 and 12).

The resulting reliability index β for the column is given by a combination of both reliability indices β_1, and β_2 that are given in Table 6. As a simple approximation the minimum of both values β_1, and β_2 may be considered as the resulting reliability index β. It follows from Table 6 that in all the study cases considered here $\beta_1 < \beta_2$; thus the first design situation with the short term imposed load being absent seems to be decisive.

10. Conclusions

Results of the reliability analysis of 12 study cases of reinforced concrete column show considerable differences in the reliability level of the column in different structural arrangements. Considering 50 years life time, wind action and long term imposed load as time variant actions (short time imposed load being absent) obtained values of the reliability index β varies within a broad range from 2,9 up to 6,1. Generally higher values of β (from 4,2 to 6,5) correspond to the reliability of columns during those days when short term imposed load is present.

It appears that the reliability level of reinforced concrete columns designed according to Eurocodes may be in some cases insufficient in other cases, depending on actual structural arrangements, it may become uneconomical. To harmonise reliability levels obtained for various structural members further research on random variable models using available experimental data and calibration of present generation of Eurocodes to existing structures is urgently needed.

References

[1] Ton Vrouwebvelder, Ansgar Neuenhofer, Jean Goyet, Hans Denver and Milan Holický: JCSS Working Document on Eurocode Random Variable Models, JCSS-VROU-06-2-95, ICSS 1995.
[2] ENV 1991-1 :Eurocode 1: Basis of design and actions on structures - Part 1: Basis of design, CEN 1994.
[3] ENV 1991-2-1 :Eurocode 1: Basis of design and actions on structures - Part 2.1: Densities, self-weight and imposed loads, CEN 1994.
[4] ENV 1991-2-4 :Eurocode 1: Basis of design and actions on structures - Part 2.4: Wind loads. CEN 1995.
[5] ENV 1992-1 :Eurocode 2: Design of concrete structures - Part 1: General rules and rules for buildings. CEN 1993.
[6] RCP - Reliability Consulting Programs: STRUREL, A Structural Reliability Analysis Program System, COMREL & SYSREL Users Manual. RCP Consult, München 1995.
[7] CIB Report: Actions on structures, self weight loads. Publication 115, CIB 1989.
[8] CIB Report: Actions on structures, live loads in buildings. Publication 116, CIB 1989.
[9] A.C.W.M. Vrouwenvelder and A.J.M. Siemens: Probabilistic calibration procedure for the derivation of partial safety factors for the Netherlands building codes. Heron, 1987/4, pp. 9 - 30.

Reliability analysis of a reinforced concrete column designed according to the Eurocodes

Milan Holický and Ton Vrouwenvelder

Abstract

Reliability analysis of reinforced concrete columns is a part of an extended research activity on Eurocode Random Variable Models supervised by the Joint Committee for Structural Safety. Submitted analysis concerns reliability of a built in reinforced concrete column designed according to Eurocodes 1 and 2. Reliability of a column of the first floor of a multi-storey frame structure is analysed using software product COMREL developed by RCP Műnchen. Preliminary results of the analysis are presented for the total of 12 study cases corresponding to different structural arrangements.

The design effects of actions are determined in accordance with Eurocode 1 considering the permanent load due to self weight and variable load due to wind, long term and short term imposed load. The column cross sections are designed using a simplified interaction diagram for axial force and bending moment and material properties specified in Eurocode 2. Dimensions b and h of rectangular cross sections rounded to 5 10^{-2} m are chosen such that $h/b = 2$ (in one study case $h/b = 3$). Symmetrical reinforcement having the theoretical area A_s rounded upward to 10^{-5} m^2, which do not necessarily correspond to any specific bar size, is considered in the reliability analysis.

Using the FORM method of probability integration results of time variant reliability analysis of columns for long term and short term actions are submitted for the all 12 study cases. Considering 50 years life time, wind action and long term imposed load as time variant actions (short time imposed load being absent) obtained values of the reliability index β varies within a broad range from 2,9 up to 6,1. Generally higher values of β (from 4,2 to 6,5) correspond to the reliability of columns during those days when short term imposed load is also present.

It appears that the reliability level of reinforced concrete columns designed according to Eurocodes may be in some cases insufficient in other cases, depending on actual structural arrangements, it may become uneconomical. To harmonise reliability levels provided for various structural members further research of random variable models using available experimental data and calibration of present generation of Eurocodes to existing structures is urgently needed.

ANHANG III:

JCSS SICHERHEITSKONZEPT FÜR BESTEHENDE TRAGWERKE

ASSESSMENT OF EXISTING STRUCTURES

1 FORWORD

1.1 SCOPE AND OBJECTIVES

The aim of the activities of the Joint Committee on Structural Safety (JCSS) is to promote matters within the field of structural reliability at an inter-association level. Among these matters the reliability based reassessment of existing structures has been identified as a topic of major importance. In fact the assessment of *existing structures* is getting more and more important also due to social and economical reasons, while most codes deal explicitly only with design situtions of *new structures.*

The assessment of an existing may however differ very much from the design of a new structure. One should bear for example in mind, that due to deterioration and damage it is general practice to inspect existing structures and if necessary to repair and/or strengthen them. While initially the various uncertainties related to loads and resistance parameters must be assessed *a priori* resulting in appropriate safe codified design provisions, actual observations update the prior assessment. Consequently regarding the *state of information* the situation in assessing *existing structures* is completely different from that during *design.* In addition special attention is paid to specific parts of the existing structure and to a probably limited part of items with a real risk of damage according to the observed behaviour of the structure. On the other hand *the interpretation and the analysis* of the additional information may not be a simple matter.

Therefore specific procedures and tools are required in order to judge the safety of existing structures. Such procedures and tools have been reviewed within this project of the JCSS and have been thoroughly discussed in various meetings during the last six years. Thereby the following basic goals for such procedures have been set:

a) to standardize methods and terminology;

b) to be operational for the practical engineers;

c) to be generally applicable for various materials and various structural types;

d) to be useful as guidelines of precodification state i.e. to build the basis of future codes, standards or code type of recommendations.

1.2 ORGANISATION OF THE REPORT

The results obtained within this project are documented in the present report. Although the nature of the report is educational, it contains practical and operational recommendations and rules for the assessment of existing structures also illustrated in characteristic examples and real case studies. The present report is organized as follows.

Chapter 2 provides guidelines on how to plan and carry out the assessment of existing structures in a systematic and economic way. It contains basic concepts and definitions, it discusses decision criteria and it classifies the various phases in the assessment process of an existing structure.

Chapter 3 gives recommendations for codification that means recommendations on how to come up with minimum requirements for judging the safety of existing structures. Consequently this chapter serves as a basis for the task of preparing national and international standards or codes of practice in accordance with current technical practice and socioeconomical preference.

Annex A summarizes the principles and procedures used in formulating and solving reliability related problems through reliability analysis techniques. The principles of limit state analysis and their application to codified design are described. Reliability methods and associated computer tools for component and system reliability analysis are briefly discussed.

Annex B describes the methodologies for decision making and for updating of additional information which are of significant importance in the assessment process of existing structures. Emphasis is given on the Bayesian statistical analysis and the associated techniques. Methodoligical aspects on optimal planning of inspection and maintenance are also included.

Annex C focusses on risk acceptance criteria for existing structures. Acceptable target reliability levels are discussed and appropriate values are proposed based on safety class differention principles.

Annex D presents a selection of illustrative examples to illustrate the applicability and efficiency of the methods and the associated tools. Such examples are in fact simple and will help the reader to follow the proposed procedures and the implemented computation techniques.

Annex E contains real case studies with the main scope to demonstrate the use of the proposed methods and procedures in practical cases. The case studies are dealing with different types of problems as faced in the practical requalification of structures or of structural elements.

It remains here the hope that this document contributes not only to improve the general knowledge and understanding within the field of reliability based assessment of existing structures but also to transform such knowledge into principles usable in practical applications.

2 GUIDELINES

2.1 INTRODUCTION

The need to assess the reliability of an existing structure may arise from a number of causes among which:

- deviations from the original project description are observed
- adverse results of a periodic investigation of its state
- doubts about the structural safety caused by evidence of damage
- unusual incidents during use (such as impact of vehicles, avalanches, fire in the building, earthquakes), which could have damaged the structure
- a clearly inadequate serviceability
- suspicion of possible impairment of the structural safety related to building materials, to construction methods or to the statical system
- the discovery of design or construction errors
- a planned change of the use of the structure
- the expiry of a residual service life granted on the basis of an earlier assessment of the structure
- simply because of doubts about the safety of the structure.

The assessment of the reliability of an existing structure aims at producing proof that it will function safely over a specified residual service life.

The reliability assessment is mainly based on the results of assessing hazards and load effects to be anticipated in the future, and of assessing material properties and geometry taking due account of the present state of the structure. Thereby several questions arise and a number of decisions must be taken, such as:

- What type of inspections are neccessary?

  It should be noted that routine inspections are not common for conventional structures but only for specific structures such as bridges, offshore structures or nuclear power plants.

- What analyses shall be performed?

  Different types of analyses might be considered including structural, reliability and cost-benefit analysis.

- What are the risks involved in further using the structure?
- What are the risk acceptance criteria to be observed?

  Could one accept a lower safety level compared to the originally assumed target safety goals at the design stage?

- What type of measures shall be taken?

  Measures may include maintaining the present state, repair, strengthening or even replacement of the structure. Answering these questions implies economical considerations .

- What is the degree of non-objectivity of statements about the safety of an existing structure?

  Structural reliability is quantified as the probability of safe structural behavior during any specified time, given the information base concerning the structure. At the design stage the reliability of the structure is evaluated conditional on the applied analysis and design methods and on the expectation that the structure will be realized according to some standard practice that possibly is formally defined in given structural codes and regulations. For an existing structure subject to safety evaluation the assessed value of the safety measure becomes more dependent on the opinion of the assessing engineer (expert). Such opinion is influenced by the observable, but often poorly investigated physical properties of the structure, by the envisaged behaviour of the structure as well as by the expected hazard scenarios in the course of the Residual Service Life of the structure. Thus a statement about the safety of an existing structure is highly person dependent and due to the lack of the comfort of beeing able to refer to a code of practice the assessment reflects, to a much larger degree as was the case at the design stage, the state of knowledge of the person that makes the statement. This is confirmed by the fact that expert opinions often differ considerably. However, as a rule in the course of discussions the views held by the experts tend to converge and experts can, eventually, even reach full agreement. As long as the safety assessment is based on rational modeling both respecting the observed facts about the structure and obeying the laws of nature it still can satisfy the requirement of objective argumentation.

The assessment of the structural reliability of an existing structure is a difficult task, because statements about its possible behaviour under conditions of extreme loading have to be made. Such conditions normally lie outside of the range of experience gained from observing the behaviour under service loads. Also critical for assessing the structural safety is the often rather poor information about the condition of certain structural elements, e.g. with respect to corrosion, or fatigue.

Thus, the structural safety of an existing structure is a question - and in many cases at the same time the result - of a close inspection of ist state and detailed analysis of ist behavior. What, for example, at first sight appears to be unsafe, may upon closer examination be found to be safe. The opposite, however, may also occur. The updating of information about a structure will influence the initially somewhat subjective opinion concerning structural safety.

Doubts about serviceability, however, are not associated to major problems, because either the structure has shown itself to be adequate or the corresponding defects are known from previous use.

As deflections, cracks, vibrations etc. may be observed under normal conditions of utilisation the serviceability of the structure is normally easier to assess. Also the owner or user may have enough experience to give his opinion on the serviceability of his structure.

With respect to the question of the durability of materials and details of existing structures, it appears in many cases possible to predict further developments based on the actual state of the structure. By specifying the measures necessary for properly maintaining the structure, problems associated to durability can be prevented.

In the design of new structures, on the other hand, in general the problems are just the opposite: The structural safety of a structural system can easily and reliably be *checked against rules laid down in codes and specifications and can be verified with relatively simple models, while, due to the difficulties of prediction and a lack of knowledge of later use, even with "exact models" only an estimate of serviceability can be given. It is even more difficult to predict the durability of structures at the planning stage.*

The evaluation of the reliability of existing structures should be based on a rational approach. The safety and economy implied by certain decisions are evaluated by means of both structural and eventually reliability analyses and economical considerations. The degree of sophistication depends on the type of structure of concern.

2.2 BASIC CONCEPTS AND DEFINITIONS

The following basic concepts play an important role in assessing the safety and reliability of existing structures. The proper use of these concepts in the procedures and in the respective documents is recommended.

2.2.1 Residual Service Life

When assessing existing structures it is essential to know for how long the structure is intended to serve its purposes. This period is termed the Residual Service Life (RSL).

The issue is twofold. The owner might wish to have the structure accepted for an indefinite residual service life, while the engineer or the authority judging the structure might rather restrict its use to some definite rather shorter time. The idea is that after assessing the structure a specified residual service life is granted. The residual service life may be also associated to a certain inspection and maintenance program. When the residual service life elapses, a new assessment is requested.

2.2.2 Utilisation Plan

It is evident that the assessment of a structure must properly take into account its use during the foreseen residual service life and any particular requirements of the owner. The respective document is the Utilisation Plan (UP).

The utilisation plan comes together as a result of discussions between the owner and the engineer assessing the structure. It contains all necessary information on the essential aspects of future use and the requirements concerning the behaviour of the structure. This utilisation plan is to be signed by the owner and the engineer.

2.2.3 Hazard Scenarios

On the basis of the utilisation plan and regarding the future use of the examined structure a list of of hazards likely to act on the structure must be defined.

The term Hazard Scenario *is a rather broad concept. It calls for imagining a situation, transient in time, that a structure might happen to undergo which would endanger its life and those of people. A Hazard Scenario is defined by a leading hazard and a number of accompanying actions, influences and settings. The hazard scenario concept is especially applicable to existing structures since a direct application of existing codes is not possible.*

Obviously, there are many such scenarios. On the basis of the utilisation plan, the engineer is expected to consider all relevant Hazard Scenarios likely to act on the structure during the residual service life envisaged.

2.2.4 Safety Plan

The Safety Plan (SP) assigns the appropriate counteracting safety measures to the defined Hazard Scenarios.

In order to render an existing and somehow deficient structure safe, measures can be drawn from a number of categories.

- *eliminating hazard scenarios at the source of its leading hazard*
- *avoiding hazard scenarios by changing intentions or structural concepts*
- *controlling hazard scenarios by safety devices, warning systems as well as by checking, supervision, inspection followed by adequate corrective measures*
- *overpowering hazard scenarios by dimensioning using adequate safety margins*
- *accepting hazard scenarios because they either cannot – without prohibitive cost – be counteracted by one or more of the above measures.*

Obviously, in most cases an appropriate combination of the above measures is optimal, e.g. counteracting hazard scenarios partly by control, partly by dimensioning and to some degree always also by accepting hazards.

Thus, the safety plan consists of a number of lists describing the relevant hazard scenarios and allocating the respective safety measures. Such lists address the different parties and functions involved in the building process, e.g. architects, contractors, and users, or, more specific, structural analysis, controlling, checking,. inspection and maintenance. Setting up this plan is in the sole responsibility of the engineer, in due co-operation, however, with the architect and other specialists.

2.2.5 List of Accepted Risks

The List of Accepted Risks (LAR) is an important document as it clarifies who profits from accepting risks and who bears the consequences. Conceptionally, both should be the same person or body.

The respective list is to be discussed with the client, resp. the owner, and possibly also the Building Authorities in order to locate the implications and to make sure that everybody knows about who will bear which consequences. The parties concerned are advised to sign the list of accepted risks.

2.2.6 Updating of Information

Updating of information about the structure and its present and future use is an important procedure in assessing the reliability of existing structures. Updating is based on <u>prior information</u> and collected observations and measurements. It results in <u>posterior information</u> that serves for assessing the structure.

2.3 INSPECTION AND MAINTENANCE

2.3.1 General Considerations

In the assessment of the structural safety of an existing structure several parties might be involved, each of them contributing with, or requiring, different types of information. The final decision is gradually reached by pooling all these aspects into one.

The aforementioned contributions come from:

- Design: the information relevant to this aspect is generally obtained from reports, existing drawings etc.
- Field experience: the experience acquired during operation improves the knowledge on the real behaviour of the structure. Data may be obtained from monitoring, inspections, etc.
- Requalification analysis: at this stage information obtained from both the design documentation and the field experience are critically reviewed and updated and then used to estimate the new conditions of the structure.
- Economical analysis: the potential consequences in terms of direct or indirect costs are evaluated.

The above listed contributions lead to the collection of information that are of very diverse nature, e.g. in terms of type of data and category of persons/deciders who provide them. Therefore the evaluation of such information becomes very important.

2.3.2 Inspection techniques

Inspection is an investigation intended to update the knowledge about the present condition of the structure. Related to inspections typically two types of interrelated decisions have to be made:

- What inspections shall be performed?

  For example which are the parameters to be inspected, how many samples and when shall be taken, what are the techniques to be used.

- What to do with the inspection results?

  For examle type of measures to be taken (repair, strengthening, etc), development of an inspection plan.

Two types of inspection can be in general distinguished:

- qualitative inspection: this type of information is related to the observion of parameters such as surface characteristics, visible deformations, cracks, spalling, corrosion etc.

  The description of possible damage of the structure will be in qualitative terms like: no damage, minor damage, moderate damage, severe damage etc. The ranges of each category shall be thereby specified. However it is possible and sometimes necessary to process the observation in a more formal way.

- quantitative inspection: this type of information results in a set of values of parameters that characterize the condition of the structural elements.

  Examples of such condition parameters are: crack depth and length, corrosion area and depth, displacements, residual stresses, damping, excentricities etc.

For both inspection types the related uncertainties such as the probability to detect some damage and/or the accuracy of the results shall be specified and taken into account.

2.3.3 Proof Loading

A special type of inspection is proof loading. Based on such tests one may draw conclusions with respect to:

- resistance of the tested member;
- resistance of other similar members;
- resistance under other conditions;
- behaviour of the system.

Based on the proof load results the reliability estimate of the structure can be updated. The inference in the first case is relatively easy. The probability density function of the load bearing capacity is simply cut off at the value of the proof load. The inference of the other conditions is more complex. It should be noted that the number of proof load tests does not need to be restricted to one.

2.3.4 Monitoring

Important parameters affecting the overall behaviour of the structure such as vibrations, deformations etc. can be continously observed with the use of technical equipment. This observation method is called monitoring; the obtained results are usually organized in data bases and are evaluated by applying appropriate software packages.

2.3.5 Maintenance

Maintenance is defined as a set of activities that are carried out to retain or restore a structure in an operable state. The following types of maintenance can be distinguished:

- <u>Corrective maintenance</u>: no inspection is carried out and repair is done after failure has occured.

  Corrective maintenance will generally be applied if the cost of failure is relatively low or if inspection costs are relatively high.

- <u>Preventive maintenance</u>: no inspection is carried out but replacement or maintenance at a time that no failure has occured.

  Preventive maintenance will generally be applied when the failure costs are high and the time of failure can be predicted in advance.

- <u>Condition based maintenance</u>: inspections are planned in advance and when measured parameters no longer meet prescribed criteria repair or replacement must be carried out.

  Inspection intervalls may be either fixed (based on a long term plan) or may depend on the measured condition at the previous inspection.

2.4 DECISION CRITERIA

Decision criteria serve as a basis of the decision regarding the requalification of an existing structure. Decision criteria may be absolute but, normally, are relative in a sense that they allow an ordering of states or possible solutions. Possible decision criteria are briefly reviewed in the following.

2.4.1 Target reliability

The selection of the target failure probability or the target safety level depends on different parameters such as type and importance of the structure, possible failure consequences, socioeconomic criteria etc.

Target safety levels (annual or lifetime) for the design of different types of structural components have been reported in various national and international standards and will be not discussed herein.

Specific efforts have been also done to define acceptable failure probabilities for existing structures and especially offshore structures. Thereby exposure measures can be defined (Bea, 1993) by considering failure costs, investment costs to reduce structural failure and platform lifetime. The analyses for several platforms around the world have shown a differentiation between exposure measure for new designs and requalifications of existing platforms. This important fact reflects a willingless to accept lower reliabilities associated with older systems and not to require that those systems have reliabilities that equal those of new systems. Such a willingness can be demonstrated to be true for a variety of engineering systems such as cars, airplanes, power plants etc. There is no question that the same holds for structures.

2.4.2 Economical considerations

Economical considerations normally take into account

- expected benefits from the residual use of the structure
- associated commitments and costs including
  - costs related to engineering and structural analyses;
  - costs related to repair work;
  - costs related to planned inspection and maintenance.

In general costs related to repair work are usually dominating the total budget.

2.4.3 Time constraints

Time is an important factor in the assessment of an existing structure and it appears under several different aspects:

- desired residual service life of the structure: it depends on type and use of the structure, on current socioeconomical conditions and reflects the requirements of the owner
- granted residual service life of the structure: it depends on type and future use of the structure and its state and reflects the requirements of the engineer and of building authorities

- mean service life of structures (based on experience i.e. 100 years for bridges, 50 to 100 years for normal buildings, 20 to 30 years for nuclear power plants and offshore platforms etc.)
- time for engineering and repair or strengthening operation
- actions of building authorities: in some cases a certification authority shall verify and approve the use of non standard analyses and code checks. This may require considerable time.

2.4.4 Socioeconomical and political preference

Besides environmental implications a major structural failure may seriously damage the image of the building profession at large. Therefore to some extent target reliability depends also on the public concern and on mass media's focus on ecological and other issues.

2.4.5 Codes and Standards

Differences and compatibility between codes and standards used at design phase of the structure under consideration and actual valid standards or judgement play an important role.

> *In many cases for example the structure has been designed according to the allowable stress format but at the reassessment stage a limit state format with factored loads and resistances might be applied.*

2.4.6 Complexity of analysis

The analysis techniques should be tailored to the complexity of the problems and of the desired results and should avoid unnecessary complications.

2.5 ASSESSMENT PROCESS

2.5.1 General Considerations

Assessment procedures for an existing structure largely depend on different parameters such as type and use of the structure, implied risks and/or costs, current economical conditions of the owner, and the degree of deterioration or damage etc.

Whatever the engineer advises, he could easily run into difficulties. This is why utmost clearness in matters of concepts and procedures is of prime importance when assessing existing structures.

2.5.2 Tasks and Responsibilities

It is clear that the owner of a doubtful structure is responsible for initiating safety investigation, since he is liable causally for damage due to the failure of his structure.

> *In such a situation he is well advised to employ the services of an experienced consulting engineer and commission him with the first investigation. The corresponding contract has to be formulated carefully by both sides and put in*

written form. Because of the difficulties of the task, it is important that the owner places complete confidence in the engineer.

The engineer is responsible for a careful execution of this commission and especially for an expert formulation of statements on the safety of the structure and the measures proposed.

The way by which he reaches his conclusions is largely his own business. In particular he will use his calculations and investigations primarily to help him to come to these conclusions. Further, it is, when appropriate, a responsible fulfilment of a consulting engineer's duties to say when he feels himself incompetent to carry out the work and consequently to refuse to take it on.

In the case of an actual court case, the judge will decide above all whether, in view of the difficulty of the task, the engineer acted objectively with due care. Here the correctness of the engineer's statements is not called into question, because nobody, not even leading experts, can make absolutely correct statements. In any case it falls to the prosecuting party to prove negligence. They have to prove that the engineer failed to take all the necessary care in fulfilling his duties.

The owner is finally responsible to comply with the provisions and measures proposed by the engineer.

If he is not willing to do this, then the engineer has to point out clearly to him the possible consequences. In the case of danger to human lives, it is the legal responsibility of the engineer to report critical facts to the responsible building authorities and/or to the public prosecutor.

As a rule the final decision de jure is taken by the owner.

In seeking the advice of an engineer the latter, de facto, makes the decision. In view of great potential danger or high costs the question arises, whether a consulting engineer alone can carry the responsibility for the necessary decisions: Is he really in a strong enough position to enforce the implementation of the necessary measures? Is it adequate to the problem at hand to leave it to his judgement to accept extraordinary risks? These issues lead to proposing a structuring of the assessment process.

2.5.3 Phases in the Assessment Procedure

Experience shows that breaking down the assessment of an existing structure into up to three phases is reasonable. Fig. 1 (atteched) visualises schematically these phases. Each of these phases should be complete in itself. It is clear that each phase should be begun with a precisely formulated contract, usually in written form. The client and the consulting engineer will have to formulate this contract together. Each phase, similarly, ends with the respective report leaving the owner with his responsibility and freedom of decision. This freedom is, to be sure, constrained by the recommendations of the engineer and the requirements of the laws governing the owner's responsibilities and the criminal code.

2.5.3.1 Phase I : Preliminary Evaluation

The purpose of Phase I is a preliminary assessment with the aim to remove existing doubts using fairly simple methods such as:

- visual inspection (qualitative inspection) of the structure in order to judge its actual condition; special attention is paid to the critical parts of the structure (critical components, etc.);
- review of existing documentation (drawings, calculations, applied codes, etc..);
- compatibility with new codes (comparison between current safety criteria and design criteria, qualitative conclusions...);
- evaluation of possible changes during the passed lifetime (new loads,...);
- simplified assessment of actual condition of the structure: this step can be for example performed based on a scoring factors (Bea, 1993) by weighting important parameters such as:
  - age of the structure;
  - condition of the structure;
  - configuration of the structure and its foundation;
  - loading modifications (change of use,..);
  - modifications in the structural system (supports, ..);
- reporting including recommendations for the owner.

A detailed inspection of the object in question is extremely important. Amongst other things the aim is the recognition of typical hazard scenarios, which could endanger the structure's residual service life. Further, it is a question of detecting defects and damage due to excessive loading. As soon as there is some evidence of danger to humans or the environment, protective measures have to be implemented straightaway.

In the case of many existing structures both the utilisation plan and the safety plan mentioned above will be missing. These plans have to be set up or amended in view of the residual service life aimed at and, as a result, form an important basis for the assessment.

In studying the available documents, an attempt must be made to gain a deep insight into the original situation: which aims were followed, which construction methods and which construction materials were used? What was the economic and organisational climate? Was the work affected by pressure to meet deadlines or due to low price? Such parameters can be called quality indicators. A study of the statical analysis, in addition, provides useful information about codes, calculation and design methods. At the same time it also shows where there are reserves of strength which, according to the present state-of-the-art, could be exploited. Based on these information any doubts about the safety of the structure can be confirmed or dismissed.

All the information gained in Phase I is summarised in a report for the owner. If the doubts that led to the commission being undertaken cannot be overcome in the course of Phase I, further investigational steps must be undertaken in Phase II.

2.5.3.2 Phase II: Detailed Investigation

The following tasks are performed in case it is decided to proceed to a more detailed assessment including

- site investigation including quantitative inspections: corrosion, r.c. amount, deformations, crack dimensions

Here, in addition, a specialist firm or agency or individual experts generally have to be called in.

- updating of information gained through inspection by using statistical procedures

  Structural investigations using updated information are typical of Phase II. It is sensible and cost-effective to build upon the knowledge gained and the questions remaining from Phase I and compile a list of points requiring further investigation and thereby to specify what still needs to be checked. The thoroughly prepared investigation should be closely supervised by the consulting engineer.

- detailed structural analysis based on conventional or advanced tools, according to the problem at hand using limit state analysis, considering nonlinear material behaviour, redundancy of the structure, etc.
- reliability analysis to determine the safety of the structure or the probability of failure of the structure or of its most critical components (see Annex A and B)

  The additional information gained from the investigations can be introduced into confirmatory calculations with the aim of finally dispelling or confirming any doubts as to whether the structure is safe – still well aware of the subjective character of this decision.

All results of Phase II are summarised in a report, which again is handed over to the owner. In particular, the report contains all neccessary information on the structural safety of the investigated structure and conclusions regarding repair and/or future maintenance.

If the safety is thought to be inadequate, then intensified monitoring, reduced loads, strengthening and, if the circumstances justify it, a possible demolition and reconstruction of the structure must be considered.

If the decision to adopt one of these measures is of little consequence, then the investigation can be brought to a close at the end of Phase II.

That would be, for instance, when no human lives are endangered and risks of damage to assets can be accepted. Ending the investigation is also acceptable if one decides upon strengthening, repair, or demolition and reconstruction, as long as this does not imply inordinate risks or financial consequences.

If these conditions, however, are not met then the consulting engineer in his report on Phase II should propose proceeding further to Phase III.

The owner, under these circumstances, should be in favour of going ahead with this step, if he is interested in a balanced and unprejudiced assessment.

2.5.3.3 Phase III: Calling a Team of Experts

For problems with large consequences in terms of risk or of cost related to a decision, a team of experts should be called in order to check carefully the proposals reached in Phase II for the pending decision.

The team should comprise, apart from the consulting engineer commissioned to do the work thus far, additional experienced engineers. The owner or the operator is not a member of the team, but should supply the team with information as required.

Such a team of experts, in assessing an existing structure, act to a certain extent as a substitute for the codes of practice, which for new structures constitute the rules to obey in a well-balanced and safe design. In particular, the acceptance of increased *risks should in principle be left to this team of experts. The engineer responsible for Phases I and II of the work should draw the attention of his team colleagues to all available documents and justify his proposals for the measures to be adopted. The team is well-advised both to inspect the structure and to confer together. The decision of the team should be unanimous and be defended as a team before the owner and, if necessary, publicly. The responsibility for the decision is carried by the team as a whole. The engineering team is not part of the judgement team (or court), which is responsible for the legal questions.*

It must be stated, however, that even the opinion of a team of experts is subjective and might be opposed by others.

3 CODIFICATION ASPECTS

3.1. STATE OF THE ART

In discussing codification related to existing structures it appears useful and practical to distinguish between:

- prenormative research including for example code committee work, documents of international associations or organizations;
- guidelines and recommendations in use;
- applicable code type documents

With respect to prenormative research considerable investigations have been performed regarding existing structures. A representative example is the work carried out by C.E.B. GTG 21 on redesign of r.c. structures. Appropriate safety elements for the requalification phase of such elements are derived on the basis of modern reliability tools, of cost considerations and of limit state formulation. Another example is the development of preliminary criteria for existing structures in Canada (Allen, 1991). The proposed safety elements allow thereby for more flexibility due to additional information.

Guidelines for existing structures exist in a large number of countries. Thereby many countries have presented documents for particular categories of structures such as bridges, towers or normal buildings and also recommendations associated to particular aspects such as seismic parameters remodelling and so on.

A review on the present situation is described by Vrouwenvelder (1993). At least in the USA, Canada, Switzerland, UK such guidelines have been prepared at a detailed level. At pesent only a few countries have a general applicable and real code type document for the assessment of existing structures (CSN, 1986 in Chechoslovakia and RBCV, 1992 in the Netherlands).

The criteria derived for example in the Netherlands are based on probabilistic methods and differ from those related to design of new structures due to:

- *costs which are involved to increase the safety of an existing structure are relatively high;*
- *the remaining lifetime is often short (or shorter than the design lifetime)*
- *additional information on loads and material properties.*

It can be concluded that, although the codification for assessment procedures for existing structures is a relatively new field of development, considerable work has been lately performed in this field and valuable documents have been issued by different national and international associations. Basic items are:

- general principles of assessment;
- procedures and different phases of assessment;
- methods for updating of additional information and appropriate evaluation of inspection results;
- format for verification;
- risk acceptance criteria.

3.2 REQUIREMENTS FOR CODES

Important requirements for a code related to assessment of existing structures are:

- Applicability: the code should be applicable to typical assessment cases.
- Compatibility to codes for new structures: the code should use the same philosophy as current codes for new structures (limit state analysis, safety factor format etc).
- Flexibility: the code should be flexible to include additional information gained by inspection.
- Ease of use: the code should be understandable to engineers and easy to use in practice.

3.3 POSSIBLE CONTENT OF A CODE

Some recommendations on the content of a code on assessment of existing structures are briefly described next:

3.3.1 Area of Application

The code should be applicable in the following cases:

- changes in the load-carrying system;
- change in the utilisation of the structure;
- extension of planned service life;
- deterioration and/or damage;
- reliability of the structure in doubt;
- clearly inadequate serviceability.

3.3.2 General Principles of Assessment

Analysis and assessment of an existing structure shall be based on the same general principles as provided by current standards for the design of new structures. Older codes valid in the period when the original structure was designed, or based on other principles, should be used only as guidance documents.

3.3.3 Criteria

Procedures and different phases of assessment depend on type and the importance of the structure. Different phases in the assessment procedure may be appropriate.

Currently applied limit state formulations for the specified hazard scenarios should provide the basis for reassessment criteria. Limit states are basically classified as for design purposes in two categories (see Annex A):

- Ultimate Limit States (ULS), which concern the maximum load carrying capacity of the structure.
- Serviceability Limit States (SLS), which concern the normal use of the structure.

For the description and the formulation of the limit states the following groups of basic variables are taken into account:

- geometry properties (such as dimensions of structural members);

- load characteristics;
- material properties;
- model uncertainties.

The variability of such variables shall be analized based on the available information.

3.3.4 Methods for Updating

Updating of information should be performed based on state-of-the art of reliability analysis. Two different routes can be distinguished:

- updating of individual random variables due to measurements, observations related to the individual variable based possibly on Bayesian techniques.
- updating of failure probability by conditioning i.e. conditional failure probabilities due to measured cracks, or due to survival of extreme loads,...

Analytical methods for updating are described in Annex B.

3.3.5 Format for Verification

The format of verification depends upon the degree of sophistication of the assessment analyses. Either a partial safety factor format or a semi- or full probabilistic format may be used.

3.3.6 Risk Acceptance Criteria

Risk acceptance criteria should be derived based on:

- implementation of a safety class differentiation principle;
- limit state classification;
- considerion of the desired residual service life;
- current tendencies in target safety levels regarding existing structures.

3.3.7 Decisions

If the degree of the reliability is too low, the code must require a decision to:

- either reduce the loads,
- or to adequately strengthen the structure,
- or to demolish the structure.

LIST OF REFERENCES

Allen, D.E., 1991, Criteria for Structural Evaluation and Upgrading of Existing Buildings, The Canadian Journal of Civil Engineering.

Bea, R.G.,1993, Reliability Based Requalification Criteria for Offshore Platforms, Proceedings of ASME-OMAE Conference, Glasgow, UK.

Vrouwenvelder, T., 1993, Codes of Practice for the Assessment of Existing Structures, Proc. of IABSE Conference, Copenhagen, Denmark.

ASSESSMENT OF EXISTING STRUCTURES
ANNEX A: RELIABILITY ANALYSIS PRINCIPLES

1. INTRODUCTION

In recent years, practical reliability methods have been developed to help engineers tackle the analysis, quantification, monitoring and assessment of structural risks, undertake sensitivity analysis of inherent uncertainties and make appropriate decisions about the performance of a structure. The structure may be at the design stage, under construction or in actual use.

This Annex summarizes the principles and procedures used in formulating and solving risk related problems via reliability analysis. It is neither as broad nor as detailed as available textbooks on this subject, some of which are included in the bibliography.

Starting from the principles of limit state analysis and its application to codified design, the link is made between unacceptable performance and probability of failure. It is important, especially in assessment, to distinguish between components and systems. System concepts are introduced and important results are summarized. The steps involved in carrying out a reliability analysis, whose main objective is to estimate the failure probability, are outlined and alternative techniques available for such an analysis are presented. Some recommendations on formulating stochastic models for commonly used variables are also included.

2. CONCEPTS

2.1. Limit States

The structural performance of a whole structure or part of it may be described with reference to a set of limit states which separate acceptable states of the structure from unacceptable states. The limit states are divided into the following two categories:

- ultimate limit states, which relate to the maximum load carrying capacity.
- serviceability limit states, which relate to normal use.

The boundary between acceptable (safe) and unacceptable (failure) states may be distinct or diffuse but, at present, deterministic codes of practice assume the former.

Thus, verification of a structure with respect to a particular limit state is carried out via a model describing the limit state in terms of a function (called the limit state function) whose value depends on all relevant design parameters. In general terms, attainment of the limit state can be expressed as

$$g\,(\mathbf{s}, \mathbf{r}) = 0 \qquad \text{(A.1)}$$

where **s** and **r** represent sets of load (actions) and resistance variables.

Conventionally, $g\,(\mathbf{s}, \mathbf{r}) \le 0$ represents failure; in other words, an adverse state.

The limit state function, $g\,(\mathbf{s}, \mathbf{r})$, can often be separated into one resistance function, *r(.)*, and one loading (or action effect) function, *s(.)*, in which case equation (1) can be expressed as

$$r\,(\mathbf{r}) - s\,(\mathbf{s}) = 0 \qquad \text{(A.2)}$$

2.2. Partial Factors and Code Formats

Within present limit state codes, **s** and **r** are treated as deterministic quantities. The particular values substituted into equations (A.1) or (A.2) -the "design" values- are based on past experience and, in some cases, on reliability calibration methods.
Typically, the design value, x_{di}, of any particular variable is given by

$$x_{di} = \gamma_i \, x_{ki} \qquad \text{(A.3a)}$$

$$x_{di} = x_{ki} / \gamma_i \qquad \text{(A.3b)}$$

where x_{ki} is a characteristic value and γ_i is a partial factor. Eqn (A.3a) is appropriate for loading variables whereas eqn (A.3b) applies to resistance variables, and in both cases γ_i has a value greater than unity. A characteristic value is strictly defined as the value of a random variable which has a prescribed probability of not being exceeded (or of being attained). In treating time-varying loads, a value other than the characteristic may be introduced. For material properties a specified or nominal value is often used as a specified characteristic value.

Partial factors account for the possibility of unfavourable deviations from the characteristic value, inaccuracies and simplifications in the assessment of the resistance or the load effect, uncertainties introduced due to the measurement of actual properties by limited testing, etc. The partial factors are an important element in controlling the safety of a structure designed to the code but there are other considerations involved in achieving this objective. It is clear from eqn (A.3a) and (A.3b) that a particular design value x_{di} may be obtained by different combinations of x_{ki} and γ_i.

The process of selecting the set of partial factors to be used in a particular code could be seen as a process of optimization such that the outcome of all designs undertaken to the code is in some sense optimal. However, such a formal optimization process is not always carried out in practice; even in cases where it has been undertaken, the values of the partial factors finally adopted may be modified to account for simplicity and ease of use.

It may be deduced from the above that in the specification of design values in limit state codes a probabilistic interpretation is possible and desirable. Even if a probabilistic design philosophy and its associated rules have not been formally used in the development of a design code, it is generally accepted that the code should not be written in a way that contradicts such principles.

Eqn (A.2), lends itself to the following partial factor safety checking code format

$$\gamma_{Sd} \, s(F_d \, , \ldots) \leq \frac{1}{\gamma_{Rd}} r(f_d \, , \ldots) \qquad \text{(A.4)}$$

where F_d, f_d are the design values, which can be obtained from characteristic values and associated partial factors, and γ_{Sd}, γ_{Rd} are partial factors related to modelling uncertainties (loading and resistance respectively). Note that altenative safety checking formats have been developed for and used in limit state codes.

The safety checking equation controls the way in which the various clauses of the code lead to the desirable level of safety of structures designed to the code. It relates to the number of design checks required, the rules for load combinations, the number of partial factors and their position in design equations, as well as whether they are single or multiple valued, and the definition of characteristic or representative values for all design variables.

In principle, there is a partial factor associated with each variable. Furthermore, the number of load combinations can become large for structures subjected to a number of permanent and variable (time-dependent) loads. In practice, it is desirable to reduce the number of partial factors and load combinations while, at the same time, ensuring an acceptable range of safety level and an acceptable economy of construction.

2.3. Structural Reliability

Load, material and geometry parameters are subject to uncertainties, which can be classified according to their nature, see section 3. They can, thus, be represented by random variables (this being the simplest possible probabilistic representation, whereas more advanced models might be appropriate in certain situations, such as random fields). The variables **S** and **R** are often referred to as "basic random variables" (where the upper case letter is used for denoting random variables) and may be collectively represented by a random vector **X**.

In this context, failure is a probabilistic event and its probability of occurrence, P_f, is given by

$$P_f = \text{Prob}\ \{\ g\ (\mathbf{X}) \leq 0\ \} = \text{Prob}\ \{\ \text{M} \leq 0\ \} \qquad \text{(A.5a)}$$

where, M = g (**X**). Note that M is also a random variable, called the safety margin.

If the limit state function is expressed in the form of eqn (A.2), eqn (A.5a) can be written as

$$P_f = \text{Prob}\ \{\ r\ (\mathbf{R}) \leq s\ (\mathbf{S})\ \} = \text{Prob}\ \{\ \text{R} \leq \text{S}\ \}$$

where R = r (**R**) and S = s (**S**) are random variables associated with resistance and loading respectively. This expression is useful in the context of the discussion in section 2.2 on code formats and partial safety factors but will not be further used herein.

The failure probability defined in eqn (A.5a) can also be expressed as follows

$$P_f = \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}\ (\mathbf{x})\ d\mathbf{x} \qquad \text{(A.5b)}$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function of **X**.

The reliability, P_s, associated with the particular limit state considered is the complementary event, i.e.

$$P_s = 1 - P_f \qquad \text{(A.6)}$$

In recent years, a standard reliability measure, the reliability index β, has been adopted which has the following relationship with the failure probability

$$\beta = -\ \Phi^{-1}(P_f) = \Phi^{-1}(P_s) \qquad \text{(A.7)}$$

where $\Phi^{-1}(.)$ is the inverse of the standard normal distribution function, see Table A.1.

Table A.1: Relationship between β and P_f

P_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	1.3	2.3	3.1	3.7	4.2	4.7	5.2

In most engineering applications, complete statistical information about the basic random variables **X** is not available and, furthermore, the function $g(.)$ is a mathematical model which idealizes the limit state. In this respect, the probability of failure evaluated from eqn (A.5a) or (A.5b) is a point estimate given a particular set of assumptions regarding probabilistic modelling and a particular mathematical model for $g(.)$. The uncertainties associated with these models can be represented in terms of a vector of random parameters **Q**, and hence the limit state function may be re-written as $g(\mathbf{X}, \mathbf{Q})$. It is important to note that the nature of uncertainties represented by the basic random variables **X** and the parameters **Q** is different. Whereas uncetainties in **X** cannot be influenced without changing the physical characteristics of the problem (e.g. changing the steel grade), uncertainties in **Q** can be influenced by the use of alternative methods and collection of additional data.

In this context, eqn (A.5b) may be recast as follows

$$P_f(\theta) = \int_{g(\mathbf{x},\theta) \leq 0} f_{\mathbf{X}|\Theta}(\mathbf{x}|\,\theta)\, d\mathbf{x} \qquad \text{(A.8)}$$

where $P_f(\mathbf{q})$ is the conditional probability of failure for a given set of values of the parameters **q** and $f_{\mathbf{X}|\mathbf{Q}}(\mathbf{x}|\,\mathbf{q})$ is the conditional probability density function of **X** for given **q**.

In order to account for the influence of parameter uncertainty on failure probability, one may evaluate the expected value of the conditional probability of failure, i.e.

$$\overline{P_f} = \mathrm{E}\,[P_f(\theta)] = \int_{\theta} P_f(\theta)\, f_{\Theta}(\theta)\, d\theta \qquad \text{(A.9a)}$$

where $f_{\mathbf{Q}}(\mathbf{q})$ is the joint probability density function of **Q**. The corresponding reliability index is given by

$$\overline{\beta} = -\,\Phi^{-1}(\overline{P_f}) \qquad \text{(A.9b)}$$

The main objective of reliability analysis is to estimate the failure probability (or, the reliability index). Hence, it replaces the deterministic safety check, e.g. eqn (A.4), with a probabilistic assessment of the safety of the structure, e.g. eqn (A.5) or eqn (A.9). Depending on the nature of the limit state considered, the uncertainty sources and their implications for probabilistic modelling, the characteristics of the calculation model and the degree of accuracy required, an appropriate methodology has to be developed. In many respects, this is similar to the considerations made in formulating a methodology for deterministic structural analysis but the problem is now set in a probabilistic framework.

2.4. System Concepts

Structural design is, at present, primarily concerned with component behaviour. Each limit state equation is, in most cases, related to a single mode of failure of a single component. However,

- most structures are an assembly of structural components
- even individual components may be susceptible to a number of possible failure modes.

In deterministic terms, the former can be tackled through a progressive collapse analysis (particularly appropriate in redundant structures), whereas the latter is usually dealt with by checking a number of limit state equations.

However, the system behaviour of structures is not well quantified in limit state codes and requires considerable innovation and initiative from the engineer. A probabilistic approach provides a better platform from which system behaviour can be explored and utilised. This can be of benefit in assessment of existing structures where strength reserves due to system effects can alleviate the need for expensive strengthening.

There are two fundamental systems, see Fig. A.1:

(1) A series system is a system which fails if one or more of its components fail.
(2) A parallel system is a system which fails when all its components have failed.

The probability of system failure is given by

$$P_{f,\,sys} = P[E_1 \cup E_2 \cup ... \cup E_n] \quad \text{for a series system} \qquad \text{(A.10a)}$$

$$P_{f,\,sys} = P[E_1 \cap E_2 \cap ... \cap E_n] \quad \text{for a parallel system} \qquad \text{(A.10b)}$$

where E_i (i=1, ...n) is the event corresponding to failure of the ith component. In the case of parallel systems, which are designed to provide some redundancy, it is important to define the state of the component after failure. In structures, this can be described in terms of a characteristic load-displacement response, see Fig. A.2, for which two convenient idealisations are the 'brittle' and the 'fully ductile' case. Intermediate, often more realistic, cases can also be defined.

The above expressions can be difficult to evaluate in the case of large systems with stochastically dependent components and, for this reason, upper and lower bounds have been developed, which may be used in practical applications. In order to appreciate the effect of system behaviour on failure probabilities, results for two special systems comprising equally correlated components with the same failure probability for each component are shown in Fig. A.3(a) and A.3(b). Note that in the case of the parallel system, it is assumed that the components are fully ductile.

More general systems can be constructed by combining the two fundamental types. It is fair to say that system methods are more developed for skeletal rather than continuous structures. Important results from system reliability theory are summarized in section 4.

3. COMPONENT RELIABILITY ANALYSIS

The framework for probabilistic modelling and reliability evaluation is outlined in this section. The focus is on the procedure to be followed in assessing the reliability of a critical component with respect to a particular failure mode.

3.1. General Steps

The main steps in a component reliability analysis are the following:

(1) select appropriate limit state function
(2) specify appropriate time reference
(3) identify basic variables and develop appropriate probabilistic models
(4) compute reliability index and failure probability
(5) perform sensitivity studies

Step (1) is essentially the same as for deterministic analysis. Step (2) should be considered carefully, since it affects the probabilistic modelling of many variables, particularly live loading. Step (3) is perhaps the most important because the considerations made in developing the probabilistic models have a major effect on the results obtained, see section 3.2. Step (4) should be undertaken with one of the methods summarized in section 3.3, depending on the application. Step (5) is necessary insofar as the sensitivity of any results (deterministic or probabilistic) should be assessed before a decision is taken.

3.2. Probabilistic Modelling

For the particular failure mode under consideration, uncertainty modelling must be undertaken with respect to those variables in the corresponding limit state function whose variability is judged to important (basic random variables). Most engineering structures are affected by the following types of uncetainty:

- intrincic physical or mechanical uncertainty; when considered at a fundamental level, this uncertainty source is often best described by stochastic processes in time and space, although it is often modelled more simply in engineering applications through random variables.
- measurement uncertainty; this may arise from random and systematic errors in the measurement of these physical quantities
- statistical uncertainty; due to reliance on limited information and finite samples
- model uncertainty; related to the predictive accuracy of calculation models used

The physical uncertainty in a basic random variable is represented by adopting a suitable probability distribution, described in terms of its type and relevant distribution parameters. The results of the reliability analysis can be very sensitive to the tail of the probability distribution, which depends primarily on the type of distribution adopted. A proper choice of distribution type is therefore important.

For most commonly encountered basic random variables there have been studies (of varying detail) that contain guidance on the choice of distribution and its parameters. If direct measurements of a particular quantity are available, then existing, so-called *a priori*, information (e.g. probabilistic models found in published studies) should be used as prior statistics with a relatively large equivalent sample size ($n' \approx 50$).

The following comments may also be helpful in selecting a suitable probabilistic model.

Material properties
- frequency of negative values is normally zero
- log-normal distribution can often be used
- distribution type and parameters should, in general, be derived from large homogeneous samples and with due account of established distributions for similar variables (e.g. for a new high strength steel grade, the information on properties of existing grades should be consulted); tests should be planned so that they are, as far as possible, a realistic description of the potential use of the material in real applications.

Geometric parameters
- variability in structural dimensions and overall geometry tends to be small
- dimensional variables can be adequately modelled by the normal or log-normal distribution
- if the variable is physically bounded, a truncated distribution may be appropriate (e.g. location of reinforcement); such bounds should always be carefully considered to avoid entering into physically inadmissible ranges
- variables linked to manufacturing can have large coefficients of variation (e.g. imperfections, misalignments, residual stresses, weld defects).

Load variables
- loads should be divided according to their time variation (permanent, variable, accidental)
- in certain cases, permanent loads consist of the sum of many individual elements; they may be represented by a normal distribution
- for single variable loads, the form of the point-in-time distribution is seldom of immediate relevance; often the important random variable is the magnitude of the largest extreme load

that occurs during a specified reference period for which the probability of failure is calculated (e.g. annual, lifetime)

- the probability distribution of the largest extreme could be approximated by one of the asymptotic extreme-value distributions (Gumbel, sometimes Frechet)
- when more than one variable loads act in combination, load modelling is often undertaken using simplified rules suitable for FORM/SORM analysis.

In selecting a distribution type to account for physical uncertainty of a basic random variable afresh, the following procedure may be followed:

- based on experience from similar type of variables and physical knowledge, choose a set of possible distributions
- obtain a reasonable sample of observations ensuring that, as far as possible, the sample points are from a homogeneous group (i.e. avoid systematic variations within the sample) and that the sampling reflects potential uses and applications
- evaluate by an appropriate method the parameters of the candidate distributions using the sample data; the method of maximum likelihood is recommended but evaluation by alternative methods (moment estimates, least-square fit, graphical methods) may also be carried out for comparison.
- compare the sample data with the resulting distributions; this can be done graphically (histogram vs. pdf, probability paper plots) or through the use of goodness-of-fit tests (Chi-square, Kolmogorov-Smirnov tests)

If more than one distributions give equally good results (or if the goodness-of-fit tests are acceptable to the same significance level), it is recommended to choose the distribution that will result in the smaller reliability. This implies choosing distributions with heavy left tails for resistance variables (material properties, geometry excluding tolerances) and heavy right tails for loading variables (manufacturing tolerances, defects and loads).

Capturing the essential features of physical uncertainty in a load or in a structure property through a random variable model is perhaps the simplest way of modelling uncertainty and quantifying its effect on failure probability. In general, loads are functions of both time and position on any particular structure. Equally, material properies and dimensions of even a single structural member, e.g. a RC floor slab, are functions which vary both in time and in space. Such random functions are usually denoted as random (or stochastic) processes when time variation is the most important factor and as random fields when spatial variation is considered.

Fig. A.4(a) shows schematically a continuous stochastic process, e.g. wind pressure at a particular point on a wall of a structure. The trace of this process over time is obtained through successive realisations of the underlying phenomenon, in this case wind speed, which is clearly a random variable taking on different values within each infinitesimally small time interval, δt.

Fig. A.4(b) depicts a two-dimensional random field, e.g. the spatial variation of concrete strength in a floor slab just after construction. Once again, a random variable, in this case describing the possible outcomes of, say, a core test obtained from any given small area, δA, is the basic kernel from which the random field is built up.

In considering either a random process or a random field, it is clear that, apart from the characteristics associated with the random variable describing uncertainty within a small unit (interval or area), laws describing stochastic dependence (or, in simpler terms, correlation) between outcomes in time and/or in space are very important.

The other three types of uncertainty mentioned above (measurement, statistical, model) also play an important role in the evaluation of reliability. As mentioned in section 2.3, these uncertainties are influenced by the particular method used in, for example, strength analysis and by the collection of additional (possibly, directly obtained) data. These uncertainties could be rigorously analysed by adopting the approach outlined by eqns (A.8) and (A.9). However, in many practical applications a simpler approach has been adopted insofar as model (and measurement) uncertainty is concerned based on the differences beween results predicted by the mathematical model adopted for $g(x)$ and some more elaborate model believed to be a closer representation of actual conditions. In such cases, a model uncertainty basic random variable X_m is introduced where

$$X_m = \frac{\text{actual value}}{\text{predicted value}}$$

and the following comments offer some general guidance in estimating the statistics of X_m:

- the mean value of the model uncertainty associated with code calculation models can be larger than unity, reflecting the in-built conservatism of code models
- the model uncertainty parameters of a particular calculation model may be evaluated vis-a-vis physical experiments or by comparing the calculation model with a more detailed model (e.g. finite element model)
- when experimental results are used, use of measured rather than nominal or characteristic quantities is preferred in calculating the predicted value
- the use of numerical experiments (e.g. finite element models) has some advantages over physical experiments, since the former ensure well-controlled input.
- the choice of a suitable probability distribution for X_m is often governed by mathematical convenience and a normal distribution has been used extensively.

3.3. Computation of Failure Probability

As mentioned above, the failure probability of a structural component with respect to a single failure mode is given by

$$P_f = \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x} \qquad \text{(A.5b)}$$

where **X** is the vector of basic random variables, $g(x)$ is the limit state (or failure) function for the failure mode considered and $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function of **X**.

An important class of limit states are those for which all the variables are treated as time independent, either by neglecting time variations in cases where this is considered acceptable or by transforming time-dependent processes into time-invariant variables (e.g. by using extreme value distributions). The methods commonly used for calculating P_f in such cases are outlined below. Guidelines on how to deal with time-dependent problems are given in section 5. Note that after calculating P_f via one of the methods outlined below, or any other valid method, a reliability index may be obtained from equation (A.7), for comparative or other purposes.

<u>Asymptotic approximate methods</u>

Altough these methods first emerged with basic random variables described through 'second-moment' information (i.e. with their mean value and standard deviation, but without assigning any probability distributions), it is nowadays possible in many cases to have a full description of the random vector **X** (as a result of data collection and probabilistic modelling studies). In such cases, the probability of failure could be calculated via first or second order reliability methods (FORM and SORM respectively). Their implementation relies on:

(1) Transformation techniques:

$$\mathbf{T}: \quad \mathbf{X} = (X_1, X_2, \dots X_n) \quad \rightarrow \quad \mathbf{U} = (U_1, U_2, \dots U_n) \qquad \text{(A.11)}$$

where $U_1, U_2, \ldots U_n$ are independent standard normal variables (i.e. with zero mean value and unit standard deviation). Hence, the basic variable space (including the limit state function) is transformed into a standard normal space, see Fig. A.5. The special properties of the standard normal space lead to several important results, as discussed below.

(2) Search techniques:

In standard normal space, the objective is to determine a suitable checking point: this is shown to be the point on the limit-state surface which is closest to the origin, the so-called 'design point'. In this rotationally symmetric space, it is the most likely failure point, in other words its co-ordinates define the combination of variables that are most likely to cause failure. This is because the joint standard normal density function, whose bell-shaped peak lies directly above the origin, decreases exponentially as the distance from the origin increases. To determine this point, a search procedure is required in all but the most simple of cases (the Rackwitz-Fiessler algorithm is commonly used).

Denoting the co-ordinates of this point by

$$\mathbf{u}^* = (u_1^*, u_2^*, \ldots u_n^*)$$

its distance from the origin is clearly equal to

$$(\sum_{i=1}^{n} u_i^{*\,2})^{1/2}$$

This scalar quantity is known as the Hasofer-Lind reliability index, β_{HL}, i.e.

$$\beta_{HL} = (\sum_{i=1}^{n} u_i^{*\,2})^{1/2} \qquad \text{(A.12)}$$

Note that $\mathbf{u}^*$ can also be written as

$$\mathbf{u}^* = \beta_{HL}\, \alpha \qquad \text{(A.13a)}$$

where $\alpha = (\alpha_1, \alpha_2, \ldots \alpha_n)$ is the unit normal vector to the limit state surface at $\mathbf{u}^*$, and, hence, α_i (i=1,...n) represent the direction cosines at the design point. These are also known as the sensitivity factors, as they provide an indication of the relative importance of the uncertainty in basic random variables on the computed reliability. Their absolute value ranges between zero and unity and the closer this is to the upper limit, the more significant the influence of the respective random variable is to the reliability. The following expression is valid for independent variables

$$\sum_{i=1}^{n} \alpha_i^2 = 1 \qquad \text{(A.13b)}$$

(3) Approximation techniques:

Once the checking point is determined, the failure probability can be approximated using results applicable to the standard normal space. Thus, in a first-order approximation, the limit state surface is approximated by its tangent hyperplane at the design point. The probability content of the failure set is then given by

$$P_{fFORM} = \Phi(-\beta_{HL}) \qquad \text{(A.14a)}$$

In some cases, a higher order approximation of the limit state surface at the design point is merited, if only to check the accuracy of FORM. The result for the probability of failure assuming a quadratic (second-order) approximation of the limit state surface is asymptotically given by

$$P_{f\text{SORM}} = \Phi(-\beta_{\text{HL}})\prod_{i=1}^{n-1} (1 - \beta_{\text{HL}}\kappa_j)^{-1/2} \qquad \text{(A.14b)}$$

for $\beta_{\text{HL}} \rightarrow \infty$, where κ_j are the principal curvatures of the limit state surface at the design point. An expression applicable to finite values of β_{HL} is also available.

Simulation Methods

In this approach, random sampling is employed to simulate a large number of (usually numerical) experiments and to observe the result. In the context of structural reliability, this means, in the simplest approach, sampling the random vector **X** to obtain a set of sample values. The limit state function is then evaluated to ascertain whether, for this set, failure (i.e. $g(\mathbf{x})\leq 0$) has occurred. The experiment is repeated many times and the probability of failure, P_f, is estimated from the fraction of trials leading to failure divided by the total number of trials. This so-called Direct or Crude Monte Carlo method is not likely to be of use in practical problems because of the large number of trials required in order to estimate with a certain degree of confidence the failure probability. Note that the number of trials increases as the failure probability decreases. Simple rules may be found, of the form $N > C/P_f$, where N is the required sample size and C is a constant related to the confidence level and the type of function being evaluated.

Thus, the objective of more advanced simulation methods, currently used for reliability evaluation, is to reduce the variance of the estimate of P_f. Such methods can be divided into two categories, namely indicator function methods and conditional expectation methods.

An example of the former is Importance Sampling, where the aim is to concentrate the distribution of the sample points in the vicinity of likely failure points, such as the design point obtained from FORM/SORM analysis. This is done by introducing a sampling function, whose choice would depend on *a priori* information available, such as the co-ordinates of the design point and/or any estimates of the failure probability. In this way, the success rate (defined here as a probability of obtaining a result in the failure region in any particular trial) is improved compared to Direct Monte Carlo. Importance Sampling is often used following an initial FORM/SORM analysis. A variant of this method is Adaptive Sampling, in which the sampling density is updated as the simulation proceeds. Importance Sampling could be performed in basic variable or standard normal space, depending on the problem and the form of prior information.

A powerful method belonging to the second category is Directional Simulation. It achieves variance reduction using conditional expectation in the standard normal space, where a special result applies pertaining to the probability bounded by a hypershere centred at the origin. Its efficiency lies in that each random trial generates precise information on where the boundary between safety and failure lies. However, the method does generally require some iterative calculations. It is particularly suited to problems where it is difficult to identify 'important' regions (perhaps due to the presence of multiple local design points).

The two methods outlined above have also been used in combination, which indicates that when simulation is chosen as the basic approach for reliability assessment, there is scope to adapt the detailed methodology to suit the particular problem in hand.

3.4. Interpretation of Results

As mentioned above, under certain conditions the design point is the most likely failure point. Since the objective of a deterministic code is to ascertain attainment of a limit state, it is clear that any check should be performed at a critical combination of loading and resistance variables and, in this respect, the design point values from a reliability analysis are a good choice. Hence, in the deterministic safety check, equation (A.4), the design values can be directly linked to the results of a reliability analysis, i.e. P_f or β and α_i 's. Thus, the partial factor associated with a basic random variable X_i, is given as

$$\gamma_{Xi} = \frac{x_{di}}{x_{ki}} = \frac{F_{Xi}^{-1}(\Phi(u_i^*))}{x_{ki}} = \frac{F_{Xi}^{-1}(\Phi(\alpha_i\beta))}{x_{ki}} \qquad \text{(A.15a)}$$

where x_{di} is the design point value and x_{ki} is a characteristic value of X_i, As can be seen, the design point value can be written using the results of section 3.3 in terms of the original distribution function $F_{x_i}(.)$, the reliability analysis results, i.e. β and α_i, and the standard normal distribution function $\Phi(.)$.

If X_i is normally distributed, eq. (A.15a) can be written as (after non-dimensionalising both x_{di} and x_{ki} with respect to the mean value)

$$\gamma_{Xi} = \frac{1 - \alpha_i \beta\, v_{Xi}}{1 + k\, v_{Xi}} \qquad \text{(A.15b)}$$

where v_{Xi} is the coefficient of variation and k is a constant related to the fractile of the distribution selected to represent the characteristic value of the random variable X_i. As shown, eq. (A.15a) and (A.15b) are used for determining partial factors of loading variables, whereas their inverse is used for determining partial factors of resistance variables. Similar expressions are available for variables described by other distributions (e.g. log-normal, Gumbel type I). Thus, partial factors could be derived or modified using FORM/SORM results.

If the reliability assessment is carried out using solely simulation, sensitivity factors are not directly obtained, though, in principle, they could be through some additional calculations.

A possibility that might arise in assessment is the reduction of uncertainty in a random variable (through measurement, monitoring, etc.). This reduces the corresponding sensitivity factor and increases the reliability index. An approximate expression that gives the factor by which the reliability index is increased if the ith basic variable is replaced by a fixed value (in fact, its median value) is as follows

$$\frac{\beta_{new}}{\beta_{old}} = \frac{1}{\sqrt{1 - \alpha_i^2}} \qquad \text{(A.16)}$$

3.5. Recommendations

As with any other analysis, choosing a particular method must be justified through experience and/or verification. Experience shows that FORM/SORM estimates are adequate for a wide range of problems. However, these approximate methods have the disadvantage of not being quantified by error estimates, except for few special cases. As indicated, simulation may be used to verify FORM/SORM results, particularly in situations where multiple design points might be suspected. Simulation results should include the variance of the estimated probability of failure, though good estimates of the variance could increase the computations required.

When using FORM/SORM, attention should be given to the ordering of dependent random variables and the choice of initial points for the search algorithm. Not least, the results for the design point should be assessed to ensure that they do not contradict physical reasoning.

4. SYSTEM RELIABILITY ANALYSIS

As discussed in section 3, individual component failure events can be represented by failure boundaries in basic variable or standard normal space. System failure events can be similarly represented, see Fig. A.6(a) and A.6(b), and, once more, certain approximate results may be derived as an extension to FORM/SORM analysis of individual components. In addition, system analysis is sometimes performed using bounding techniques and some relevant results are given below.

4.1. Series systems

The probability of failure of a series system with *m* components is defined as

$$P_{f\,sys} = \mathrm{P}\left[\bigcup_{j=1}^{m} \mathrm{F}_j\right] \qquad \text{(A.17)}$$

where, F_j is the event corresponding to the failure of the *j*th component. By describing this event in terms of a safety margin M_j

$$\mathrm{P}[\mathrm{F}_j] = \mathrm{P}[\mathrm{M}_j \leq 0] \approx \Phi(-\beta_j) \qquad \text{(A.18)}$$

where β_j is its corresponding FORM reliability index, it can be shown that in a first-order approximation

$$P_{f\,sys} = 1 - \Phi_{\mathrm{m}}\left[\tilde{\beta}\,;\,\tilde{\tilde{\rho}}\right] \qquad \text{(A.19a)}$$

where $\Phi_m[.]$ is the multi-variate standard normal distribution function, $\tilde{\beta}$ is the $(m \times 1)$ vector of component reliability indices and $\tilde{\tilde{\rho}}$ is the $(m \times m)$ correlation matrix between safety margins with elements given by

$$\rho_{jk} = \sum_{i=1}^{n} \alpha_{ij}\alpha_{ik} \quad j, k = 1, 2, \ldots, \mathrm{m} \qquad \text{(A.19b)}$$

where α_{ij} is the sensitivity factor corresponding to the *i*th random variable in the *j*th margin.

In some cases, especially when the number of components becomes large, evaluation of equation (A.19) becomes cumbersome and bounds to the system failure probability may prove sufficient.

Simple first-order linear bounds are given by

$$\operatorname*{Max}_{j=1}^{m}[\mathrm{P}(\mathrm{F}_j)] \leq P_{f\,sys} \leq \mathrm{Min}\left[\left(\sum_{j=1}^{m} \mathrm{P}(\mathrm{F}_j)\right), 1\right] \qquad \text{(A.20a)}$$

but these are likely to be rather wide, especially for large *m*, in which case second-order linear bounds (Ditlevsen bounds) may be needed. These are given by

$$P[F_1] + \sum_{j=2}^{m} \text{Max}\left\{\left[P(F_j) - \sum_{k=1}^{j-1} P\left(F_j \cap F_k\right)\right], 0\right\}$$
$$\leq P_{f\,sys} \leq \qquad \text{(A.20b)}$$
$$P[F_1] + \sum_{j=2}^{m} \left\{P[F_j] - \underset{k<j}{\text{Max}}\left[P\left(F_j \cap F_k\right)\right]\right\}$$

The narrowness of these bounds depends in part on the ordering of the events. The optimal ordering may differ between the lower and the upper bound. In general, these bounds are much narrower than the simple first-order linear bounds given by equation (A.20a). The bisections of events may be calculated using a first-order approximation, which appears below in the presentation of results for parallel systems.

4.2. Parallel Systems

Following the same approach and notation as above, the failure probability of a parallel system with m components is given by

$$P_{f\,sys} = P\left[\bigcap_{j=1}^{m} (F_j)\right] = P\left[\bigcap_{j=1}^{m} (M_j \leq 0)\right] \qquad \text{(A.21)}$$

and the corresponding first-order approximation is

$$P_{f\,sys} = \Phi_m\left[-\tilde{\beta} \,;\, \tilde{\tilde{\rho}}\right] \qquad \text{(A.22)}$$

Simple bounds are given by

$$0 \leq P_{f\,sys} \leq \underset{j=1}{\overset{m}{\text{Min}}}\,[P(F_j)] \qquad \text{(A.23a)}$$

These are usually too wide for practical applications. An improved upper bound is

$$P_{f\,sys} \leq \underset{j,\,k=1}{\overset{m}{\text{Min}}}\left[P\left(F_j \cap F_k\right)\right] \qquad \text{(A.23b)}$$

The error involved in the first-order evaluation of the intersections, $P[F_j \cap F_k]$, is, to a large extent, influenced by the non-linearity of the margins at their respective design points. In order to obtain a better estimate of the intersection probabilities, an improvement on the selection of linearisation points has been suggested.

5. TIME-DEPENDENT RELIABILITY

5.1. General Remarks

Even in considering a relatively simple safety margin for component reliability analysis such as $M = R - S$, where R is the resistance at a critical section in a structural member and S is the corresponding load effect at the same section, it is generally the case that both S and resistance R are functions of time. Changes in both mean values and standard deviations could occur for either $R(t)$ or $S(t)$. For example, the mean value of $R(t)$ may change as a result of deterioration (e.g. corrosion of reinforcement in an RC bridge implies loss of area, hence a reduction in the mean resistance) and its standard deviation may also change (e.g. uncertainty in predicting the effect of corrosion on loss of area may increase as the periods considered become longer). On the other hand, the mean value of $S(t)$ may increase over time (e.g. due to higher traffic flow and/or higher individual vehicle weights) and, equally, the estimate of its standard deviation may increase due to lower confidence in predicting the correct mix of traffic for longer periods. The general time-dependent reliability problem could thus be schematically represented as in Fig. A.6(a), the diagram clearly implying that the reliability decreases with time. Although this situation is usual, the converse could also occur in reliability assessment of existing structures (through variable or margin updating, as described in Annex B).

Another important class of problems calling for a time-dependent reliability analysis are those related to damage accumulation, such as fatigue and fracture. This case is depicted in Fig. A.6(b) via a fixed threshold (e.g. allowable crack size) and a monotonically increasing time-dependent load effect (e.g. actual crack size at any given time).

5.2. Transformation to Time-Independent Formulations

Although time variations are likely to be present in most structural reliability problems, the methods outlined in Sections 3 and 4 have gained wide acceptance, partly due to the fact that, in many cases, it is possible to transform a time dependent failure mode into a corresponding time independent mode. This is especially so in the case of overload failure, where individual time-varying actions, which are essentially random processes, $p(t)$, can be modelled by the distribution of the maximum value within a given reference period T, i.e. $X = \max_T\{ p(t)\}$ rather than the point-in-time distribution. For continuous processes, the probability distribution of the maximum value (i.e. the largest extreme) is often approximated by one of the asymptotic extreme value distributions. Hence, for structures subjected to a single time-varying action, a random process model is replaced by a random variable model and the principles and methods given previously may be applied.

The theory of stochastic load combination is used in situations where a structure is subjected to two or more time-varying actions acting simultaneously. When these actions are independent, perhaps the most important observation is that it is highly unlikely that each action will reach its peak lifetime value at the same moment in time. Thus, considering two time-varying load processes $p_1(t)$, $p_2(t)$, $0 \le t \le T$, acting simultaneously, for which their combined effect may be expressed as a linear combination $p_1(t) + p_2(t)$, the random variable of interest is

$$X = \max_T\{ p_1(t) + p_2(t)\} \tag{A.24a}$$

If the loads are independent, replacing X by $\max_T\{ p_1(t)\} + \max_T\{p_2(t)\}$ leads to very conservative results. However, the distribution of X can be derived in few cases only. One possible way of dealing with this problem, which also leads to a relatively simple deterministic code format, is to replace X with the following

$$X' = \max_T \begin{cases} \max_T\{ p_1(t)\} + p_2(t) \\ p_1(t) + \max_T\{ p_2(t)\} \end{cases} \tag{A.24b}$$

This rule (Turkstra's rule) suggests that the maximum value of the sum of two independent load processes occurs when one of the processes attains its maximum value. This result may be generalised for several independent time-varying loads. The conditions which render this rule adequate for failure probability estimation are discussed in standard texts. Note that the failure probability associated with the sum of a special type of independent identically distributed processes (rectangular pulse or FBC process) can be calculated in a more accurate way.

The FBC (Ferry Borges-Castanheta) process is generated by a sequence of independent identically distributed random variables, each acting over a given (deterministic) time interval. This is shown in Fig. A.7 where the total reference period T is made up of n_i repetitions, where $n_i = T/\tau_i$. Because of independence, the maximum value in the reference period T is given by

$$F_{\max_T X_i}(x_i) = [F_{X_i}(x_i)]^{n_i} \tag{A.25}$$

When a number of FBC processes act in combination and the ratios of their repetition numbers within a given reference period are given by positive integers it is, in principle, possible to obtain the extreme value distribution of the combination through a recursive formula. More importantly, it is possible to deal with the sum of FBC processes by implementing the Rackwitz-Fiessler algorithm in a FORM/SORM analysis.

A deterministic code format, compatible with the above rules, leads to the introduction of combination factors, ψ_{0i}, for each time-varying load i. In principle, these factors express ratios between fractiles in the extreme value and point-in-time distributions so that the probability of exceeding the design value arising from a combination of loads is of the same order as the probability of exceeding the design value caused by one load. For time-varying loads, they would depend on distribution parameters, target reliability and FORM/SORM sensitivity factors and on the frequency characteristics (i.e. the base period assumed for stationary events) of loads considered within any particular combination.

5.3. Introduction to Crossing Theory

In considering a time-dependent safety margin, i.e. $M(t) = g(\mathbf{X}(t))$, the problem is to establish the probability that $M(t)$ becomes zero or less in a reference time period, T. This constitutes a so-called 'crossing' problem. The time at which $M(t)$ becomes less than zero for the first time is called the 'time to failure' and is a random variable, see Fig. A.8(a), or, in a basic variable space, Fig. A.8(b). The probability that $M(t) \leq 0$ occurs during T is called the'first-passage' ~~or 'outcrossing' probability)~~ Clearly, it is identical to the probability of failure during time T.

The determination of the first passage probability requires an understanding of the theory of random processes. Herein, only some basic concepts are introduced in order to see how the methods described above have to be modified in dealing with crossing problems.

With reference to Fig. A.8(b), the first-passage probability, $P_f(t)$ during a period $[0,t]$ is

$$P_f(t) = 1 - \mathrm{P}[\, N(t)=0 \mid \mathbf{X}(0) \in D]\, P[\mathbf{X}(0) \in D] \qquad \text{(A.26a)}$$

where $\mathbf{X}(0)\in D$ signifies that the process $\mathbf{X}(t)$ starts in the safe domain and $N(t)$ is the number of outcrossings in the interval $[0,t]$. The second probability term is equivalent to $1 - P_f(0)$, where $P_f(0)$ is the probability of failure at $t=0$. Equation (A.25a) can be re-written as

$$P_f(t) = P_f(0) + (1 - P_f(0))\,(1 - \mathrm{P}[N(t)=0]) \qquad \text{(A.26b)}$$

from which different approximations may be derived depending on the relative magnitude of the terms. A useful bound is

$$P_f(t) \leq P_f(0) + \mathrm{E}[N(t)] \qquad \text{(A.27)}$$

where the first term may be calculated by FORM/SORM and the expected number of outcrossings, $\mathrm{E}[N(t)]$, is calculated by Rice's formula or one of its generalisations. Alternatively, parallel system concepts can be employed.

notion of outcrossing rate !

Bibliography

/A1/ Ang A H S and Tang W H, *Probability Concepts in Engineering Planning and Design*, Vol. I & II, John Wiley, 1984.

/A2/ Augusti G, Baratta A and Casciati F, *Probabilistic Methods in Structural Engineering*, Chapman and Hall, 1984.

/A3/ Benjamin J R and Cornell C A, *Probability, Statistics and Decision for Civil Engineers*, McGraw Hill, 1970.

/A4/ Bolotin V V, *Statistical Methods in Structural Mechanics*, Holden-Day, 1969.

/A5 Borges J F and Castanheta M, *Structural Safety*, Laboratorio Nacional de Engenharia Civil, Lisboa, 1985.

/A6/ Ditlevsen O, *Uncertainty Modelling*, McGraw Hill, 1981.

/A7/ Ditlevsen O and Madsen H O, *Structural Reliability Methods*, J Wiley, 1996.

/A8/ Madsen H O, Krenk S and Lind N C, *Methods of Structural Safety*, Prentice-Hall, 1986.

/A9/ Melchers R E, *Structural Reliability: Analysis and Prediction*, Ellis Horwood/J Wiley, 1987.

/A10/ Thoft-Christensen P and Baker M J, *Structural Reliability Theory and its Applications*, Springer-Verlag, 1982.

/A11/ Thoft-Christensen P and Murotsu Y, *Application of Structural Systems Reliability Theory*, Springer-Verlag, 1986.

/A12/ CEB, *First Order Concepts for Design Codes*, CEB Bulletin No. 112, 1976.

/A13/ CEB, *Common Unified Rules for Different Types of Construction and Materials, Vol. 1*, CEB Bulletin No. 116, 1976.

/A14/ Construction Industry Research and Information Association (CIRIA), *Rationalisation of Safety and Serviceability Factors in Structural Codes*, Report 63, London, 1977.

/A15/ International Organization for Standardization (ISO), *General Principles on Reliability for Structures*, ISO 2394, Second edition, 1986.

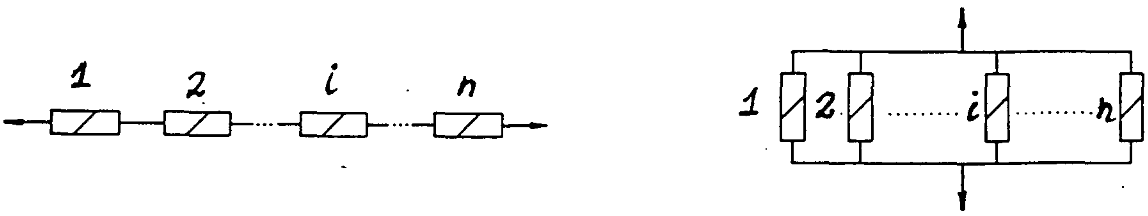


Figure 1: Schematic representation of series and parallel systems

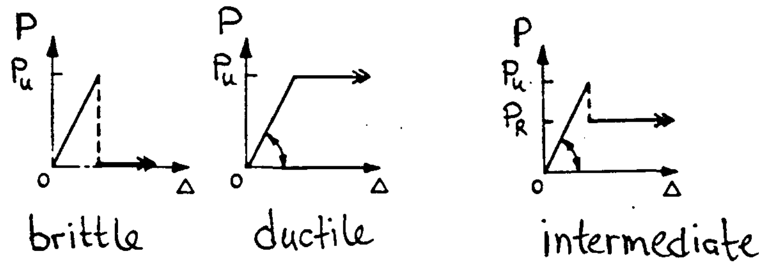


Figure 2: Idealised load-displacement response of structural elements

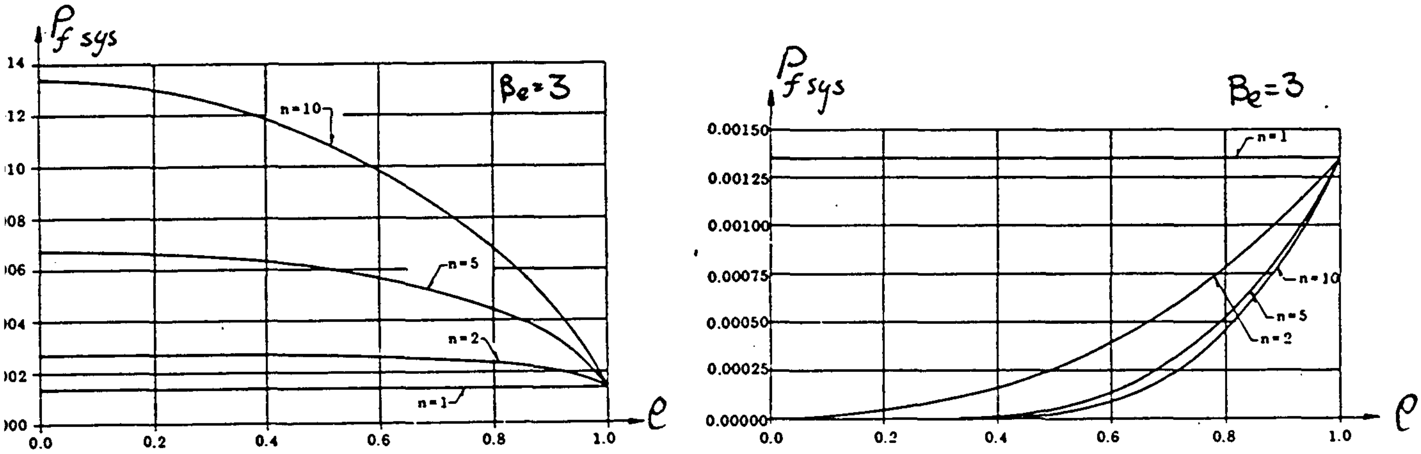


Figure 3: Effect of element correlation and system size on failure probability
(a) series system (b) parallel system

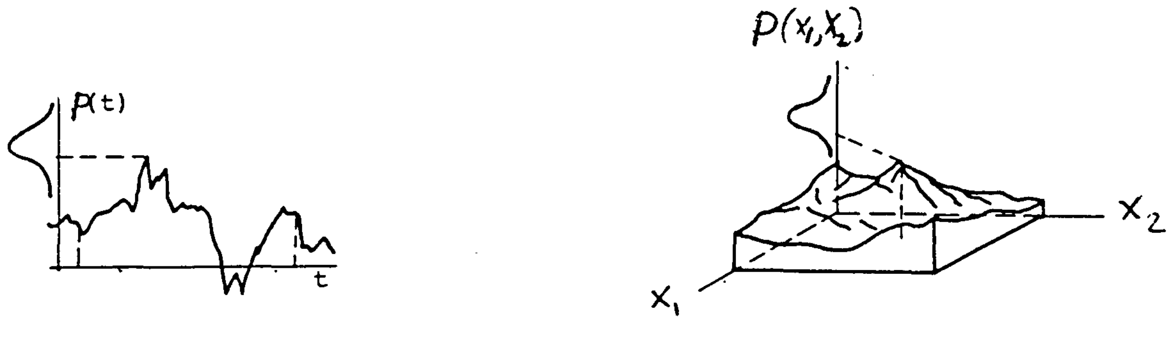


Figure 4: Schematic representations
(a) random process (b) random field

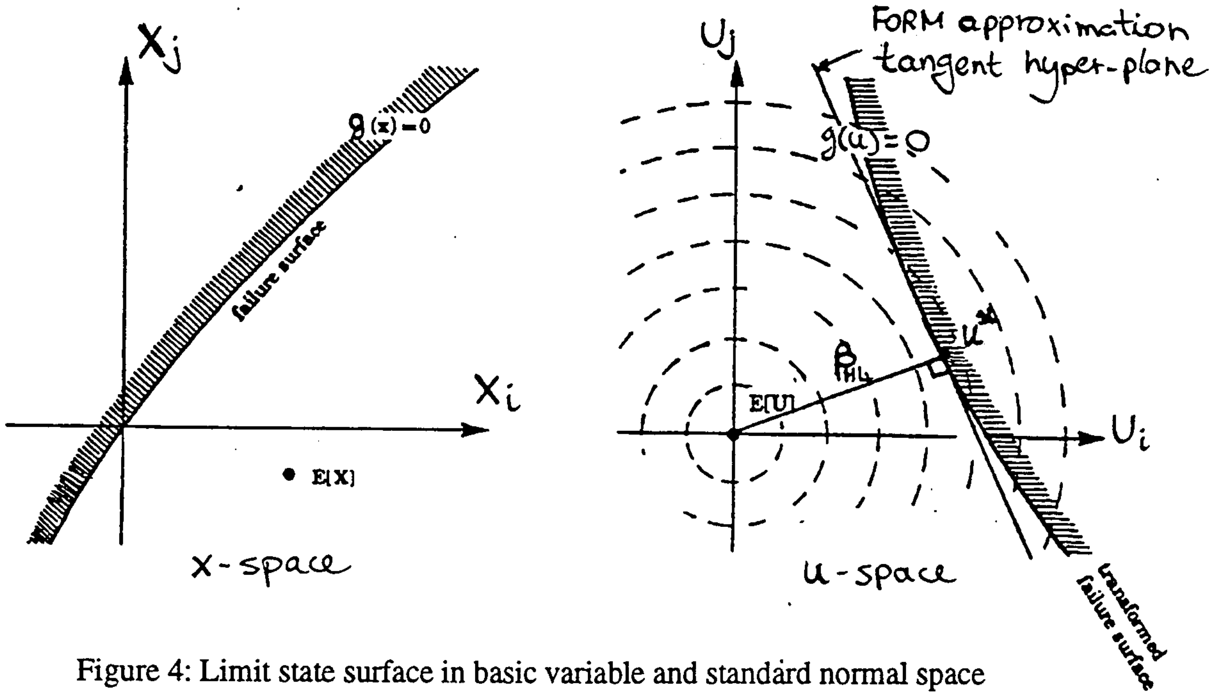


Figure 4: Limit state surface in basic variable and standard normal space

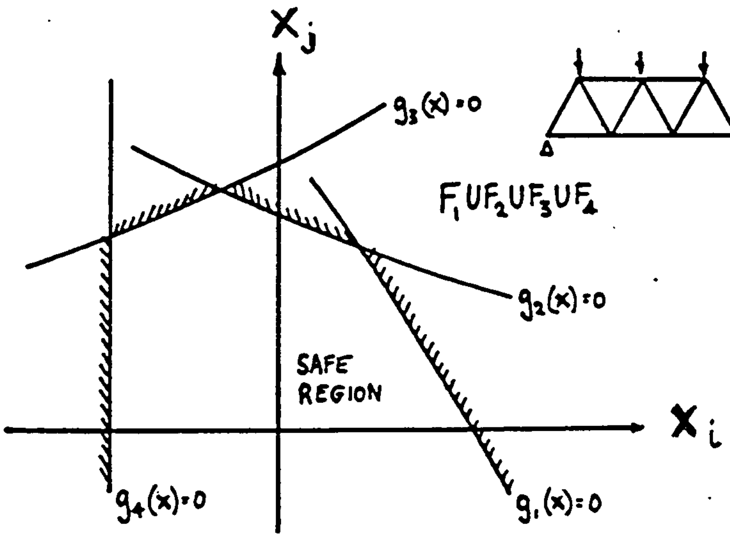


Figure 5(a): Failure region as union of component failure events for series system

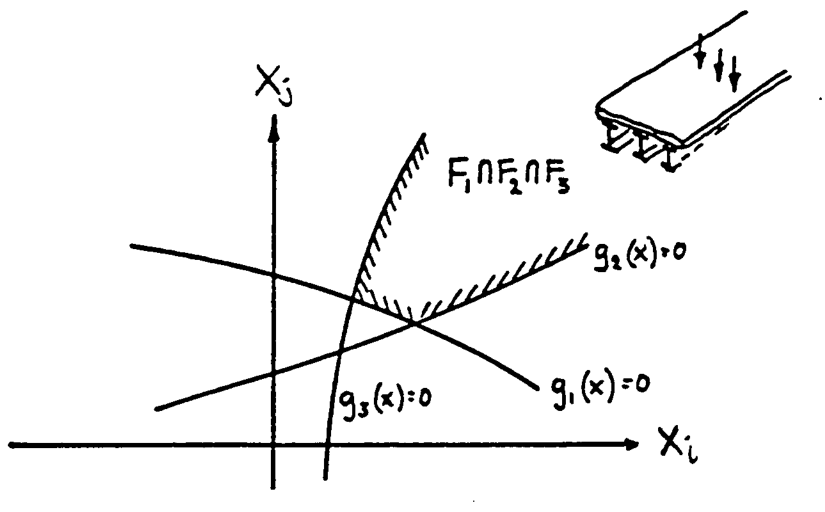


Figure 5(b): Failure region as intersection of component failure events for parallel system

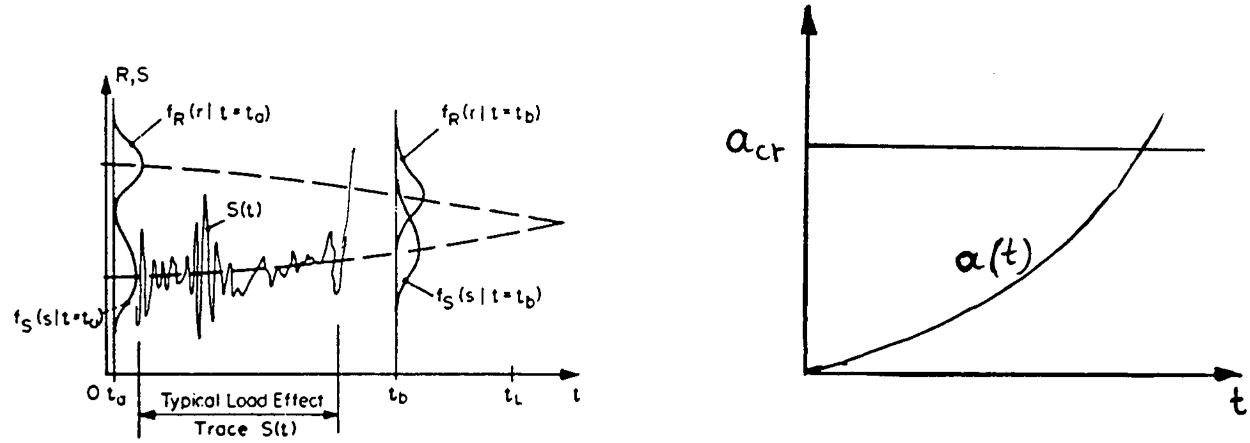


Figure 6: Time-dependent reliability analysis
(a) general case (b) damage accumulation problem

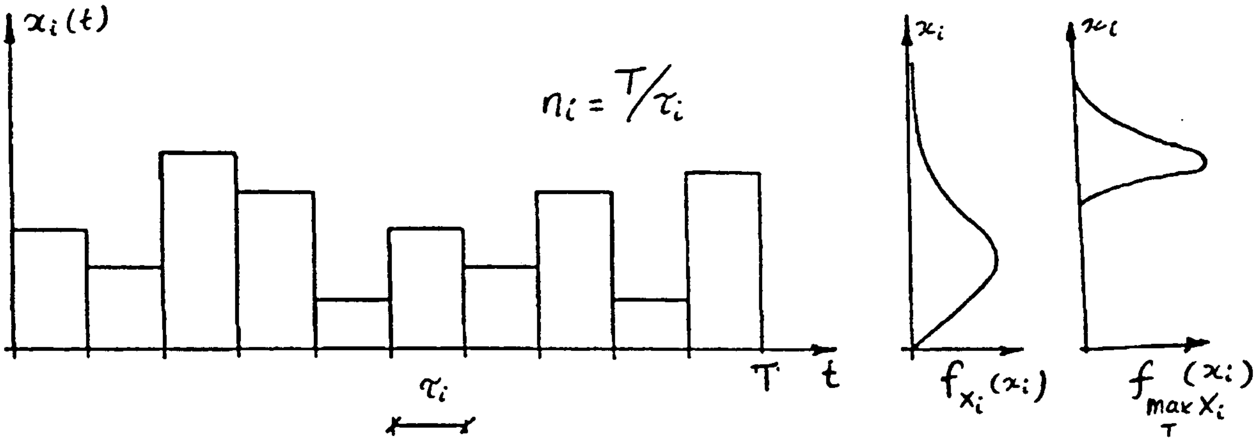


Figure 7: Realization of an FBC process

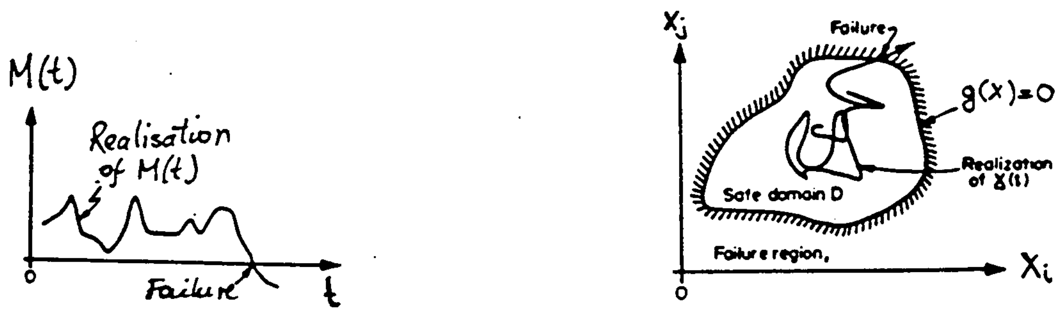


Figure 8: Time-dependent safety margin and schematic representation of vector outcrossing

JCSS

Reliability Updating Framework for Structural Reassessment

Annexes II & III

1995-10-3

ANNEX B:

RELIABILITY UPDATING FRAMEWORK

1 Introduction

When a structure is designed the knowledge about the structure 'as build' is associated with uncertainty regarding geometry, material properties, loading and environmental conditions.

A part of this uncertainty is due to inherent randomness which may be present for e.g. material properties and loading characteristics, but a substantial part of the uncertainty arise from extrapolation of information. In this way the uncertainty associated with e.g. material properties in the design phase contains a significant contribution from the fact that the materials manufacturer may not be known and because the material batch characteristics may not be known.

The probabilistic models used in the design and in the assessment of a structure are henge merely reflecting the imperfect knowledge about the structure and this knowledge may be updated as soon as the structure has been realized.

An important task in the assessment of existing structures is therefore to perform a successive process of collecting and utilizing information about the condition of the structure, behaviour of the structure and the loading on the structure.

Given that the requirements regarding the present and future use of a structure are specified the reassessment process is a decision process of identifying the measures which will lead to the most economical fulfilment of these requirements.

Such measures may be to inspect and collect information regarding the geometry of the structure, the material properties, the deterioration of the structure, the static and dynamic behaviour of the structure and the loading on the structure.

Measures may also be taken to repair or strengthen the structure or even to replace the structure.

Whatever measure is taken it must be evaluated and compared to alternative measures in terms of its monetary value throughout the required service life.

The present document describes the probabilistic and decision theoretical framework for reassessment. The more practical aspects are described in detail in the Basic Document on Probabilistic Reassessment and the detailed information on the probabilistic modelling of uncertainties and the evaluation of probabilities is described in Annex 1.

2 The Decision Theoretical Framework

In practical decision problems such as reassessment, inspection and maintenance planning for structures the number of alternative actions can be extremely large and a framework for the systematic analysis of the corresponding consequences is therefore expedient. A framework suitable for this purpose which allows for the inclusion of subjective information is the Bayesian decision analysis, and therefore a short presentation of this will be given in the following. A more complete treatment of the decision theory can e found in Raiffa & Schleifer [1]. Decision problems are conveniently represented by decision trees as illustrated in Figure 2.1.

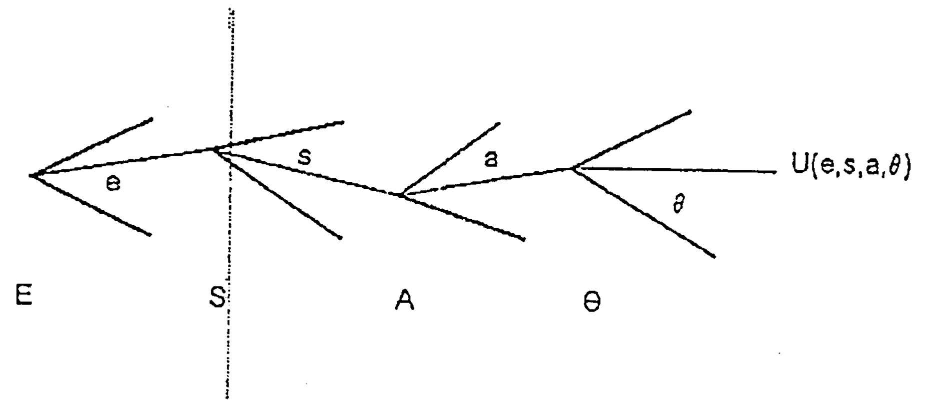


Figure 2.1 Decision tree used in Bayesian decision analysis

Generally speaking the decision making problem is to choose an experiment e from the space of possible experiments E yielding a random outcome s of possible experiment outcomes S which can be used by the decision maker to take an action a out of the possible available actions A. When the decision maker has taken an action this will result in a random outcome of the nature Θ out of the possible states of the nature θ. The performed experiment and the chosen action together with the outcome of the experiment and nature determines the corresponding utility. The part of the decision tree starting with choosing an action a based on the experiment outcome s is also called a terminal analysis or posterior analysis because the statistics of the utility can be estimated using known statistics about the nature whereas the complete analysis is called a preposterior analysis because the experiment outcomes are still unknown.

In order to perform a decision analysis the following information and operations must be included:

Information concerning the alternative "experiments" E and actions A. In reassessment this information regards the possible inspection methods, test procedures and preventive maintenance and repair actions.

Assignment of a utility function $u(e,s,a,\theta)$ on the space $E \times S \times A \times \Theta$, such that any selection of "experiments" and "actions" together with the associated outcomes can be associated with costs or whatever measure of utility is appropriate in the case of interest.

Assignment of probability $P_{\Theta,S}(\theta,s \mid e)$ on the space $\Theta \times S$. This joint probability measure determines the four probabilities of importance :

a The marginal probability measure on the state of the nature $P'_{\Theta}(\Theta)$. This is normally referred to as a prior probability in the sense that the decision maker assigns the probability measure to Θ prior to knowing the outcome s of the experiment e.

b The conditional probability measure on the outcome of the nature $P_S(s \mid \Theta,e)$ referred to as the sample likelihood representing the new information obtained by the experiment.

c New information can be combined with prior probabilities of the state of the nature by application of Bayes' rule

$$P''_{\theta}(\theta|s) = \frac{P_S(s|\theta,e)P'_{\theta}(\theta)}{\sum_{\theta} P_S(s|\theta,e)\ P'_{\theta}(\theta)} \tag{2.1}$$

The conditional probability measure on the state of the nature is called the posterior probability. Posterior in the sense that the probability measure is assigned to θ after (posterior) to knowing the outcome s of the experiment e.

d The marginal probability measure on the outcome of the nature $P\,S(s \mid e)$ of a given experiment e.

The decision problem can then be stated as ; Given E,S,A,θ,u and $P\theta,S(\Theta,s \mid e)$ how must one choose an experiment e yielding an outcome s based on which action a is taken, in such a way that the utility u is maximized.

There are two equivalent ways how the analysis leading to the maximum utility can be formulated, namely the so-called extensive and the normal form of the analysis.

In the following the normal form will be shortly explained as this is often is the most convenient formulation for practical applications. The extensive form formulation can be found e.g. in Raiffa & Schleifer [1].

In the normal form of decision analysis a decision rule d is specified which prescribes the action which must be taken for all possible outcomes of the experiment e. For every experiment e the optimal decision rule d can be selected. By doing this for all possible experiments e the optimal experiment can be selected. The decision rule for a specific experiment e is a mapping carrying s in S into $d(s)$ in A. For a selected experiment e the expected utility is

$$\begin{aligned} u(e,d) &= E_{\Theta,S \mid e}[u(e,s,d(s),\Theta)] \\ &= E'_{\Theta}(E_{S \mid e,\theta}[u(e,s,d(s),\Theta)]] \end{aligned} \tag{2.2}$$

the optimal experiment e and the optimal decision rule can now be identified by solving

$$\max_{e} \max_{d} E'_{\Theta}[E_{S \mid e,\theta}[u(e,s,d(s),\Theta)]] \tag{2.3}$$

The complete analysis is called pre-posterior because a number of posterior analysis are performed conditional upon the experiment and the outcome of the experiment.

In section 6 the application of equation (2.3) for decision making in reassessment of existing structures is considered in more detail.

3 Target Failure Probabilities

The individual decision makers may have very different preferences depending on personal factors like carrier stage and role in society. Consequently different decision makers may have quite different views on the costs of failure.

In order to avoid structures which are unacceptable to society even though optimal for a particular decision maker it is necessary to introduce a requirement on the maximum acceptable probability of failure - a target failure probability. This requirement also makes it possible together with other measures to assure that requirements regarding maximum risk exposure for personnel are maintained.

Constraints on the failure probability should be imposed twofold, namely in terms of failure rates i.e. a maximum failure probability corresponding to a specific reference period (one year) and a maximum failure probability corresponding to the design lifetime of the structure.

The failure rate related requirement assures that a reasonable level of risk can be maintained and controlled by society at all times. The lifetime failure probability related constraint may have the effect to assure that the investment in the structure is secured. In most cases the failure rate related constraint will be the active constraint.

The acceptable failure rates and probabilities are related to the type of structure considered. Hereunder its exposure to and use by the public as well as its role and value for the society.

Values for the required failure rates and probabilities corresponding to the above mentioned different structure types can be constructed from established optimal codes of practice in terms of the risk weighted average failure probability.

The appropriate approach for identification of target failure probabilities is to postulate (more or less based on physical evidence) a probabilistic model describing the uncertainty structure for the parameters defining the loading, materials and calculation models. Based on this probabilistic model the "formal" target failure probabilities are calibrated by repeated reliability analysis such that structures which are designed according to the probabilistic model and corresponding target failure probabilities are consistent with structures designed according to the existing semi-probabilistic design codes.

The approach is described in detail in Ditlevsen [2].

This task should however closely follow the calibration of the probabilistic model code.

4 Assessment of Utility

The utility measure serves as a consistent basis for taking into account the preferences of the decision maker when two or more decision alternatives are compared. Even though it may in some cases appear inappropriate, it is necessary and unavoidably that the preferences of the decision maker are expressed in terms of utility (benefit/disbenefit) whereby the different preferences are transformed into the same units.

In decision analysis for reassessment of structures, utility has to be associated with decisions which influence the future performance of the structure with respect to safety, deterioration, availability etc.. It is therefore necessary that relationships are defined such that the performance of the structure can be associated with utility. Such functions are normally denoted by utility functions.

The optimal reassessment decision is identified in terms of the expected value of the utility. Having identified all utility generating events in the decision problem, the next step is, for each decision alternative to associate to these events the corresponding marginal utilities. Thereafter the expected utility associated with each decision alternative may be evaluated by the sum over the products of the marginal utilities and the corresponding marginal probabilities of the utility generating events.

As an example consider the situation where the considered marginal utilities are associated with the events of failure, repair and inspection. In this case the expected utility for one particular decision alternative may be expressed as

$$\begin{aligned} E[C_t(t_{insp})] &= E[C_f]+E[C_r]+E[C_i] \\ &= P_f\,C_f + P_r\,C_r + C_i \end{aligned} \tag{4.1}$$

where P_f, P_r and C_f, C_r, C_i are the marginal probabilities of the utility generating events (appropriately represented in terms of limit state functions, see e.g. Annex 1 and costs associated with failure, repair and inspection respectively.

The outcome of the decision analysis is to a large extent depending on the utility representation of the preferences of the decision maker. Therefore extreme care must be exercised when the utility functions are defined. Important aspects in this connection is a careful identification of the decision maker and his preferences e.g. with respect to risk adverseness, value of reputation etc..

One of the most crucial points is to maintain that the decision process leading to a specific reassessment action is transparent to the decision maker. This effectively means that the decision basis is clear and traceable.

As a general guideline transparency may be maintained by clearly separating all the consequence generating events from each other such that all events which generate different marginal consequences are taken into account individually. This will in many cases also eliminate the need for non-linear utility functions.

5 Quantification of Information

When assessing a structure according to phase III (as described in the Basic Document on Reassessment) all available information about the properties and behaviour of the structure may lead to a better estimation of the structural capacity and thus to a reduction of the uncertainties. In principle, it does not matter whether the information is quantitative or qualitative as both types of information can be treated. Formal reliability assessment, however, requires a quantitative type of statement as a starting point for further processing.

For this reason, "qualitative" statements like "the structure looks fine" should be translated into "quantitative statements" like: no visible cracking, no visible deflection and so on. If one also knows, from other experiments, what the threshold values are for visual crack and deflection observation, these statements can be used in the formal procedure.

Several sources of uncertainty influence the consequences of selecting a specific reassessment action. These uncertainties are the uncertainties over which the expectation operation in equation (2.3) must be performed when the optimal reassessment scheme is identified.

The uncertainties which must be considered are uncertainties associated with the loading environment, the geometry of the structure, the material properties, the inspection methods and the repair qualities.

Modern reliability methods allow for a very general representation of these uncertainties ranging from non-stationary Gaussian stochastic processes and fields to time-invariant random variables, see Annex 1.

The basis for the uncertainty model may be any mixture of frequentistic and subjective information. It should, however, be emphasized that on the one hand the model should aim for simplicity in the formulation of the uncertainty modelling and on the other hand the model should be close enough to reality to allow for including important information collected during the lifetime of the structure, thus allowing for updating the uncertainty structure in the considered problem. In this way uncertainty models which initially are based entirely on subjective information will as new information is collected eventually be based on frequentistic information.

When discussing updating techniques for structural reliability two types of quantitative information should be distinguished:

- information of the equality type
- information of the inequality type

When information of the <u>equality</u> type is present, it means that for some basic or response variables the value has been measured. Examples are: the stress equals 200 MPa, the crack length is 3.2 mm. Of course, these equality measurements are seldom perfect and may suffer from some kind of measurement error. In a probabilistic evaluation procedure, measurement errors should be modelled as random variables, having means (zero for unbiased estimates), standard deviations and some correlation pattern. The standard deviation is a property of the measurement technique, but may also depend on the circumstances. An important but very difficult modelling part is the degree of correlation between observations on different places and different points in time.

The information of the inequality type refers to observations where it is only known that the observed variable is greater than or less than some limit: a crack may be less than the observation threshold, a limit state of collapse may be reached (or not). Uncertainty in the threshold value should be taken into account. The distribution function for the minimum threshold level is often referred to as the Probability of Detection curve (POD curve). Also here, correlations for the probability of detection in various observations should be known.

Mathematically the two types of information can be denoted as:

equality type: $g(x) = 0$ (5.1)

inequality type: $g(x) < 0$ (5.2)

x = vector of basic variables

In this notation measurement values and threshold values are considered as components of the vector x.

5.1 Updating of Random Variables

Inspection results relating directly to realizations of random variables may be used in the updating. This is done by assuming the distribution parameters of the distributions used in the probabilistic modelling to be uncertain themselves. New samples or observations of realizations of the random variables are then used to update the probability distribution functions of these distribution parameters.

The distribution parameters are initially (and prior to any update) modelled by prior distribution functions. The prior distribution functions is best updated by Bayesian reasoning which, however, requires that a weight is given to the information contained in the prior distribution functions e.g. in terms of equivalent sample sizes if conjugate priors are used. Unfortunately the latter are only available for some distribution functions which nevertheless belong to the set of those models most commonly in use. By application of Bayes theorem the prior distribution functions, assessed by any mixture of frequentistic and subjective information, are updated and transformed into posterior distribution functions. A set of conjugte priors with associated posteriors and predictive distributions may be found in Raiffa & Schleifer [1] and in the Basic Note on Bayesian updating. The general scheme for the updating is

$$f''(q|x) = \frac{f'(q)\ L(q|x)}{\int f'(q)\ L(q|x)\ dq} \qquad (5.3)$$

where "''" denotes the posterior, "'" the prior and L(q | x) the likelihood function for q given the observation x. For discrete distributions the integral is replaces by summation. In principle, it is possible to operate with arbitrary priors when FORM/SORM techniques are applied, see Annex 1. An example regarding the updating of concrete compression strength can be found in annex 4.

5.2 Event Updating

Given an inspection result of a quantity which is given as a functional relationship between several basic variables, probabilities may be updated by direct updating of the relevant failure probabilities, using the definition of conditional probability:

$$P\{F \mid I\} = \frac{P\{F \cap I\}}{P\{I\}} \tag{5.4}$$

F = Failure
I = Inspection result

For a further evaluation of (5.4) it is important to distinguish between the two types of inspection results mentioned in section 2.

The inequality type information "g(x) < 0 " may be elaborated in a very straight forward way. Let F be represented by M(x) < 0, where M denotes the event margin. We then have:

$$P\{F \mid I\} = \frac{P\{M(x) < 0 \cap g(x) < 0\}}{P\{g(x) < 0\}} \tag{5.5}$$

x = vector of random variables having the prior distribution $f_x(\xi)$

This procedure can easily be extended to complex failure modes and to a set of inspection results $\cap g_i(x) < 0$.

For further calculation, software packages are available such as STRUREL by RCP-GmbH and PROBAN by Det Norske Veritas (DNV).

For (5.5) the equality sign inspection type gives more difficulties. One possibility is to replace "g(x) = 0" by "g(x) > 0 and g(x) < ε" where ε is some convenient small number. Another possibility is to use the theory of conditional Gaussian distributions. In that case M and g should be approximated by Gaussian random variables with parameters μ(M), σ(M), μ(g), σ(g) and ρ(Mg). Then:

$$P\{F \mid I\} = P\{M < 0 \mid g = 0\} = P\{M' < 0\} \tag{5.6}$$

Here M′ is the conditional distribution of M given g = 0. This distribution is Gaussian with:

$$\mu(M') = \mu(M) + \rho(Mg)\ \mu(g)\ \sigma(M)/\sigma(g) \tag{5.7}$$

$$\sigma(M') = \sigma(M)\sqrt{(1-\rho^2(Mg))} \tag{5.8}$$

This system can also be extended to complex failure modes and multiple inspection results. What additionally is needed then is the, coefficient of correlation between M_1 and M_2 for given g:

$$\rho(M_1' \; M_2') = \frac{\rho(M_1M_2) - \rho(M_1g)\,\rho(M_2g)}{\sqrt{\{1-\rho^2(M_1g)\}}\,\sqrt{\{1-\rho^2(M_2g)\}}} \tag{5.9}$$

In reality, of course, inspection results can also be of a mixed type: partly equality, partly inequality type.

Finally it should be mentioned that individual random variables may also be updated by inspections of events involving the outcomes of several random variables. This should nevertheless be done with care. For instance it is important to realise that all the random variables that are present in g(X) (and all the variables correlated to X) are affected by the inspection. For instance, if we measure a crack length in one point of an offshore structure, this affects the distributions of the load parameters, the stress concentration factors, the residual stresses, and the parameters of the fatigue model. Moreover, all these parameters become correlated, even if they were independent before inspection.

6 Optimal Reassessment and Maintenance Planning

The reassessment scheme and corresponding inspection and maintenance plan yielding the maximum utility or equivalently the minimum expected total costs for maintaining the structure throughout its anticipated lifetime is the so called optimal scheme.

As the number of numerical operations necessary in order to estimate the expected costs associated with a particular reassessment scheme can be extremely large and rather time consuming it is important to use a model of the decision problem which on one hand reflects the features in the real decision problem and on the other hand is practical applicable seen from a computational point of view.

One such model is to represent the impact of future inspections and maintenance actions in the assessment scheme by the so called adaptive scheme. In this scheme only the next inspection time t_{insp} is taken into account together with the inspection method *i* and the repair action *d*. When the next inspection has been performed the next inspection is planned taking the most resent inspection observations (if there are any) into due account. The corresponding decision tree is shown in Figure 6.1.

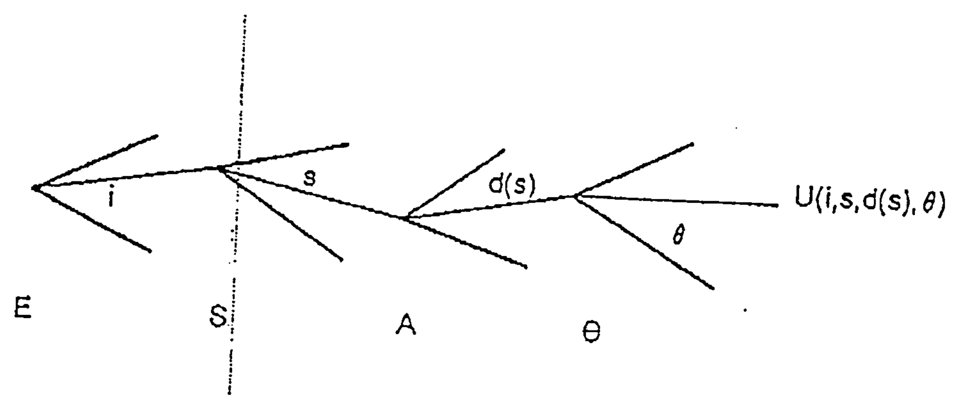


Figure 6.1 Decision tree for the adaptive scheme

Corresponding to equation (2.3) the optimal assessment scheme is identified as the scheme minimizing the total costs C_p associated with the structure throughout its anticipated lifetime. This scheme can be identified by solving

$$\min \min E_{\Theta,S \mid i}[C_t(i,s,d(s),\Theta)] \tag{6.1}$$

In general the total cost C_t which must be considered are the costs associated with performing the inspections, the costs associated with the repair actions, the costs associated with the failure events and the costs associated with loss of fulfilment of its requirements.

The expected total costs $E[C_t]$ associated with a particular assessment scheme are calculated by the following expression

$$E[C_t(t_{insp})] = E[C_f] + E[C_r] + E[C_i] \tag{6.2}$$

where $E[C_f]$ is the expected cost of failure , $E[C_r]$ is the expected cost of repair and $E[C_i]$ is the costs of inspection.

Even though the failure costs are included in the cost term it may be necessary to impose requirements on the reliability (see section 3) of the structure corresponding to a certain specified time interval (normally one year or the anticipated lifetime t_s of the structure) such that this does not decrease below a certain minimum value. This may be taken into account by reformulating equation (6.1) as

$$\begin{aligned} &\min \min E_{\Theta,S \mid i}[C_t(i,s,d(s),\Theta)] \\ &\text{s.t.} \qquad \beta(t_s) \geq \beta_{min} \end{aligned} \tag{6.3}$$

where β is the generalized reliability index defined as $\beta = -\Phi^{-1}(P_f)$ with P_f the failure probability.

7 Examples

7.1 Optimal Inspection and Maintenance Planning for Offshore Structures

The following example considers the basic framework for the optimal planning of inspections and maintenance for an offshore structure subject to fatigue crack growth.

7.1.1 Inspection Modelling

Inspection, testing and instrumentation of structures with respect to the assessment of the damage state, the static and dynamic properties of the structure as well as characteristics of the loading and the environment may be associated with considerable uncertainty. In order to extract the information correctly from observations and measurement the uncertainties associated with these must be taken properly into account.

The reliability of inspections, tests and measurements in terms of their ability to detect as well as their accuracy in sizing is therefore an important quantity to take into account when an assessment scheme together with an inspection and maintenance plan is evaluated.

At the time of inspection, tests and measurements basically 4 different observations are possible;

1. The quantity of interest is not observed - e.g. the damage is non-existing or smaller than the detectable damage size A_d - i.e. a pure detection problem.
2. The observed quantity has a size equal to A_{obs}.
3. The observed quantity has a size smaller than A_{obs} - e.g. damage is observed but the only sizing possible shows that the damage is smaller than A_{obs}.
4. The observed quantity has a size larger than A_{obs} - similar to 3.

These observations are represented by their corresponding event margins

1. $M_{obs} = A_d - A(t_{insp}) > 0$
2. $M_{obs} = A_{obs} + \epsilon_{insp} - A(t_{insp}) = 0$
3. $M_{obs} = A_{obs} + \epsilon_{insp} - A(t_{insp}) > 0$
4. $M_{obs} = A_{obs} + \epsilon_{insp} - A(t_{insp}) < 0$

where ϵ_{insp} represents the sizing uncertainty associated with the inspection, test or measurement performed and $A(t_{insp})$ is the actual size of the observed quantity at the time t_{insp}. It can normally be assumed that the sizing uncertainty variable ϵ_{insp} has zero mean and standard deviation σ, but this matter must be considered from case to case.

The qualities of different inspection methods with respect to their ability to detect the quantities of interest are modelled by assigning different distributions (P.O.D.) for the detectable size A_d of the considered quantity and for the measurement uncertainty ϵ_{insp} to the individual methods. Often A_d is modelled by the exponential distribution and ϵ_{insp} by the Normal distribution. The distribution parameters of A_d and ϵ_{insp} differ from inspection method to inspection method. Examples of P.O.D. curves are shown in Figure 7.1.

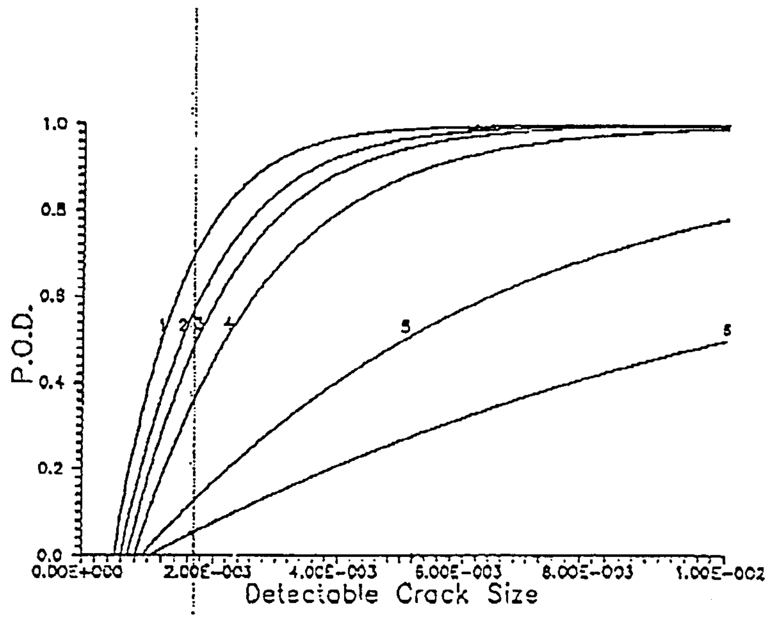


Figure 7.1 *Exponential P.O.D curves corresponding to different inspection methods P.O.D.(a) = 1-exp(-λ_i (a-a_{oi})), i=1,6.*

7.1.2 Modelling of Repairs

In the assessment situation decisions must be taken regarding both immediate and future repairs. These decisions may be either condition dependent or condition independent.

It is important that the probabilistic model of the structure after repair reflects the uncertainty associated with the particular repair method selected. Furthermore it is important to include the dependence or maybe rather independence between the behaviour of the structure before and after the repair.

In case of damage dependent repair actions the events of repair could for example be modelled by

Repair event 1 $$M_{r1} = A_1 + \epsilon_{insp} - A(t_{insp}) > 0 \tag{7.1}$$

Repair event 2 $$M_{r2} = A_2 + \epsilon_{insp} - A(t_{insp}) \leq 0 \tag{7.2}$$

If the damage size is larger than A_1 a repair of type 1 will take place and otherwise the damage will be repaired using repair type 2.

7.1.3 Modelling of Failure

Structural failure may result from extreme events such as excessive overload, long term excitation and environmental conditions such as accumulated fatigue and corrosion or combinations of these such as e.g. crack instability.

The failure event is represented by the failure margin $M_f(t)$ which for the simple situation of one structural component subject to extreme loading may be represented by

$$M_f(t) = R(t) - X(t) \leq 0 \tag{7.3}$$

where R(t) is the strength of the component and X(t) is the excitation on the component.

It is important that the modelling of the strength of a component whatever the failure mode reflects the characteristic of the previously performed repairs on the component.

If for example a repair of type 1 has taken place the failure margin is denoted

$$M_{f1}(t) = R_1(t) - X(t) \leq 0 \tag{7.4}$$

and correspondingly if a weld repair of type 2 has taken place the failure margin is denoted

$$M_{f2}(t) = R_2(t) - X(t) \leq 0 \tag{7.5}$$

Semilarly failure margins may be given corresponding to accumulated damage (e.g. fatigue) by

$$M_f(t) = A_{crit} - A(t) \leq 0 \tag{7.6}$$

where A_{crit} is a critical damage size and A(t) is the damage size at time t.

Mixed failure margins such as e.g. the R6 failure criteria may be defined as e.g.

$$M_f(t) = R(t, A_{crit}, A(t)) - X(t) \leq 0 \tag{7.7}$$

7.1.4 Calculation of Costs and Sensitivities

As explained in section 6 the optimal assessment scheme and corresponding inspection and maintenance strategy can be identified by estimating and comparing the expected total costs corresponding to the possible options of repair, inspection, testing and measuring. According to equation (6.2) this requires that the expected failure costs, the expected repair costs and the costs associated with inspection, testing and measurements are calculated. As this assessment must be considered as being a long term investment it is necessary to take the real rate of interest into account when these cost are calculated.

Using as an example the previously discussed condition based repair criteria the expected cost of failure $E[C_f\}$ capitalized to the date of construction of is calculated as

$$E[C_f] = P_f(t_{insp})\ C_f/(1+r)^{t_{insp}} + P_i(t_s - t_{insp})\ C_f/(1-r)^{t_s} \tag{7.8}$$

where r is the real rate of interest, t_s-t_{insp} refers to the period between the inspection and the end of the service life. The expected cost of repair $E[C_r]$ is calculated as

$$E[C_r] = (P_1(t_{insp})\ C_1 + P_2(t_{insp})\ c_2)/(1+r)^{t_{insp}} \tag{7.9}$$

where $P_1(t_{insp})$ is the probability that repair type 1 will take place after the inspection (test or measurement) and $P_2(t_{insp})$ is the probability that repair type 2 will take place after the inspection. C_1 and C_2 are the associated cost, respectively. The cost of inspection (test or measurement) C_i are calculated as

$$C_i = P(M_f(t_{insp}) > 0)\ C_i/(1+r)^{t_{insp}} \tag{7.10}$$

The probabilities of failure (assuming accumulated damage failure modes) are calculated as

$$\begin{aligned}
&P_f(t_s - t_{insp}) = \\
&P(M_f(t_{insp}) > 0 \cap M_{obs}(t_{insp}) > 0 \cap M_f(t_s - t_{insp}) \le 0 | M_{obs}(t_{insp}*)) \\
&+ P(M_f(t_{insp}) > 0 \cap M_{obs}(t_{insp}) \le 0 \cap M_{r1}(t_{insp}) > 0 \\
&\qquad \cap M_{f1}(t_s - t_{insp}) \le 0 | M_{obs}(t_{insp}) \le 0 | M_{obs}(t_{insp}*)) \\
&+ P(M_f(t_{insp}) > 0 \cap M_{obs}(t_{insp}) \le 0 \cap M_{r2}(t_{insp}) \le 0 \\
&\qquad \cap M_{f2}(t_s - t_{insp}) \le 0 | M_{obs}(t_{insp}*))
\end{aligned} \tag{7.11}$$

where $M_{obs}(t_{insp}^*)$ refers to an inspection (test or measurement) event at the last performed inspection (test or measurement).

Similarly the probabilities of repair are calculated as

$$P_{r1}(t_{insp}) = P(M_f(t_{insp}) > 0 \cap M_{obs}(t_{insp}) \leq 0 \cap M_{r1}(t_{insp}) > 0) \tag{7.12}$$

$$P_{r2}(t_{insp}) = P(M_f(t_{insp}) > 0 \cap M_{obs}(t_{insp}) \leq 0 \cap M_{r2}(t_{insp}) \leq 0 | M_{obs}(t_{insp}*)) \tag{7.13}$$

It may in some cases be difficult to assess the cost entering equation (7.8) - (7.10) with certainty and therefore it is interesting to see how important the individual costs are for the total expected cost. This can be seen by examining the first order partial derivatives of the total expected cost with respect to the cost of failure C_f, the cost of the various repair types C_{r1} and C_{r2} and finally the cost of inspections, tests or measurements C_I. From equation (7.8) - (7.10) it is seen that these first order partial derivatives are readily available as

$$\frac{\partial E[C_t]}{\partial C_f} = P_f(t_{insp}) / (l + r)^{t_{insp}} + P_f(t_s - t_{insp}) / (l + r)^{t_s} \tag{7.14}$$

$$\frac{\partial E[C_t]}{\partial C_{r1}} = P_{r1}(t_{insp}) / (l + r)^{t_{insp}} \tag{7.15}$$

$$\frac{\partial E[C_t]}{\partial C_{r2}} = P_{r2}(t_{insp}) / (l + r)^{t_{insp}} \tag{7.16}$$

$$\frac{\partial E[C_t]}{\partial C_I} = P(M_f(t_{insp}) > 0) / (l + r)^{t_{insp}} \tag{7.17}$$

The estimation of the probabilities in equations (7.14) - (7.17) can be undertaken as described in equation (7.11) - (7.13) and Annex I.

The expected cost and the corresponding derivatives are shown in figures 7.2 - 7.3 for a particular assessment scheme taken from the offshore industry. As the costs associated with the assessment scheme depend on the corresponding inspection and maintenance plan the expected costs are shown as function of the time to the next inspection t_{insp} varied in the interval $[t_{insp}^* ; t_s]$.

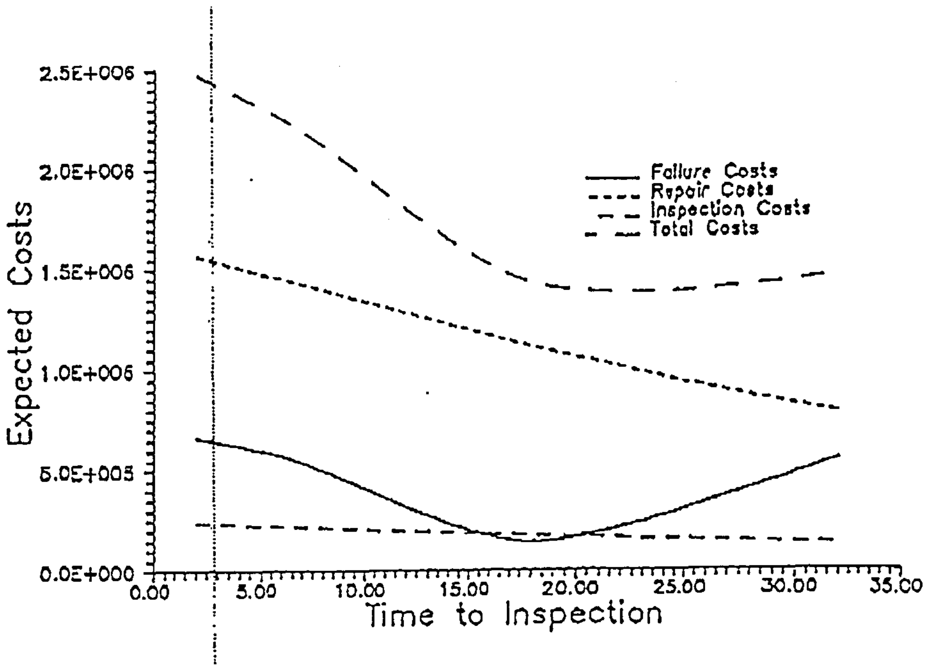


Figure 7.2 *Expected cost corresponding to a specific assessment scheme with corresponding inspection and repair strategy as a function of the time to the next inspection.*

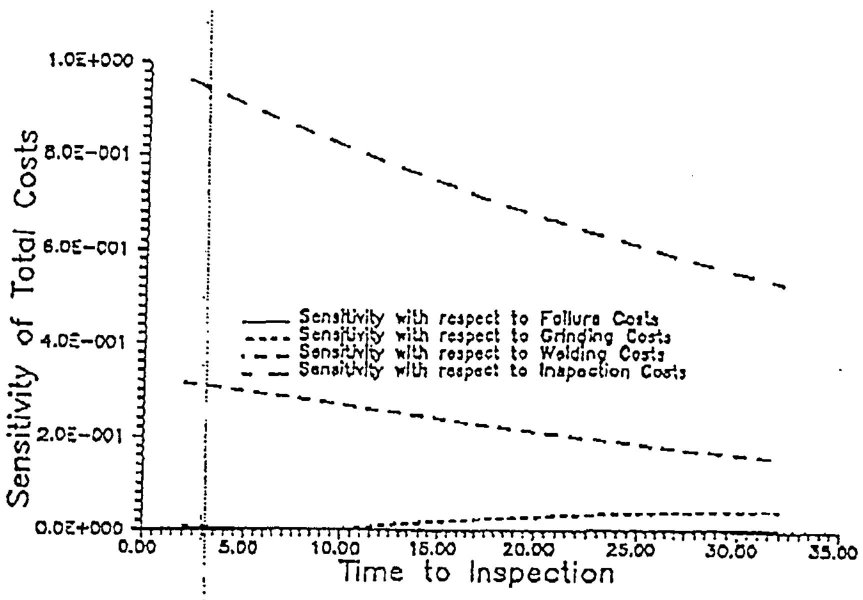


Figure 7.3 *First order partial derivatives of the total expected cost as function of the time to the next inspection.*

7.1.5 Estimation of Updated Reliabilities

In order to assure that the reliability of the considered structure is acceptable and meets the requirements (see e.g. equation (3.3)) the reliability index $\beta(t)$ as function of time t for the structure must be calculated.

If the considered structure has been inspected, tested or measured previously at the time t_{insp}^* the corresponding observations $M_{obs}(t_{insp}^*)$ are taken into account when $\beta(t)$ is estimated as

$$\beta(t) = \Phi^{-1}(P_f(t)) = \Phi^{-1}(P(M_f(t) \leq 0 \mid M_{obs}(t_{insp}^*))) \tag{7.18}$$

where t is varied in the interval $[t_{insp}^* ; t_s]$.

A plot corresponding to equation (7.19) is shown in Figure 7.4.

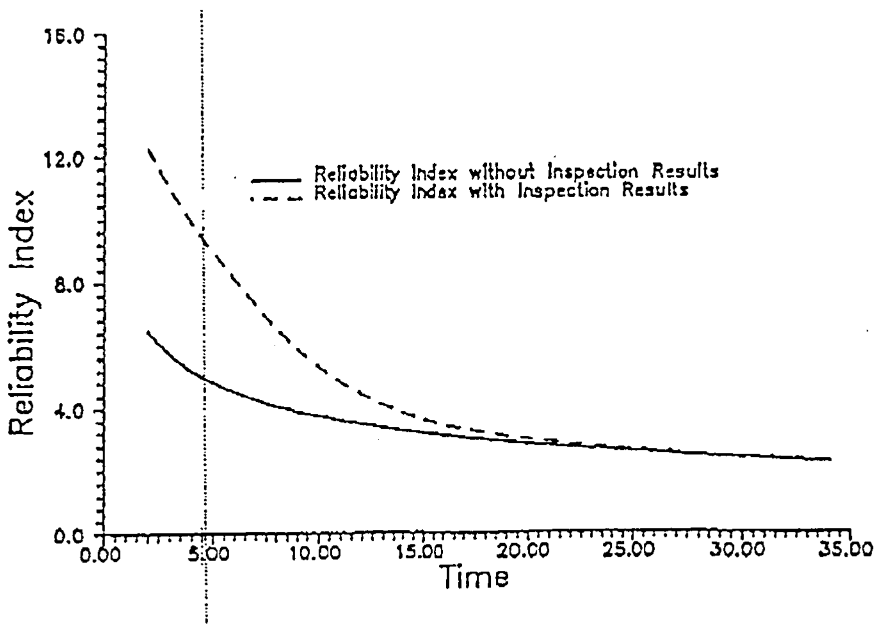


Figure 7.4 Plots of the reliability index with and without updating as function of the lifetime of the structure.

Also the predicted failure probability may be calculated corresponding to a specific assessment scheme and corresponding inspection and maintenance plan given as

$$P_f = P_f(t_{insp}) + P_f(t_s - t_{insp}) \quad (7.19)$$

where $P_f(t_{insp})$ and $P_f(t_s-t_{insp})$ are calculated with t_{insp} varied in the interval $[t_{insp}^* ; t_s]$. An example of such a calculation is shown in Figure 7.5.

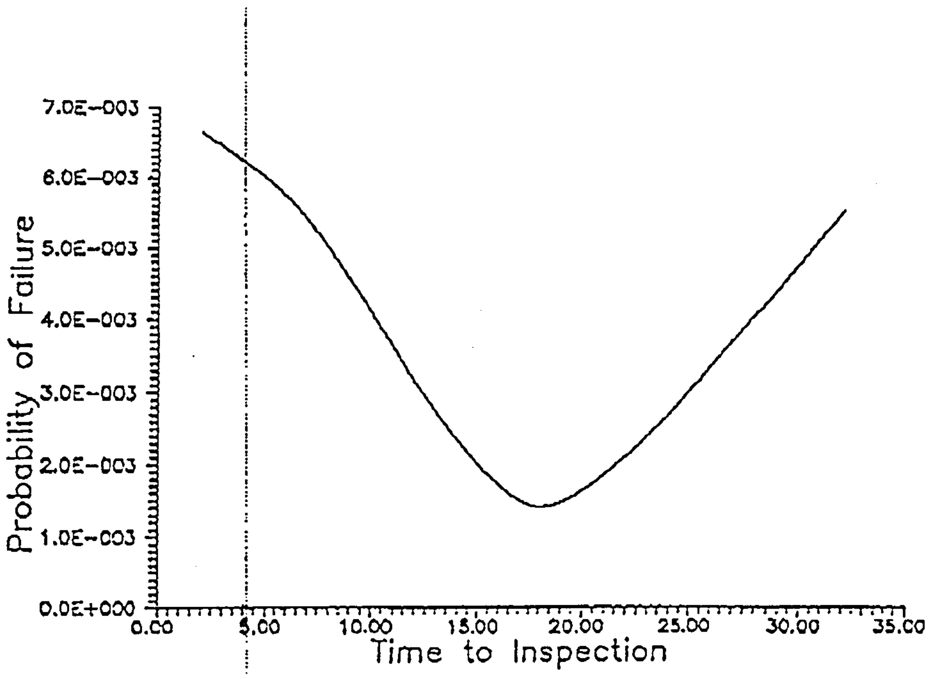


Figure 7.5 *Plot of the predicted failure probability corresponding to a particular assessment scheme and corresponding inspection and maintenance plan as function of the time to the first inspection.*

ANNEX C: TARGET RELIABILITY LEVEL FOR EXISTING STRUCTURES

By T. Arnjberg-Nielsen and D. Diamantidis

1. Introduction

In terms of a reliability based approach the structural risk acceptance criteria correspond to a required minimum reliability herein defined as *target reliability*. The requirements to the safety of the structure are consequently expressed in terms of *the accepted minimum reliability index* or *the accepted maximum failure probability*.

For the practical application of the reliability theory it is therefore important to define target safety levels which should be fulfilled by a specific design or redesign of a structure. In relation to fulfilling such requirement the existing structures will differ from the structures at the design stage. At the design stage the expenses involved in increasing the structural reliability will in many cases be relatively marginal as the structures relatively easily can be altered. The same increase in reliability for an existing structure will, however, in general be relatively expensive or even not economically feasible.

As a consequence of this difference, the treatment of reliability requirements for existing structures may in some situations call for other measures than the treatment at the design stage. In addition this raises the question about whether or not to accept lower reliability level in relation to existing structures than for similar structures at the design stage.

This Annex C deals with two approaches to the target reliability level. The first is a simple approach in which a direct requirement to the target reliability level is specified (see Chapter 2). In a more detailed approach the target reliability level can be found by means of a decision analysis as described in Chapter 3.

2. Simplified approach to the target reliability level

2.1 General Aspects

In a simplified approach the target reliability level is taken equal to the reliability level at the design stage. Although this in general does not lead to the cost optimal decision, it can in many cases serve as the basis for obtaining an acceptable decision in terms of structural safety.

In a practical approach the required reliability of the structure at the design stage is controlled by:

i) a set of assumptions about *quality assurance* and *quality management* measures; these measures are for example related to design and construction supervision and are intended to avoid *gross errors*.

ii) formal failure probability requirements, *conditional upon these assumptions*, defined by specified target values for the various classes of structures and structural members.

The formal target reliability levels to be used have by various national and international associations been proposed as either target reliability levels for the whole structure or as target reliability levels for the structural members.

The approach taken in the following is that the reliability is to be evaluated for the structure as a whole under the influence of the parameters described in Section 2.2, e.g. failure consequence, type of failure and type of limit state. The requirement to safety is under normal circumstances insured by requiring the individual structural elements to have sufficient safety, i.e. reliability level larger than or equal to the target reliability level. Further, the requirements to robustness, refer to Annex A, shall be met in order to insure the safety of the structure as a whole.

Recommended target values for structural elements are specified in Section 2.4.

2.2 Influencing Parameters

The main parameters affecting the choice of the target reliability are described next.

Degree of failure consequences

Whole structures as well as structural components maybe classified according to the consequences of failure. Generally, a classification according to the following is sufficient:

Class 1 Minor Consequences: Risk to life, given a failure, is low and also economic and social consequences are small or negligible (e.g. agricultural structures, silos, masts).

Class 2 Moderate Consequences: Risk to life, given a failure, is medium or economic and social consequences are considerable (e.g. office buildings, industrial buildings, apartment buildings).

Class 3 Large Consequences: Risk to life, given a failure, is high, or economic or social consequences are significant (e.g. main bridges, theaters, hospitals, high rise buildings).

Class 4 Extreme Consequences: Risk to life, given a failure, is extreme as well as social and economic impact (e.g. nuclear power plants, important dam structures).

At a similar way the relative costs of safety measures can be subdivided into classes, e.g. low, moderate and high.

System behaviour

Apart from the classification of structures a classification of structural elements is needed. The failure consequences of elements in one structure may differ quite substantially. This means that one should take into account *the system behaviour* as characterized by the type of systems e.g.: *redundant systems* and *non-redundant systems* as identified in Annex A.

Types of failure

The following types of failure can be classified:

a) ductile failure with reserve strength capacity resulting from strain hardening
b) ductile failure with no reserve capacity
c) brittle failure.

Consequently a structural element which would be likely to collapse suddenly without warning should be designed for a higher level of reliability than one for which a collapse is preceded by some kind of warning which enables measures to be taken to avoid severe consequences.

Limit State Type

Ultimate and *serviceability limit states* are considered. For specific cases a limit state between those two can be distinguished defined here as „reversable limit state". In case of earthquake or accidental loading affecting a plant such limit state is for example associated to the safe shut-down of the plant.

2.3 Time of Reference

The formal target reliability levels to be used have by various national and international associations been proposed as either lifetime target safety levels or as target safety levels for a given reference period, typical of one year.

The relationship between the reliability index associated to two reference periods t and T, e.g. lifetimes and one year, can be approximated as:

$$\beta(t) = - \Phi^{-1} (\Phi (- \beta(T)) (t/T)) \qquad (1)$$

where:

$\beta(t)$: reliability index associated to reference time t
$\beta(T)$: reliability index associated to reference time T
Φ : standard normal distribution function

In the following Section 2.4 recommended target reliability values are specified corresponding to a one year reference period. The use of a one year reference period is a formal choice, and it reflects the point of view that the target reliability in general should be independent of the lifetime requirement. This approach in relation to reference time is consistent with the conclusions in Rackwitz (1998) that the optimal solution for building facilities with a systematic rebuilding policy is based on failure rates and not on time dependent failure probabilities.

Some exceptions to the use of yearly failure rates as target values may exist. However, these should under normal circumstances be limited to situations with very low risk to life given a failure.

2.4 Recommended Target Values

Tentative target values for ultimate limit state are presented in Table 1. The values correspond to individual structural elements and a one year reference period. These values shall be considered in reliability analyses in association with the stochastic models for the influencing variables as described in the probabilistic model code. In case of structures with extreme failure consequences the target values shall be defined based on risk-benefit studies.

Table 1: Tentative target reliability indices, one year reference period - ultimate limit states

Relative Cost of Safety Measure	Minor consequences of failure	Moderate consequences of failure	Large consequences of failure
Large	**3.2**	**3.7**	**4.2**
Moderate	**3.7**	**4.2**	**4.7**
Low	**4.2**	**4.7**	**5.2**

When setting target values for serviceability limit states (SLS) it is important to distinguish between irreversible and reversible serviceability limit states. The methods based on decision analysis as described in chapter 3 are well suitable for serviceability limit states, as these normally do not involve risk of life.

For irreversible serviceability limit states tentative target values are, with reference to Rackwitz 1998, given in Table 2.

Table 2: Tentative target reliability indices, one year reference period - irreversible serviceability limit states

Relative Cost of Safety Measure	Target Index (irreversible SLS)
Large	**1.3**
Moderate	**1.7**
Low	**2.3**

For reversible serviceability limit states no general values are given.

3. Approach to the target reliability level by decision analysis

3.1 General aspects

In decision analysis for existing structures, utility has to be associated with the decisions which influence the future performance of the structure. It is therefore necessary that relationships are defined such that the performance of the structure can be associated with utility. In relation to existing structures utility is in general related to economy, as economic considerations play an important role in decisions on repair, maintenance and inspection. The costs that should be taken into account are direct costs of measures or actions such as repair, maintenance and inspection, costs of exploitation following from failure or bad behavior of the structure. Decisions should be in general taken in such a way that the total expected cost reaches a minimum.

The type of decision analysis is mainly possible when economic loss dominate over the loss of life and limb. When the expected loss of life and limb is dominating, the decisions become more controversional. In order to overcome this problem the utility value specifications should, at least in an initial phase, be calibrated to such values that the decision pointed out by the decision analysis by and large leads to the reliability levels approved by the society and applied in the existing codes (Ditlevsen, 1997).

3.2 Optimal Decision

The optimal decision is that which minimizes the total expected cost of a given structure. In general this decision is obtained by solving an optimization problem, as for example described in Rosenblueth and Mendoza (1971) or as discussed with repect to existing structures in Vrouwenvelder, 1991. In a simple form the optimization problem can be written as:

$$\text{Min } C_T(p) = C_D(p) + C_M(p) + C_Fp(p)p_F \qquad (2)$$

C_T: total cost

C_D : cost of direct measure (strengthening or upgrading)

C_M : cost of maintenance

C_F : cost of failure including cost of repair or replacement, cost due to loss of contents, cost due to business interruption, cost of injury and cost of fatality

p_F: probability of failure

p : a parameter vector associated to redesign/strengthening of the structure

Without loss of generality all cost quantities will be measured in monetary units. The aforementioned formulation is a so called cost based, unconstrained optimization problem.

In addition to the cost optimization requirements maybe formulated with respect to:

a) minimum safety i.e. $p_F < p_P$

b) maximum budget i.e. $C_D < C_P$

This leads to the so-called constrained optimization problem results, where p_P and C_D are prescribed values. One should avoid to have both constraints active, since this may lead to an unsolvable problem.

The requirement a) is especially used in cases where human safety or large environmental consequences are involved. Budget constraints i.e. requirement b) is often set if the damage is of materialistic nature.

The optimization should be made for a defined period of time. Such period can correspond to the residual lifetime of the structure or to the time until the next inspection. The decision about the redesign parameter p has to be made at the initial time t = 0. This requires to capitalize all cost. A continuous capitalization function is proposed:

$$d(t) = \exp(-r\,t)$$

with r the interest rate. Usually, a yeary interest rate is defined and d(t) can be approximated as:

$$d(t) = (1 + r)^{-t}$$

with r here the yearly interest rate. In many cases it might be even possible to neglect the interest effect. An example on the influence of cost capitalisation on optimal inspection and maintence planning is shown in Annex D.

References

Ditlevsen, O., 1997, Structural Reliability Codes for Probabilistic Design- A debate Paper based on Elementary Reliability and Decision Analysis Concepts.

Rosenblueth, E. and E. Mendoza, 1971, Reliability Optimization in Isostatic Structures, ASCE Engineering Mechanics Division, EM6, Vol. 97, pp. 1625-1642.

Vrouwenvelder, A., 1991, Assessment of existing structures, Comet advanced course on structural reliability and load modelling, Technical University of Denmark, Lyngby.

Rackwitz, R., 1998, Optimization - The Basis of Code-making and Reliability Verification.

JCSS document Reliability of Existing Structures

Annex D: Examples (second draft)

Example D1: Timber beam, deflection measurement

Example D2: Inspection for fatigue cracks

Example D3: Concrete strength evaluation

by A. Vrouwenvelder TNO/The Netherlands

Example D1: Timber beam, deflection measurement

Statement of the problem

Consider a timber beam as presented in figure D1.1. First the reliability of this beam without any inspection will be estimated. Then the updating of this reliability will be demonstrated if the beam deflection is measured.

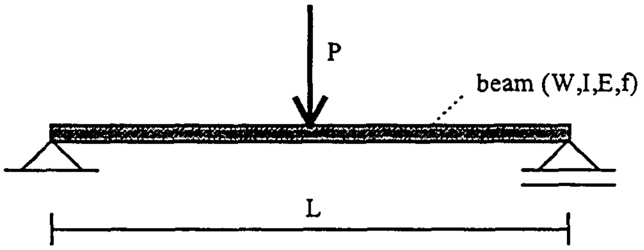


Figure D1.1: Simply supported timber beam with concentrated load

The limit state function for failure is defined as:

$$M = W f - 0.25 P L$$

For the meaning of the variables and their respective probability models reference is made to Table D1.1. All random variables are assumed to be normal for simplicity. The yield stress f and the modulus of Elasticity E are correlated.

Note: the statistical properties of f and E, including their correlation, depend heavily on the classification procedure of the timber. The present numbers, however, have no pretention to correspond to some specific procedure.

Table D1.1: data

Variable	**note**	**Designation**	**μ**	**V**
L		span	4 m	-
W		plastic section modulus	0.01 m^3	-
I		moment of inertia	0.0002 m^4	-
P	(1)	variable load	100 kN	0.20
Pt		test load	50 kN	-
f	(2)	yield stress	20 MPa	0.15
E	(2)	modulus of elasticity	30 GPa	0.15

Notes (1) yearly maximum,
(2) $\rho(E,f) = 0.5$

Calculation of the failure probability

Given the data we my calculate the failure probability according to (see Annex A):

$$\mu(M) = W\,\mu(f) - 0.25\,\mu(P)\,L = 0.01 * 20000 - 0.25 * 100 * 4 = 100 \text{ kN}$$

$$\sigma(M) = \sqrt{(W^2\,\sigma^2(f) + 0.25^2\,\sigma^2(P)\,L^2)} = \sqrt{(0.01^2 * 3000^2 + 0.25^2 * 20^2 * 4^2)} = 36 \text{ kN}$$

$$\beta = 100/36 = 2.8$$

$$P_f = 0.0026$$

Measurement

Assume next that we do a measurement of the deflection u under a deterministic load of P_t = 50 kN. We would expect to have a deflection equal to:

$$u = P_t L^3/48\mu(E)I = 0.0011 \text{ m} = 11 \text{ mm}$$

Suppose the test gives u = 9 mm. In that case we may conclude that the beam is better than expected. Given the positive correlation between stiffness and strength this should lead to an increase of the beam reliability. We will make this calculation by the two possible alternative procedures.

Procedure (1) Direct updating

For the direct calculation we introduce a so called “artificial limit state function" for the measurement event, which is given by:

$$g = 48\,EI\,u - P_t\,L^3$$

The corresponding β_g can be calculated as follows:

$$\mu(g) = 48\,\mu(E)I - P_t\,L^3 = 48 * 30\,000\,000 * 2.10^{-4} * 0.009 - 50\,4^3 = -\,608 \text{ kNm}^3$$

$$\sigma(g) = 48\,\sigma(E)I = 518 \text{ kNm}^3$$

$$\beta_g = -1.2$$

The negative β_g corresponds to the fact that beam behaves better than expected.

We now use the standard formulas (5.7) and (5.8) from Annex B for direct updating:

$$\mu(M|g=0) = \mu(M) + \rho\sigma(M)\frac{0-\mu(g)}{\sigma(g)} \qquad (5.7) \text{ (Annex B)}$$

$$\sigma(M|g=0) = \sigma(M)\sqrt{1-\rho^2} \qquad (5.8)\text{(Annex B)}$$

The basic data, which follow form the previous calculations, are:

$$\mu(M) = 100 \text{ kN},\ \sigma(M) = 36 \text{ kN},\ \mu(g) = -608 \text{ kNm}^3,\ \sigma(g) = 518 \text{ kNm}^3$$

In order to find the coefficient of correlation we first calculate the covariance using a first order approximation:

$$\text{cov}(M,g) = \Sigma\Sigma \frac{\partial M}{\partial X_i}\frac{\partial g}{\partial X_j}\sigma(X_i)\sigma(X_j)\rho(X_i X_j) =$$

In this summation only $\partial M/\partial f$ and $\partial g/\partial E$ give a contribution:

$$\text{cov}(M,g) = \{W\}\{48Iu\}\sigma(f)\sigma(E)\rho(E,f) = 776\ (\text{kN})^2\ \text{m}^3$$

And so:

$$\rho(M_i g) = \frac{\text{cov}(M_i g)}{\sigma(M)\sigma(g)} = \frac{7776}{36 * 518} = 0.42$$

Inserting the numbers in the basic equations (5.7) and (5.8) of Annex B leads to:

$$\mu(M|g) = 117$$

$$\sigma(M|g) = 33$$

and this leads to an updated reliability index $\beta(M|g)$ equal to:

$$\beta(M|g) = 117/33 = 3.5$$

This means that the good inspection result has increased β from 2.8 to 3.5. If, for instance, we would have started from u = 14 mm we would have found the updated β to be 2.4. In that case the beam has low E and probably a corresponding low f, leading to a reduction of the reliability.

Procedure (2) Updating of individual random variables

As an alternative we could also update the random variables following the formulas (5.7) and (5.8) from Annex B with f in stead of M and E in stead of g. From u=0.009 m we may derive that E = 37.000.000 kN/m^2 deterministically. We now may update the mean and standard deviation of f according to:

$$\mu(f\,|E = 37000000) = \mu(f) + \rho\sigma(f)\frac{37000000 - \mu(E)}{\sigma(E)} = 21.8 \text{ MPa}$$

$$\sigma(f\,|E{=}37000) = \sigma(f)\sqrt{1-\rho^2} \;=\; 3\sqrt{1-0.5^2} = 2.6 \text{ MPa}$$

If we redo the limit state reliability analysis using this new model for f we find:

$$\mu(M|E) = W\,\mu(f\,|E) - 0.25\,\mu(P)\,L = 0.01 * 21800 - 0.25 * 100 * 4 = 118 \text{ kN}$$

$$\sigma(M|E) = \sqrt{(W^2\,\sigma^2(f\,|E)+0.25\,\sigma^2(P)\,L^2)} = \sqrt{(\,0.01^2*2600^2 + 0.25^2*20^2*4^2)} = 33 \text{ kN}$$

$$\beta = 118/33 = 3.5$$

In this case the procedure is relatively easy, because only one variable is involved. In general, however, the first procedure is to be preferred.

Semi-probabilistic verification

We could even have a semi-probabilistic updating and telling that the characteristic value for the strength has increased from:

$$f_k = \mu(f) - 1.64\,\sigma(f) \;=\; 20\;\; 1.64 * 3 = 15 \text{ MPa}$$

to $$f_{k|E} = \mu(f\,|E) - 1.64\,\sigma(f\,|E) \;=\; 21.8 - 1.64 * 2.6 = \;17.5 \text{ MPa}$$

and perform an updated level I analysis.

Example D2: Inspection for fatigue cracks

Statement of the problem

Consider an offshore structure where various nodes are inspected for fatigue cracks. Failure in those cases will happen if the inspection results are considered as satisfactory but the failure event (nevertheless) occurs, assuming that some adequate action is taken if the inspection is not satisfactory.

Let the fatigue crack for some selected node grow as indicated in figure D2.1. Fatigue failure will occur as soon as the crack a (t) reaches a random critical length a_c, so the failure probability for a period t can be written as:

$$P_F(t) = P\{M_f < 0\} = P\{a_c - a(t) < 0\}$$

Note that a_c is considered as time-independent; if a_c is considered as time dependent (as it is in reality) this equation becomes more complex.

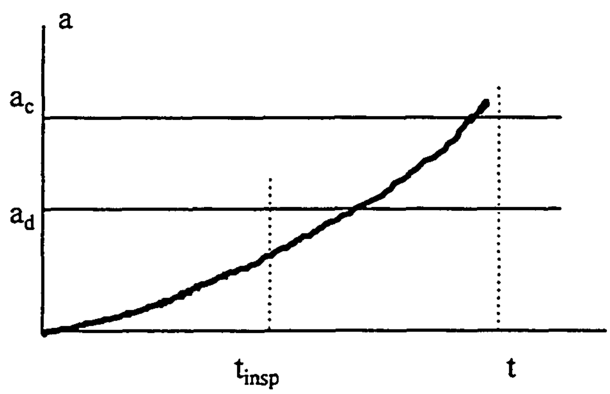


Figure D2.1: Fatigue failure before time t occurs if at inspection the crack length is smaller than a_d and at time t the crack length is larger than a_c

Let the reliability be considered as inadequate. For this reason an inspection is planned at some point t_{insp} during the life time. Let the decision rule be that the structure will be repaired if the measured crack a_m is larger than random detection limit a_d . Of course $a_m(t_{insp})$ may be different form the actual $a(t_{insp})$ due to measurement errors or even lack of detection.

Updated reliability analysis

The probability of failure, given a positive inspection can be written as:

$$P_F(t) = P\ (a(t) > a_c \mid a_m\ (t_{insp}) < a_d)$$

The first event represents “failure” and the second one "fit at inspection". In terms of the standard limit state functions this may be rewritten as:

$P_F(t) = P\{M_f < 0 \mid M_i > 0\}$

$M_f = a_c - a(t) < 0$

$M_i = a_d - a_m\,(t_{insp}) < 0$

We now want to do a hand calculation, using the equations (5.7) and (5.8) from Annex B. To do so we need to approximate $M_i > 0$ by $M_i = 0$. This, however, is not very accurate. An approximation of this type is accurate only if the inequality corresponds to a small probability. This situation may be easily be reached using Bayes Theorem:

$$P_F(t) = P\{M_f < 0 \mid M_i > 0\} = P\{M_i > 0 \mid M_f < 0\}\ P\{M_f < 0\}/P\{M_i > 0\}$$

We now may approximate the condition ($M_f < 0$) in the first factor by ($M_f = 0$). In that case we have for the marginal normal distribution of M_i (see Annex B, formulas (5.7) and (5.8)):

$$\mu(M_i|M_f = 0) = \mu(M_i) + \rho\sigma(M_i)\frac{0 - \mu(M_f)}{\sigma(M_f)}$$

$$\sigma(M_i|M_f = 0) = \sigma(M_i)\sqrt{1-\rho^2}$$

So for the event ($M_i < 0 \mid M_f = 0$) the reliability index is:

$$\beta(M_i|M_f = 0) = \frac{\mu(M_i|M_f = 0)}{\sigma(M_i|M_f = 0)} = \frac{\beta_i - \rho\beta_f}{\sqrt{1-\rho^2}}$$

and from there:

$$P_F(t) = P(M_i > 0|M_f = 0)\ P(M_i < 0)/P\{Z_i > 0\} = \Phi(+\frac{\beta_i - \rho\beta_f}{\sqrt{1-\rho^2}})\Phi(-\beta_f)/\Phi(\beta_i)$$

As a numerical example, take $\beta_f = 2$, $\rho = 0.8$ and $\beta_i = 1$. The value of β_i corresponds to 16% probability of finding a crack a_m larger than a_d. Then:

$$P_F(t) = \Phi\{\frac{1 - 0.8 * 2}{0.6}\}\Phi(-2)/\Phi(1) = \Phi(-1.0)\,\Phi(-2)/\Phi(+1)) = 0.16 * 0.023/0.84 = 0.004$$

So the inspection raises the reliability index from 2.0 to 2.8.

Complete event tree

It is also interesting to observe the total event tree for this case, standing at t=0 This tree is given in figure D2.2:

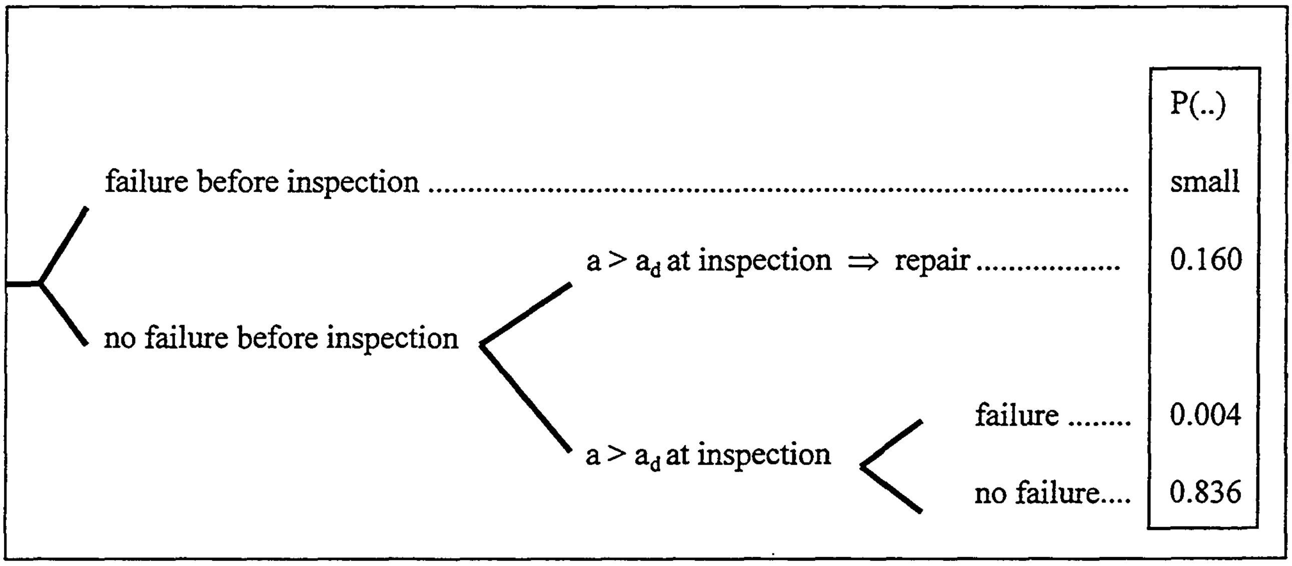


Figure D2.2: Event tree for inspection and failure events

Standing at t=0 we have first the possibility that failure occurs before the inspection is planned. Let us assume here that the rime of inspection has been chosen in such a way that this probability is small. If no failure prior or inspection occurs, this inspection may reveal a defect, which lead to a repair action. In this example the probability for this branch in the event tree is $\Phi(-1) = 0.16$. If inspection is okay, we then may have either failure or no failure in the period between inspection and the desired life time (or next inspection in a more advanced example). the probability of failure is 0.004.

If also cost values are attached to inspection and failure, the optimal time of inspection and repair level a_d can be found.

Examples D6: Concrete strength evaluation

Statement of the problem

For a set of existing buildings the concrete quality is doubted on the basis of a visual inspection. It is decided to determine the cylinder strength of a number of cores. In total 54 cores, divided over 8 buildings are tested. The means and standard deviations of the test results for each building are presented in Table D6.1.

The various buildings clearly show a great variety in strength. The problem is to find the characteristic concrete strength for every building. This strength is intended to be used in a limit state verification procedure with the aim to decide if a building should be demolished because of insufficient strength.

Table D61: CONCRETE CUBE TESTING

building	n=number of tests	mean [N/mm2]	stand dev. [N/mm2]	x_k [N/mm2]	
				no prior	prior
A	9	15.7	4.46	7.0	7.6
B	9	17.5	2.80	12.0	11.7
C	11	17.3	5.11	7.6	8.1
D	4	17.9	4.91	5.0	7.8
E	2	22.5	0.71	17.0	9.8
F	5	13.2	1.44	9.8	8.4
G	9	13.9	3.65	6.8	7.2
H	5	10.9	3.66	3.4	2.1

Analysis without prior knowledge

Using the standard statistical methods one may easily show that the differences between the various buildings are significant. This means that it is meaningless to calculate one overall characteristic value for the total population.

For that reason a characteristic value for each individual building is calculated, based on the data for the building itself. The following standard Bayesian Prediction formula was used:

$$R_d = m - t_v \, s \sqrt{(1 + \frac{1}{n})}$$

n = number of tests
m = mean test result
s = standard deviation of test
ν = n-1
t = 5 % value for the t-distribution

The resulting characteristic values are presented in Table D6.1 under the heading "no prior".

Use of other buildings as prior information

It is interesting, however, to consider also an alternative. Suppose that the test results of the buildings A to G are available and that the building H is to be tested. In that case, knowing that building H has been built in the same period, by the same contractor, using the same manufacturing procedures, there is a reason to assume that the results for building H will be in the same range as the other buildings. So one might consider the means of buildings A-G as a "prior information" for the mean of building H. A similar argument holds for the standard deviation. Given this prior information and the data for building H a posterior distribution for the mean and standard deviation can be derived using standard Bayesian updating procedures.

Let us elaborate this idea, making the following simplifying assumptions:

(1) The means and standard deviations of all buildings A to H is used to construct a prior for building H. This means that the data of building H is used for constructing the prior as well as for updating. This is not correct, but it opens the possibility to use the same prior of all buildings. So, the prior is derived on the means and standard deviations for all buildings, and this prior is used to find posteriors and predictive distributions for all individual buildings.

(2) The prior distribution of the mean and standard deviation is considered to be the classical conjugate distribution, that is, the mean is normally distributed (conditional upon the standard deviation) and the standard deviation is distributed according to a Inverse-Gamma distribution.

(3) All buildings have equal weight, regardless the number of observations; uncertainties following from the limited number of buildings or the limited number of observations for each building are neglected.

Of course, the procedure can be improved on the above points. The influence is believed, however, to be small.

For the actual analysis one has to calculate the average and standard deviation of the 8 mean values and the 8 standard deviations of Table D6.1. The result is:

$\mu(m) = 15.4$ kN/m′ $\sigma(m) = 3.98$ kN/n′
$\mu(s) = 3.34$ kN/s′ $\sigma(s) = 1.60$ kN/v′

Using the relations between the parameters of the normal-gamma2 distribution and the corresponding means and standard deviations, the following parameters may be derived:

m’ = 15.4 kN/m_2 n’ = 0.7
s’ = 3.34 kN/m_2 ν‘ = 2.2

These parameters can be interpreted as:

s' = hypothetical sample average for the standard deviation
ν' = hypothetical number of degrees of freedom for s'
m' = hypothetical sample average for the mean
n' = hypothetical number of observations for m'

Given these parameters for the prior distribution, the corresponding parameters for the posterior distribution can be obtained from:

$$n'' = n' + n$$

$$\nu'' = \nu' + \nu + \delta\{n'\}$$

$$m''n'' = n'm' + nm$$

$$[\nu'' (s'')^2 + n'' (m'')^2] = [\nu' (s')^2 + n' (m')^2] + [\nu (s)^2 + n (m)^2]$$

With: ν = n-1; δ(n') = 0 for n' = 0 and δ(n') = 1 otherwise.

So finally the characteristic value is obtained from:

$$R_d = m'' - t_{\nu''} s'' \sqrt{(1 + \frac{1}{n''})}$$

The results are printed in Table D6.1. It can be concluded that most values are in the same order of magnitude. The major difference is for building E where the small scatter lead to 17 N/mm^2 in the first case. In the second case, however, the relative very small scatter for this building is overruled by the "average scatter". The point of course is that for this building only 2 observations were available.

Influence of the prior information

It is interesting to observe the influence of the prior information on the result. As an example, let us take building D. For this building we have 4 observations with mean 17.9 N/mm^2 and standard deviation 4.9 N/mm^2. If there would be no statistical uncertainty the characteristic value would be 17.9 - 1.64*4.9 = 9.9 N/mm2. Let us now vary the "prior information form other buildings". Table D6.2 gives the results.

Table 6.2: Influence of prior

case	n	μ	ν	σ	R_k [N/mm2]
1	0	-	0	-	5.0
2	1	18	1	5	6.5
3	100	18	100	5	9.9
4	100	15	100	7	3.5
5	100	20	100	3	15.0

The first case is the result without any prior. The results are identical to those of Table D6.1.

Cases 2 and 3 represent cases where the mean and standard deviation are the same for data and prior. In case 2 the prior is weak (n=ν=1) and in case 3 the prior is strong (n=ν=100). The resulting value for the characteristic value increases and reaches already the limit value for case 3.

In the cases 4 and 5 we have also strong priors, but now the information between prior and data is conflicting. It can be seen that in both cases the prior dominates the answer. The data have almost no influence on the result.

ANNEX E: CASE STUDIES

RELIABILITY EVALUATION OF EXISTING STRUCTURES - OFFSHORE GEOTECHNICAL EXAMPLE

by F. Nadim and S. Lacasse, Norwegian Geotechnical Institute

1. Problem description

This example presents the deterministic and probabilistic analyses of an offshore pile foundation at two times in the platform lifetime.

1) In 1975, before platform installation, when limited information and limited methods of interpretation of the soil data were available

2) In 1993, after a reinterpretation of the available data using the geotechnical improvements attained in the interim additional and more advanced laboratory tests, a reanalysis of the loads, and an analysis of the installation records

The reanalysis in 1993 was prompted because the environmental loads had been revised and the operators hoped to increase the gravity loads on deck. The structure consists of a steel jacket installed in 110 m of water in the North Sea. The jacket was installed in 1976. The jacket rests on four pile groups, one at each corner. Each pile group consists of six piles (Fig. 1). The piles in the groups are 60" diameter tubulars, with wall thicknesses of 3 and 2.5".

2. Site conditions

The soil profile consists of mainly stiff to hard clay layers, with relatively thinner layers of very dense sand in between.

In 1975, two soil borings were done at the jacket location. The two borings indicated somewhat comparable soil profiles, although the horizon and the thickness of the sand layers differed. Based on the information obtained from the borings, the soil characteristics in Fig. 2 were derived from the standard "strength index" types of tests in common use at the time (torvane, pocket penetrometer, unconfined compression test, unconsolidated undrained (UU) test), and an interpretation of the results based on the judgement and experience of the geotechnical consultant at the time. The friction angle of the dense sand was based on the results of consolidated drained triaxial compression tests on recompacted specimens. The friction angle for the specimens compacted to the highest density possible was measured between 38 and 40 degrees.

In 1993, new samples were taken and more advanced strength tests were run, including consolidated undrained triaxial compression tests.

During pile installation, records were made of the blow count during driving. The pile driving records were evaluated by a consultant in 1993 and used to adjust the soil stratigraphy. The result of this "educated" adjustment and a reevaluation of the borings and laboratory test results using normalised soil characteristics, new soil samples and the running of more advanced laboratory tests (direct simple shear tests, consolidated undrained triaxial tests) led to the adjusted soil shear strengths in the stiff to firm clay shown in Fig. 3, where a fairly narrow range of soil strengths are suggested.

3. Analysis methods

The deterministic analyses were done with the API RP2A recommended practice in use at the time of the analysis. The design requirement at both times in the platform lifetime was a factor of safety of 1.50 under extreme loading and 2.0 under operation loading. The axial pile capacity is a summation of the skin friction on the pile shaft and end bearing on the pile tip.

The probabilistic analyses were done with first-order reliability method (FORM), where the deterministic axial pile capacity model was formulated in terms of random variables in each layer. In the present example, only the results of the analysis of the capacity of most loaded pile are considered.

4. Model description

Soil parameters

Table 1 gives the uncertainties associated with the soil parameters in two of the more important soil layers. The selected coefficients of variation reflect uncertainties in the laboratory test results, possible measurement errors, spatial variability and the uncertainty in degradation due to cyclic loading. Cyclic degradation is important for an overconsolidated clay subjected to a fairly high ratio of cyclic loading. The effect of cyclic loading is expected to be minor for the dense sand.

Very little data were available for the different soil parameters. The mean and coefficients of variation were obtained as follows:

Submerged unit weight, γ':

No measurements were available. The mean value and coefficient of variation were based on experience acquired for similar soils where many measurements have been taken. For stiff clays, the mean submerged unit weight is 8.5 to 9.0 kN/m^3 (as stiffness increases); for very dense sand, the mean submerged unit weight is normally 10 kN/m^3. A COV of 5% is a common value for scatter in submerged unit weight.

Depth, z:

The layer thickness can vary. Since only two borings are available, the values used in the analysis are uncertain. The mean layer thickness is based on the measured values from the site investigations. The COV of 10% is based on engineering judgement.

The position and thickness of Layer 7 were quite uncertain in 1975. For this reason the COV was increased for this layer from 10 to 20%.

Undrained shear strength in stiff clay, s_u

In 1975, the undrained shear strength was based on punctual measurements from index strength tests, known to give a relatively poor estimate of the undrained shear strength. The data points are shown in Fig. 2, which explain the high COV.

In 1993, the undrained shear strength profile was based on

(1) results of consolidated-undrained triaxial compression tests at effective stresses relevant for the in situ values (Fig. 3)

(2) a recalculation of the soil shear strength based on the normalised strength ratio for similar clays within the same geographical area and with similar geological history.

This led to two soil strength profiles (Fig. 3) and COV's of 10 or 15%.

Friction angles, ϕ' *and* δ *and coefficient of earth pressure, K, in very dense sand*

Very little information was available for the very dense sand layers. A friction angle, ϕ', of 40° (and soil friction angle δ of 35°) is typical for a very dense sand. In 1975 there was little known about this angle and the COV was set to 15%. In 1993, considerable research contributed to reducing this COV to about 5%. Lacasse and Goulois (1989) collated the opinion of 40 international experts who suggested that the uncertainty about the mean is quite small.

For the coefficient of earth pressure, K, values are again undocumented, but based on engineering judgement, experience and the results of the Lacasse and Goulois (1989) study.

Pile capacity parameter in clay, α

The prediction of the axial capacity of a pile in clay is done with the friction parameter, α, times the undrained shear strength. The mean value is based on the API RP2 A guideline. The COV is based on engineering judgement and the experience gathered for piles in stiff clay.

Pile capacity parameter in sand, f_{lim}

The mean value of f_{lim} is specified by the API RP2 A design guidelines. The decrease in the COV of f_{lim} from 25 to 15% between 1975 and 1993 reflects the understanding acquired over the year on pile friction in sand and the results of the expert opinion pooling summarised in Lacasse and Goulois (1989).

Table 1 Examples of uncertainty in soil parameters in Layers 5, 7 and 8 1975 and 1993 analyses (Figs 2 and 3)

Layer	Variable	Coefficient of Variation 1975-analyses	1993-analyses	PDF
5	γ'	5 %	5%	N
	z	10 %	10%	N
	s_u	25 %	15%	LN
	α	10 %	10%	LN
7	γ'	5 %	5%	N
	z	20 %	10%	N
	K	15 %	10%	N
	δ	15 %	5%	N
	f_{lim}	25 %	15%	N
8	γ'	5 %	5%	N
	z	10 %	10%	N
	s_u	25 %	10%	LN
	α	10 %	10%	LN
	N_c	15 %	15%	N

Notation:

γ' = submerged unit weight
s_u = undrained shear strength
N_c = bearing capacity factor
δ = soil-pile friction angle = ϕ' - 5°
ϕ' = friction angle (of sand)

z = depth to bottom of layer
α = skin friction factor
K = coefficient of lateral earth pressure
f_{lim} = limiting skin friction (sand)

PDF = probability distribution function (N = normal, LN = lognormal)

Loads

The characteristic load used for deterministic foundation design of fixed offshore structure on the Norwegian Continental Shelf is defined as the load with an annual occurrence probability of 1% (i.e. the maximum load associated with the 100-year storm).

The extreme axial load on the most loaded pile is the sum of a permanent component resisting the submerged platform weight and a transient (cyclic) component resisting the storm, current and wind-induced forces. The key parameters entering the load calculations are the environmental characteristics, the platform weight, and the model used for estimating the response of the platform to the environmental loads.

The main environmental parameters for the foundation loads of the platform under consideration were the significant wave height (H_s) and the spectral peak period corresponding to the significant wave height ($T_p|H_s$). Environmental parameters of secondary importance were the mean wind speed, wind gust factor, current speed, and water level occurring simultaneously with the largest storm peak. Data on storm characteristics were gathered during almost two decades of platform operation. A 100-year value for the significant wave height of 13.5 - 14.5m was expected for the area of the North Sea around the site, so a storm threshold of $H_s = 7$m was used in the calculations. A total of 130 events exceeding this threshold were observed during the time period summer 1975 - summer 1992. Seven of these events (all during the seventies) occurred during periods with no measurement and hindcast data were adopted for these periods. A truncated Weibull distribution was used for the significant wave height and a lognormal distribution (conditional on H_s) was adopted for the spectral peak period. To quantify the uncertainty in the significant wave height for the 100-year event, the fitted Weibull model parameters were treated as random variables. Following the procedure described by Haver and Gudmestad (1992), modelling uncertainty and statistical uncertainty were treated separately.

The procedure used for estimating the foundation loads in the design phase was deterministic. To make a comparison, similar types of distributions were assumed for the environmental parameters in 1975, but the site specific data were not used in fitting the distribution parameters. Rather, the distributions were chosen to be representative of the general area of northern North Sea, which meant that there was a larger dispersion in the parameters.

A lognormal distribution with coefficient of variation of 3% was assumed for the submerged platform weight both in 1975 and in 1993. As mentioned earlier, the main reason for the reanalyses was the planned increase of the deck weight, which would lead to 14% increase in the (mean) submerged platform weight.

To obtain the unconditional distribution of the 100-year axial load on the most loaded pile (Leg A5), a number of calculations were performed where the probability of exceeding a given load level was estimated using the FORM/SORM approach. The load level was varied and the results were plotted on the Gumbel scale as shown in Fig. 4. As seen, a Gumbel distribution with mean of 20MN and coefficient of variation of 10% provides a good fit to the extreme axial pile load based on the 1993 information, whereas with the information available in 1975, the same load has a mean of 19MN and a coefficient of variation of 15%. In both situations, the significant wave height was the dominant random variable contributing about 80% of the uncertainty in axial pile load. Modelling uncertainty was the next most important parameter. The contribution of other random variables such as the spectral peak period, submerged platform weight, and wind characteristics was negligible. The cyclic component (due to design storm) represented about 40%, and the static component (due to submerged weight) represented the remaining 60% of the extreme axial load in 1975. In 1993, the cyclic component represented about 35% of the revised axial load. The reduction of uncertainty in the extreme axial pile load reflects the change in knowledge with increased research, almost two decades of site-specific wave data, and the increased proportion of the gravity load on the total axial load.

Uncertainty in axial pile capacity model

In the probabilistic pile capacity analysis, a variable describing the uncertainty in side friction calculation in each layer was used. An independent model uncertainty variable in each layer is required because the soil type can vary from one layer to the other and different resistance mechanisms need then to be considered. In the bottom layer, two model uncertainty variables should be considered: the first applying to the side friction calculation and the second to the end bearing calculation. The duality of model uncertainty in the last layer is important because side friction and end bearing are two different resistance mechanisms which are modelled by different equations. The model uncertainty variables were taken as normally distributed.

In a dense to very dense sand, the uncertainties due the calculation model can be very large, and the bias is believed to show a lot of conservatism in the API RP2A method. The uncertainties are believed to be far greater for piles in sand than for piles in clay. The model uncertainty values used in the analyses were based on the study by Lacasse and Goulois (1989) for sand, and on several NGI research projects for clay (Lacasse and Nadim, 1996). The mean of the model uncertainty for side friction in sand was taken as 1.10 to reflect the conservatism of the API procedure for this soil type. A mean value of 1.0 was used for the clay layers. The COV of the model uncertainty for side friction varied between 0.10 and 0.15 in the different layers.

For end bearing in very dense sand, the existing calculation model is generally believed to be conservative. For this reason, the mean of the model uncertainty was taken as 1.20, with a coefficient of variation of 0.15 to reflect the lack of good reference pile load tests with comparable pile size as used offshore.

5. Results and conclusions

The results of the analyses are summarised in Table 2. In 1975, only deterministic calculations were carried out. The 1975-probabilistic calculations were run in 1994 for the purpose of this example calculation.

Table 2 Results of 1975 and 1993 deterministic and probabilistic analyses Pile P2 in Leg A5 (penetration depth = 40.8 m)

Soil Profile	Deterministic factor of safety	β Reliability index	Probability of failure, P_f
1975	1.73	2.06	$2.0 \cdot 10^{-2}$
1993	1.39	2.41	$0.8 \cdot 10^{-2}$

The values of P_f and β in Table 2 are conditional values given the 100-year storm occurs. They should not be confused with the annual failure probability and reliability index.

Figure 5 illustrates schematically the results of the reliability analysis of the most loaded pile for the offshore jacket used in this example. The newer deterministic analysis gave a low safety factor (FS), a situation of major concern since the safety factor was below the minimum required factor of safety under extreme loads of 1.50. However the added information reduced the uncertainty in both soil and load parameters. The pile with a safety factor of 1.39 is nominally safer than the pile was believed to be in 1975 where the safety factor was 1.73. The probabilistic analyses showed that the pile, although with a lower safety factor, had higher safety margin than perceived at the time of design. The lower uncertainty in the parameters in the newer analysis caused a reduction in the probability of failure (P_f) by a factor of 2.5.

The factor of safety is therefore not a sufficient indicator of safety margin because the uncertainties in the analysis parameters affect probability of failure, but these uncertainties do not intervene in the deterministic calculation of safety factor. As for deterministic calculations, the essential components of reliability estimates in geotechnics are (1) a clear understanding of the physical aspects of the geotechnical behaviour to model and (2) the experience and engineering judgement that enter into all decisions at any level, whether for parameter selection, choice of most realistic analysis model, or decision-making on the viability of a concept. The most important contribution of reliability concepts to geotechnical engineering is increasing the engineer's awareness of the existing uncertainties and their consequences.

6. References

API (1993)
Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms - Load and Resistance Factor Design.
API RP2A-WSD, American Petroleum Institute, 20th edition, Dallas, Texas, Sept. 1993.

Haver, S. and O.T. Gudmestad (1992)
Uncertainties Related to the Kinematics of Ocean Waves and Their Effects on Structural Reliability.
Statoil report No. 91053, Contract No. 96735, ID: RA-7423, Version 3: March 1992.

Lacasse, S. and A. Goulois (1989)
Uncertainty in API Parameters for Predictions of Axial Capacity of Driven Pile in Sand.
21st OTC Houston, Texas, USA. Paper 6001, pp. 353-358.

Lacasse, S. and F. Nadim (1996)
Model Uncertainty in Pile Axial Capacity Calculations.
28th OTC Houston, Texas, USA. Paper 7996.

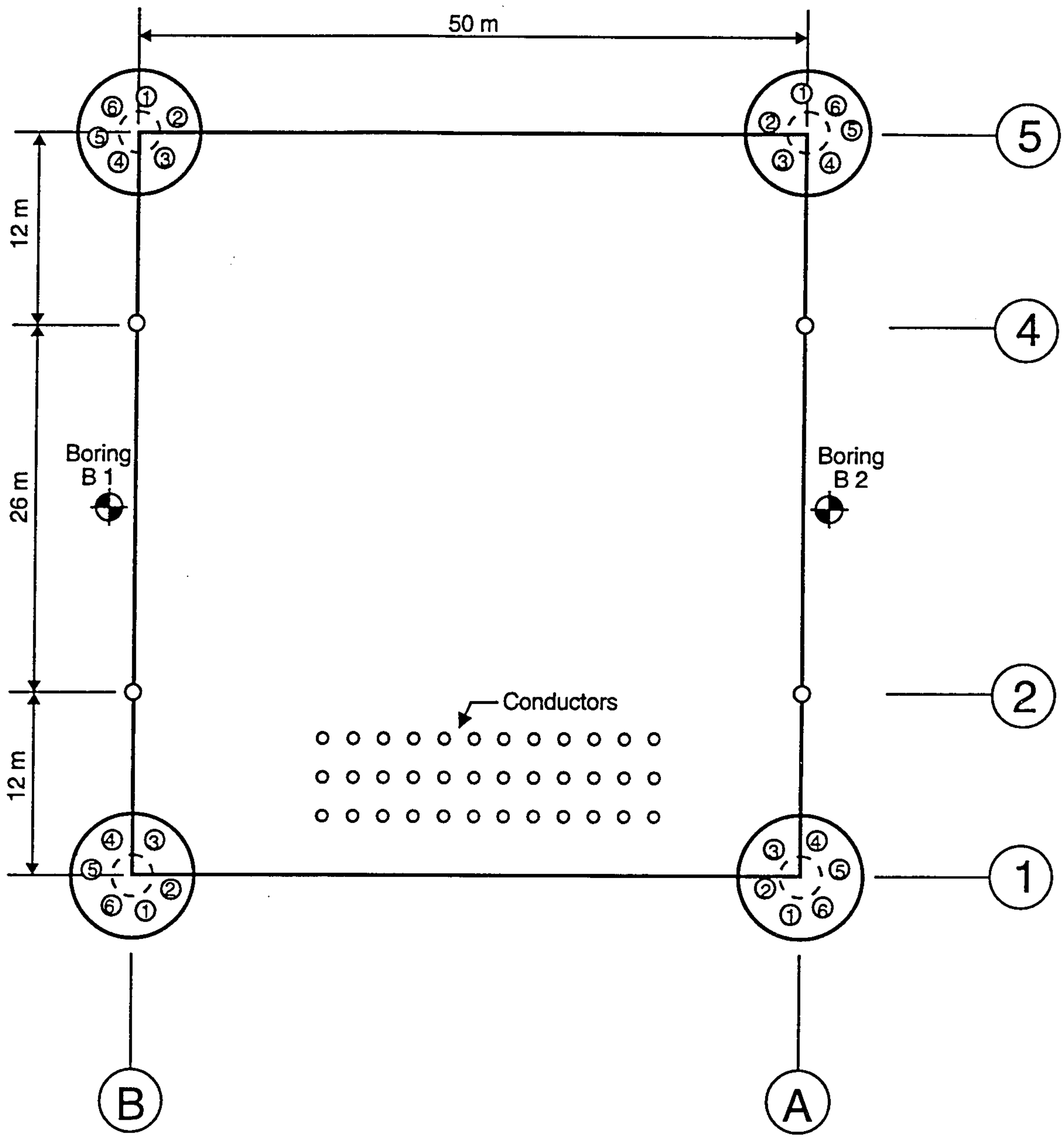


Figure 1. Foundation layout

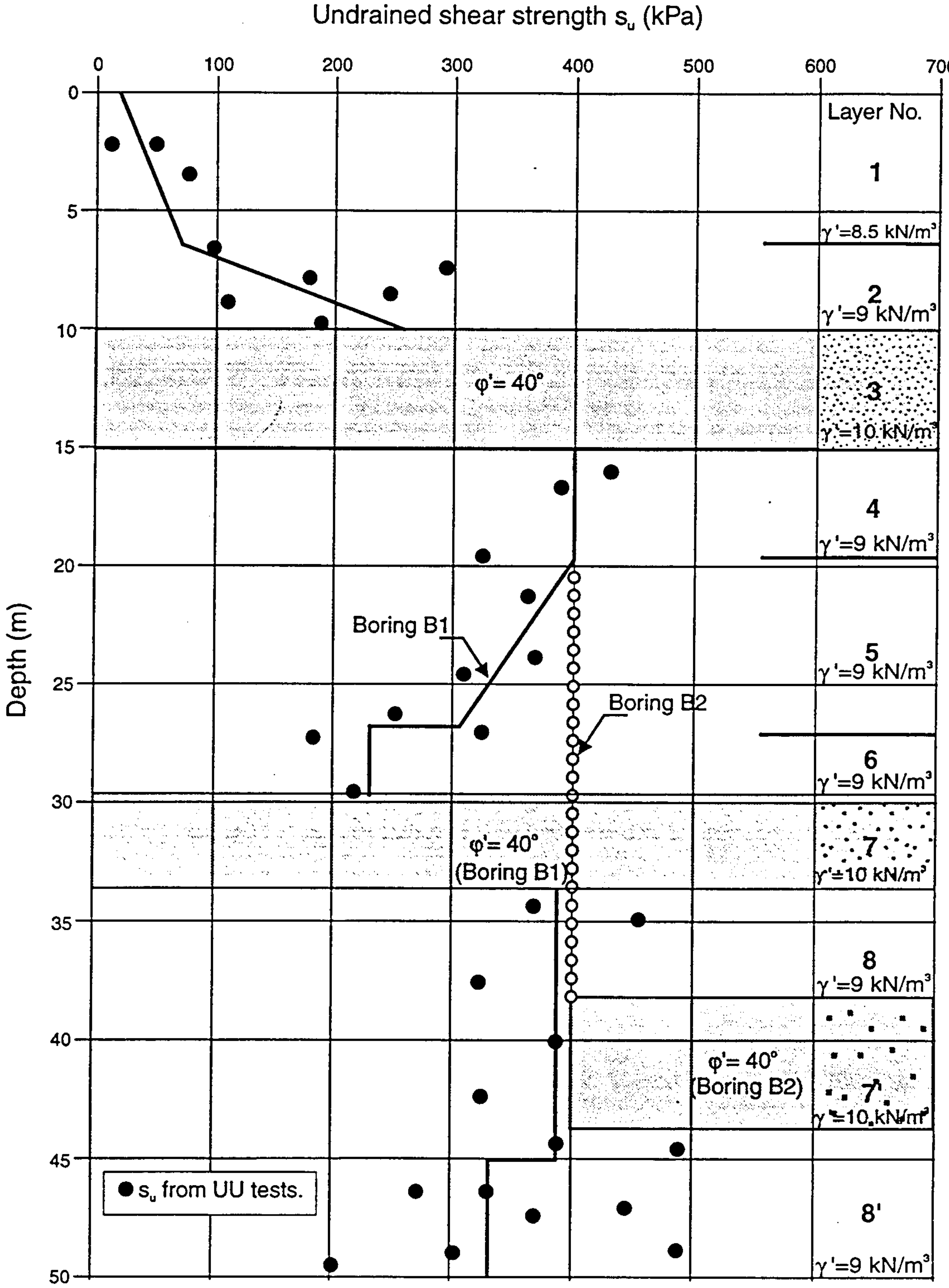


Figure 2. Soil profiles for axial pile capacity calculations, 1975-analysis

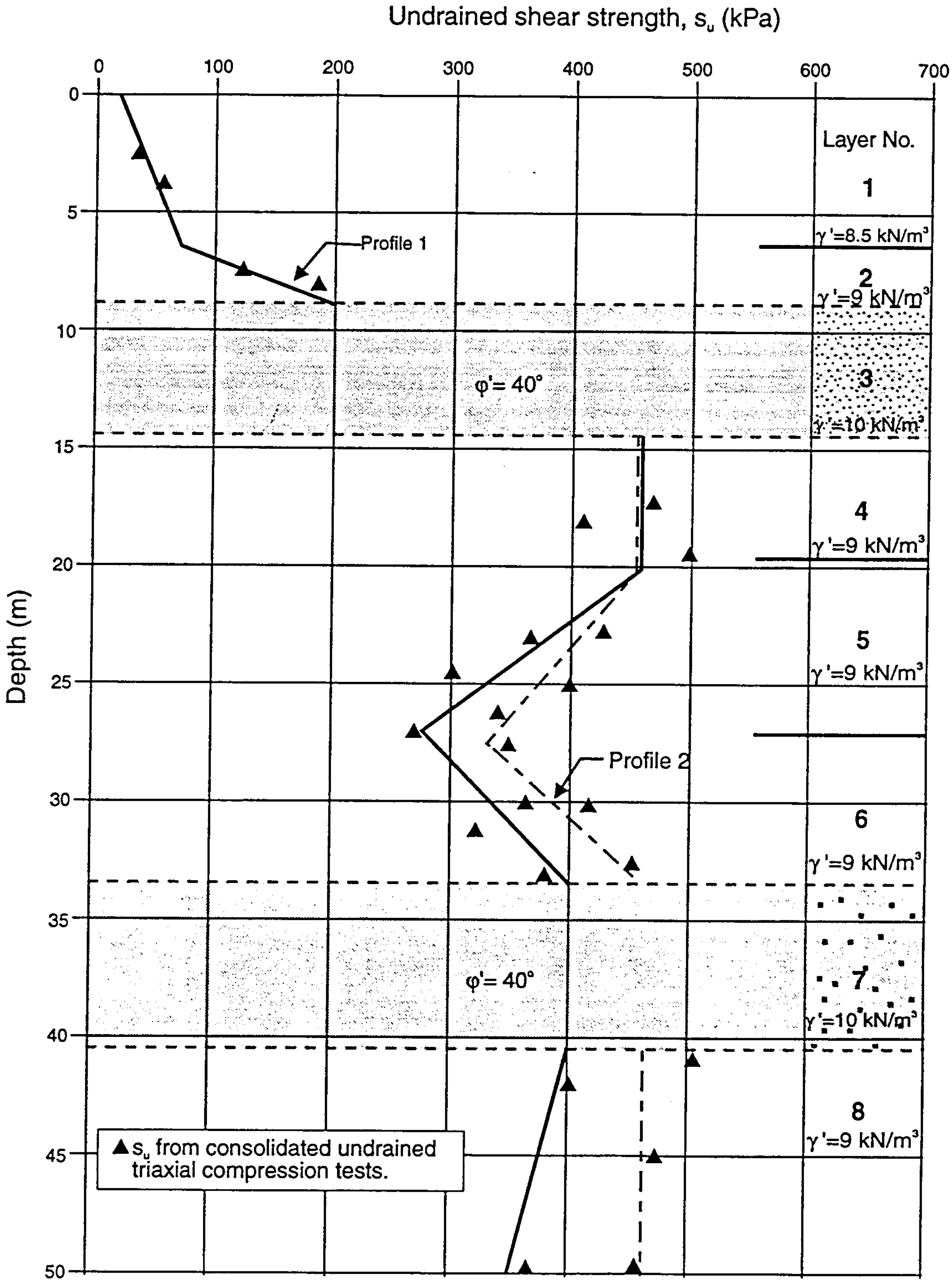


Figure 3. Example of soil profiles for axial pile capacity calculations, 1993-analysis.

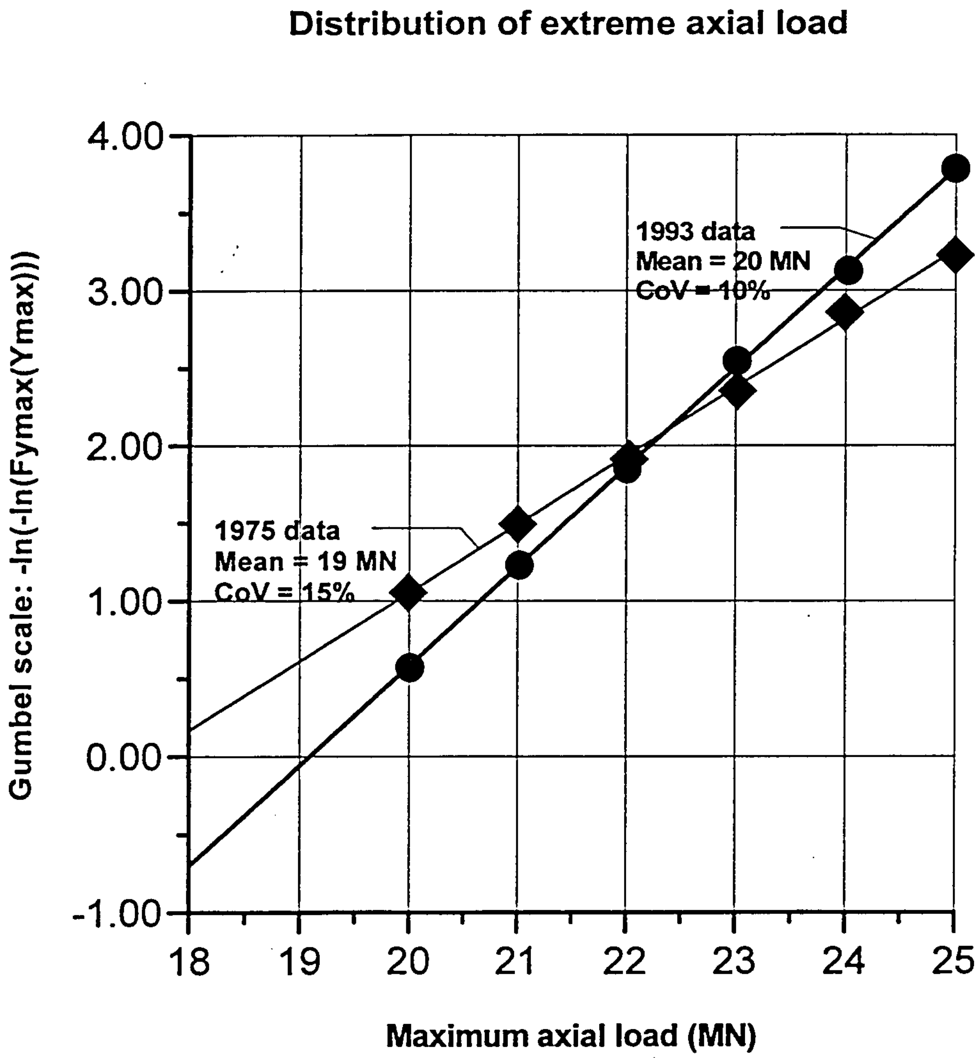
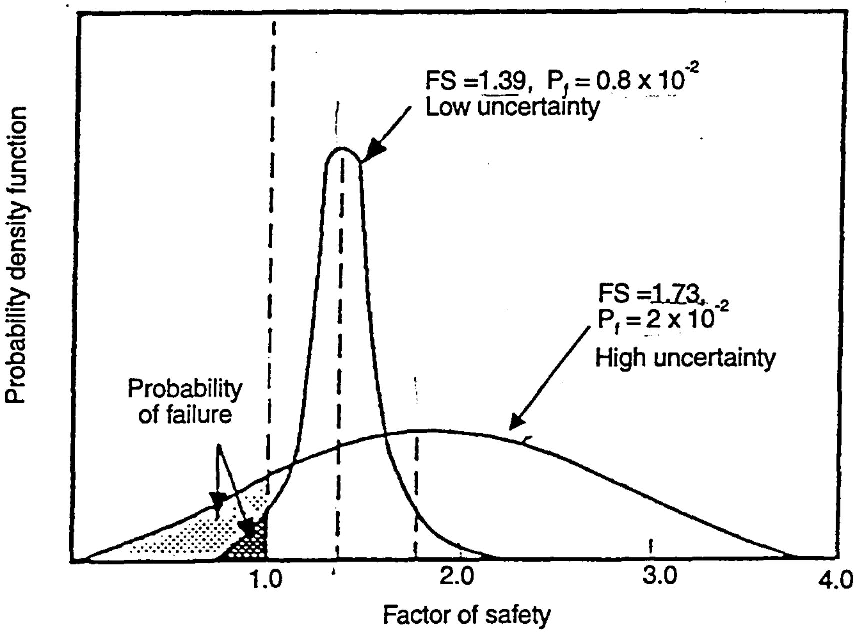


Figure 4. *Unconditional distribution of 100-year extreme axial load on pile P2 in Leg A5 plotted on Gumbel scale. Data points show results of calculations with FORM approximation.*



Note: Density functions not to scale

Figure 5. Illustration of safety factor and probability of failure of most loaded pile in example jacket.

JCSS document Reliability of Existing Structures

Annex E : Case Studies

Case Study E3 : Fatigue Crack (second draft)

by J.Goyet Bureau Veritas

1. INTRODUCTION

Even a well designed structure has to be monitored throughout its working life. It is in particular the case when design includes some checks dealing with the structural damaging process due to the action of time (where the uncertainty is large). Then inspections have to be performed leading to corrective actions if defects exist and are likely to affect structural integrity. These considerations are of the highest importance to offshore structures with fatigue sensitive structural details. During the last decade, an important amount of research has been made in order to describe how optimal reliability based inspection and repair strategies may be modelled [1], [2]. In such an approach, the parameters are the number of inspections, the inspection intervals, the inspection techniques with their associated qualities, the repair strategies and the repair techniques with their associated performance.

One optional methodology for optimal planning is presented in section 2 which gives some details about the main topics to be taken into account. The fatigue crack growth model is presented in section 3. The methodology is illustrated by one example in section 4. This example is extracted from a real situation which was previously solved by a deterministic approach using the usual safety requirements.

2. OPTIMAL INSPECTION AND REPAIR PLAN

The optimal reliability based inspection and repair plan is the least cost plan which permits the maintenance of the structural element at an adequate safety level throughout its anticiped remaining working life. The problem in question is an optimization problem insofar as we have to minimize a cost function under reliability constraints. Instead of trying to solve this problem by using complex numerical algorithms, the selected methodology (Figure 1) assumes a particular inspection and repair scheme and computes the cost function associated with a set of plans within the framework of this particular scheme. Then the decision maker chooses the plan with the smallest cost function value.

The cost function is the expected total cost defined by formula (1) :

$$E(C) = \sum_i C(S_i)\, P(S_i) \qquad (1)$$

where :

$C(S_i)$ = cost associated with the i^{th} scenario

$P(S_i)$ = probability that the i^{th} scenario occurs.

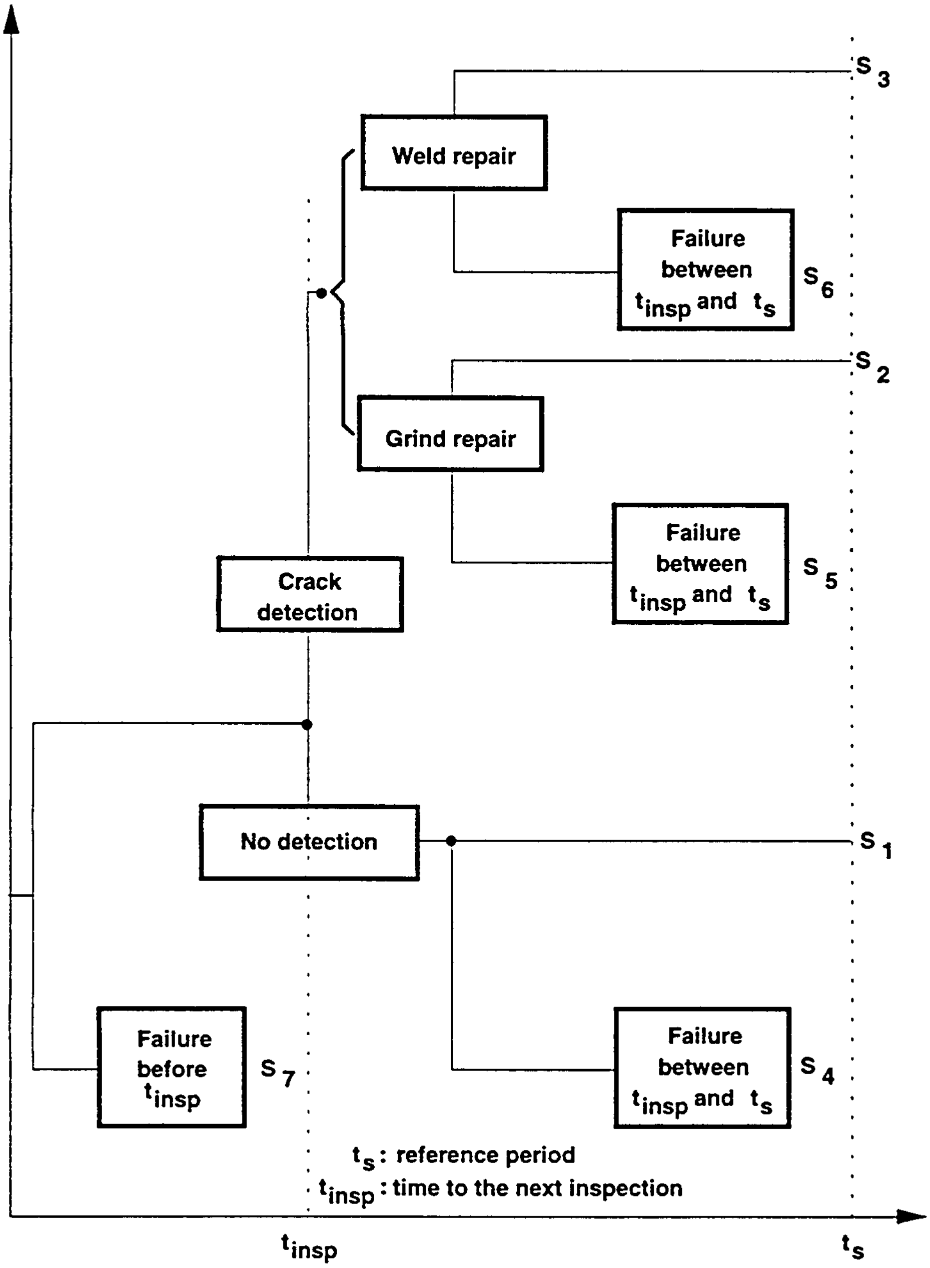


Figure 1 - Simplified IMR scheme : single inspection and automatic repair

As shown in Figure 1, each plan is characterized by :

- the time t_{insp} to the next inspection
- an inspection technique and the associated performance in terms of Probability of Detection (PoD) and Probability of Sizing (PoS) distributions
- a repair strategy : grind or weld repair according to the measured crack depth
- a repair method and the associated performance : crack depth and crack length after grind or weld repair, crack growth parameter (the Paris equation parameter) C after grind or weld repair.

A description of this scheme can be found in Faber [4].

It is also possible to take the most recent inspection observations into account through event updating. In this case, the probabilities $P(S_i / E_k(t^*))$ have to be used as a substitute for the $P(S_i)$'s where $E_k(t^*)$ is one of the possible observations at the inspection time $t = t^*(t^*<t_{insp})$.

Calculation of (1) needs to define the fatigue crack growth model to be used in analysis and to evaluate the $P(S_i)$ *or* $P(S_i / E_k(t^*))$ probabilities. Once the crack growth model has been defined, the calculation procedure is as follows :

a) define the relevant safety and event margins
b) compute component reliabilities (probability of failure for safety margins and probability of occurence for event margins)
c) compute cut-sets reliabilities (see below)
d) attribute a value to the various cost categories (cost of failure, cost of inspection, cost of repair)
e) Compute the expected total cost value by (1).

Insofar as each scenario may be modelled as a logical sequence of events, it may be also modelled as a parallel system. Then we have :

$$S_i = \bigcap_j E_{ij} \qquad \textbf{(2.1)}$$

$$P(S_i) = P\left(\bigcap_j E_{ij}\right) \qquad \textbf{(2.2)}$$

$$P(S_i / E_k(t^*)) = \frac{P(S_i \cap E_k(t^*))}{P(E_k(t^*))} = \frac{P\left(\bigcap_j E_{ij} \cap E_k(t^*)\right)}{P(E_k(t^*))} \qquad \textbf{(2.3)}$$

The probabilities (2.2) and (2.3) are estimated using the usual component (FORM) and system (SYSREL) reliability methods.

3. FATIGUE CRACK GROWTH MODEL

For offshore structures with tubular members, it is a common practice to define accumulated damage failure as the event that the crack grows through the thickness of the tubular members. The corresponding safety margin is :

$$M_F(t) = N(T) - N(t) \qquad \textbf{(3)}$$

where :

T is the chord thickness
$N(T)$is the number of cycles for the crack to grow from its initial size to T
$N(t)$is the number of cycles at time t.

$N(.)$ is estimated using the FACTS software package [6] which is a software package especially designed for fast and accurate fatigue fracture mechanics analysis of tubular welded joints. The case study which will be presented in section 4 is based on :

- the Paris crack growth equation $\frac{da}{dn} = C(\Delta K)^m$
- a multi-segment da/dn versus ΔK relationship (Figure 2)

- the choice of TPM model to express the stress intensity factor

- the WASR (Weighted Average Stress Range) concept to summarize the long term stress range distribution.

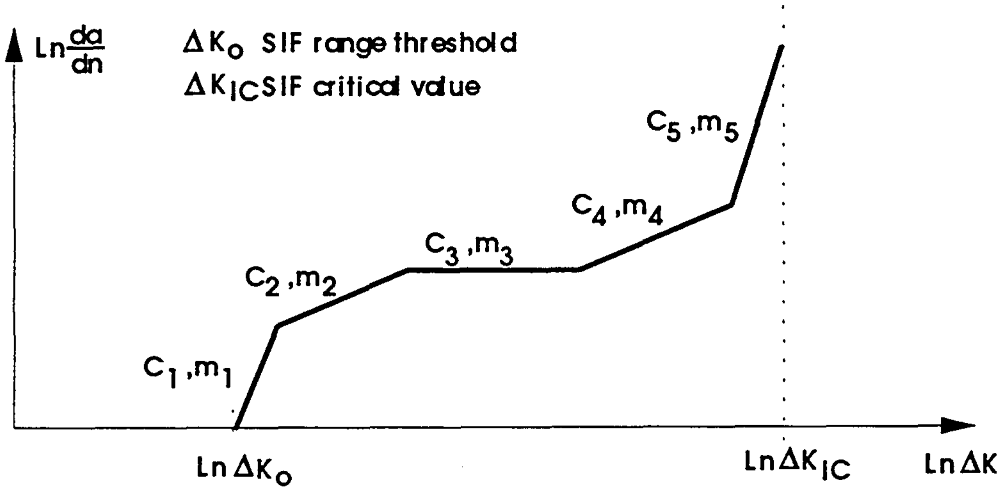


Figure 2 - Crack growth model : multi-segment Ln(da/dn) versus Ln ΔK relationship

FACTS is basically a one dimensional (crack depth) crack growth integration methodology.

4. CASE STUDY

4.1. Case description

◆ Joint geometry and initial crack depth (See Figure 3 below).

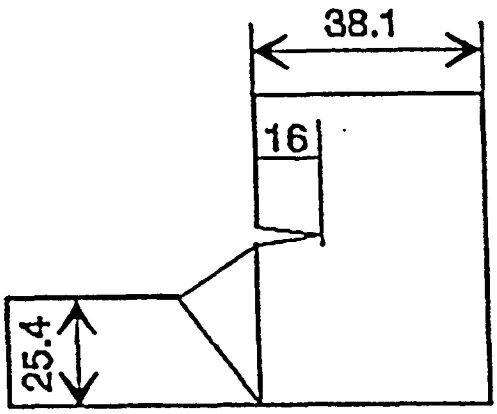


Figure 3 - Joint 46 : geometry and initial crack depth

◆ Long term stress range statistics (See Table 1 below)

Table 1 - Joint 46 : long term stress range histogram

Δσ(MPa)	Number of cycles	Δσ(MPa)	Number of cycles
1	2305190	44	2967
3	1221149	52	1397
6	830021	62	447
9	310062	72	196
12	137692	82	88
16	121281	96	49
22	42429	119	12
28	17726	148	2
35	7814		

◆ Material properties and fatigue crack growth modelling
Steel grade : E36
Modulus of elasticity : E = 21000 MPa
Poisson coefficient : ν = 0.3

◆ WASR
The expected value of the Weighted Average Stress Range is equal to 10 MPa.

4.2. References analyses

Basic random variables for reliability and cost analyses are given in Table 2.
In case of detection, weld repair takes place if the measured crack depth is larger than 20 mm.

Costs dealing with failure, inspection and repair are as follows :

Cost of failure $C_F = 100x10^6$ (French Francs)

Cost of weld repair $C_W = 5 \times 10^6$ (FF)

Cost of grind repair $C_G = 0{,}25 \times 10^6$ (FF)

Cost of UCWI inspection $C_I = 0{,}1 x\ 10^6$ (FF)

- Reference period is : $t_s = 20$ years

- Description of PoD curve and sizing uncertainty (ε) as given in Table 2 fits Underwater Close Visual Inspection technique (UCVI) and refers to inspection to be performed at $t = t_{insp}$.

Table 2 - Joint 46 : Basic Reliability and Cost Analyses: list of random variables

Nr	Symbol	Definition	Distribution law	Expected value (1st parameter)	Standard deviation (2nd parameter)	Coefficient of variation
1	T	Chord thickness	LN	38,1	5,72	0,15
2	a_o	Initial crack depth	LN	16	1	0,06
3	T_W	Chord thickness after weld repair	LN	38,1	5,72	0,15
4	a_{oW}	Initial crack depth after weld repair	LN	2	1	0,50
5	T_G	Chord thickness after grind repair	LN	38,1	5,72	0,15
6	a_{oG}	Initial crack depth after grind repair	LN	2	1	0,50
7	WASR	Weighted average stress range	N	10	3	0,30
8	C	Paris law parameter for initial material	LN	5×10^{-12}	$2,5\times10^{-12}$	0,50
9	C_W	Paris law parameter after weld repair	LN	5×10^{-12}	$2,5\times10^{-12}$	0,50
10	ε	Sizing uncertainty	N	0	2,5	-
11	PoD	Probability of detection	EXP	(10^{-5})	(76,9)	-

Distribution laws :
LN : Lognormal
N : Normal
EXP : Exponential

RELIABILITY ANALYSIS

The β-value decreases from 6.20 (1 year) to 2.01 (20 years), with a critical time to the next inspection equal to 9 years insofar as after this time the β-value goes under the selected threshold ($\beta_o = 3$). Results are shown on Figure 4 and given further in Table 5.

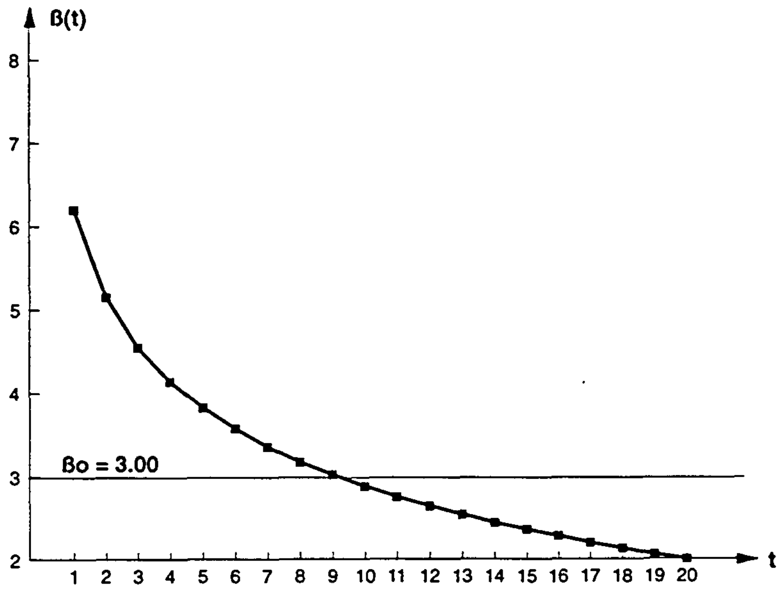


Figure 4 - Joint 46 : Reliability as function of time

COST ANALYSIS

In this basic cost analysis, each IMR plan differs from another exclusively by the time t_{insp} to the next inspection. Other parameters dealing with inspection technique, repair strategy and repair method are the same for all plans. Then, the cost analysis consists in calculating E(C) by (1) for various t_{insp} values and choosing the optimal one. Here t_{insp} varies from 1 to 20 years.
Calculation of expected cost value E(C) by (1) needs some rearrangement leading to an easier computation. The process is as follows :

$$E(C) = \sum_i \{ C_I(S_i) + C_R(S_i) + C_F(S_i) \} P(S_i) \tag{4.1}$$

$$E(C) = E(C_I) + E(C_R) + E(C_F) \tag{4.2}$$

where :

$$E(C_I) = \sum_{1to6} C_I(S_i)P(S_i) = C_I \sum_{1to6} P(Si) \tag{4.3}$$

$$E(C_R) = \sum_{2,3,5,6} C_R(S_i)P(S_i) = C_G \sum_{2,5} P(S_i) + C_W \sum_{3,6} P(S_i) \tag{4.4}$$

$$E(C_F) = \sum_{4to7} C_F(S_i)P(S_i) = C_F \sum_{4to7} P(Si) \tag{4.5}$$

where $P(Si)$ is evaluated by (2.2)

$E(C_I)$, $E(C_R)$, $E(C_F)$ and $E(C)$ are calculated for each t_{insp} value. Results are given in Table 4 and shown in Figure 5. The optimal time to the next inspection is

$$t_{insp,\, opt} = 15 \text{ (years)}$$

with an expected total cost value equal to :

$$E\{ C(t_{insp,opt}) \} = 2\,596 \text{ (KF)}$$

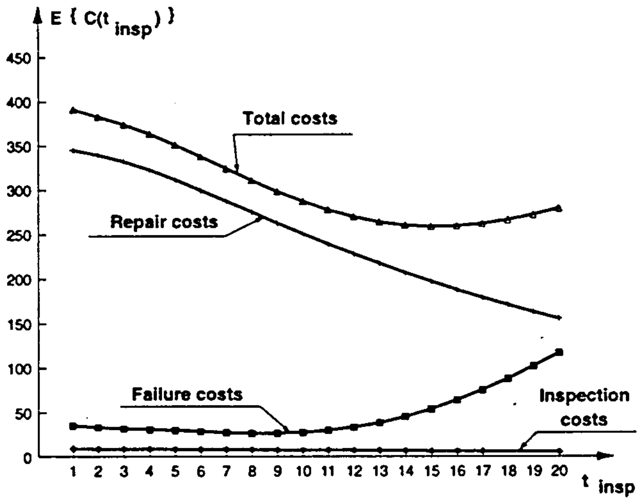


Figure 5 - Joint 46 : Expected cost value versus time t_{insp} to the next inspection (II)

Table 4 - Joint 46 : Expected cost (Failure, Repair, Inspection and Total) value versus time t_{insp} to the next inspection (I) (Unit : KF)

t_{insp}	Failure costs	Repaircosts	Inspection costs	Total costs
1	357	3 457	98	3 912
2	338	3 400	96	3 834
3	324	3 326	94	3 745
4	314	3 233	92	3 640
5	303	3 124	91	3 518
6	291	3 006	89	3 385
7	280	2 883	87	3 250
8	272	2 759	85	3 116
9	272	2 636	84	2 991
10	282	2 515	82	2 879
11	304	2 399	80	2 783
12	342	2 286	79	2 707
13	389	2 179	77	2 646
14	458	2 076	76	2 610
15	543	1 978	74	2 596
16	644	1 885	73	2 602
17	758	1 797	71	2 627
18	885	1 714	70	2 669
19	1 024	1 643	69	2 727
20	1 173	1 559	67	2 800

CONCLUSION

In view of previous considerations, the final optimal time t_{inst} to the next inspection is t = 9 years. This value of t_{insp} assures that required safety level will be maintained. The corresponding expected cost value is equal to 2 991 KF.

4.3 Other analyses

We suppose now that an inspection was performed at t* = 8 (years) and crack depth was found to be equal to 32 mm. Before making a decision about what strategy (repair or not) would be adequate, the operator decides to perform a full updated reliability and cost analysis. As mentioned in Section 2, a relevant methodology would be able to take into account the most recent observation, namely :

$E_1(t^*)$ = "the crack, as calculated by mechanical model, is equal to the observed crack a_{obs}"

$E_4(t^*)$ = "the crack, as calculated by mechanical model, is smaller than the smallest detectable crack depth a_d"

$E_2(t^*)$, $E_3(t^*)$ = "the crack is smaller or larger than the observed crack a_{obs}"

Here, we consider :

E_1(8 years) = "the crack is equal to 32 mm"

That means we have to introduce two additional basic random variables characterizing the reliability of inspection technique at time t*.

Then Table 2 has to be completed by :

Nr	Symbol	Definition	Distribution law	Expected value (1st parameter)	Standard deviation (2nd parameter)
12	ε (t*)	Sizing uncertainty	N	0	2,5
13	PoD (t*)	Probability of detection	EXP	(10^{-5})	(76,9)

Inspection was performed at t* = 8 years instead of the optimal time t_{insp} to the next inspection found by the basic cost analysis (9 years) because of some inspection effort already planned at this time at a neighbouring location.
Reliability and cost analyses results are given in Table 5 and Figure 6 for reliability analysis and in Table 6 and Figure 7 for cost analysis

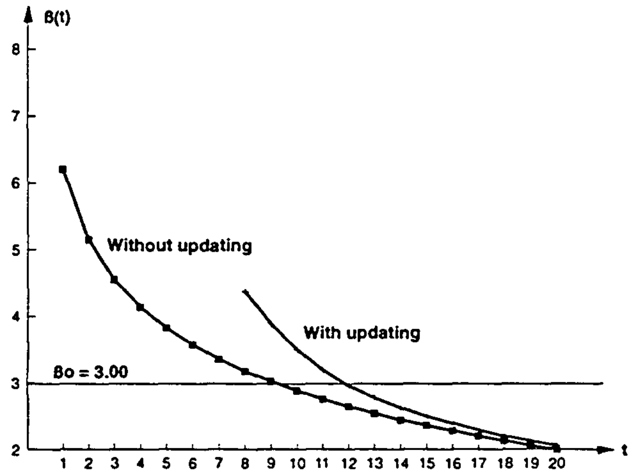


Figure 6 - Joint 46 : Reliability as function of time with (and without) updating (I)

Table 5 - Joint 46 : Reliability as function of time with (and without) updating (II)

	without updating	with updating		without updating	with updating
β1	6.20	-	β11	2.76	3.20
β2	5.15	-	β12	2.65	2.96
β3	4.55	-	β13	2.55	2.78
β4	4.14	-	β14	2.45	2.63
β5	3.83	-	β15	2.37	2.50
β6	3.58	-	β16	2.29	2.40
β7	3.36	-	β17	2.21	2.30
β8	3.18	4.38	β18	2.14	2.21
β9	3.03	3.89	β19	2.07	2.14
β10	2.89	3.50	β20	2.01	2.07

Table 6 - Joint 46 : Expected total cost value versus time t_{insp} to the next inspection, after updating at time t* = 8 years (I) (Unit : KF)

t_{insp}	Expected total cost value	t_{insp}	Expected total cost value
8	3 137	15	2 663
9	3 002	16	2 689
10	2 880	17	2 732
11	2 782	18	2 787
12	2 713	19	2 856
13	2 670	20	2 937
14	**2 656**	-	-

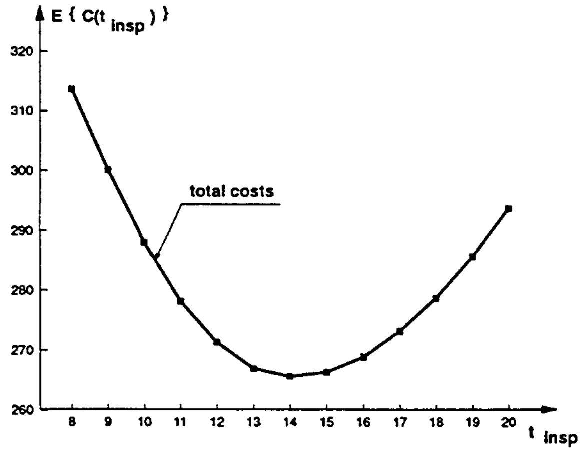


Figure 7 - Joint 46 : Expected total cost value versus time t_{insp} to the next inspection, after updating at time t* = 8 years (II)

Results are quite similar to those obtained from the basic reliability and cost analysis. The final optimal time t_{insp} to the next inspection, given a 32 mm observed crack at t* = 8 years, is :

$$t = 11 \text{ years}$$

with an expected total cost value equal to :

$$E\{\, C(t)\,\} = 2\,782 \quad \text{(KF)}$$

This value ensures that required safety level will be maintained from t = 8 years to t = 11 years.

Insofar as there is need to perform an extra inspection at t = 11 years, the question is whether it would be better (or not) to perform an immediate corrective action (repair). The behaviour of the joint, given this repair, can be evaluated by rerunning the standard reliability and cost analysis. The new stochastic modelling for *T*, a_o and *C* is exactly the same as the one used in basic or updated cost analysis (Table 2) :

$$T = T_W \qquad a_o = a_{oW} \qquad C = C_W$$

Figure 8 gives the reliability of the joint versus time after repair. As shown, β-value remains above the selected threshold until the end of reference period.

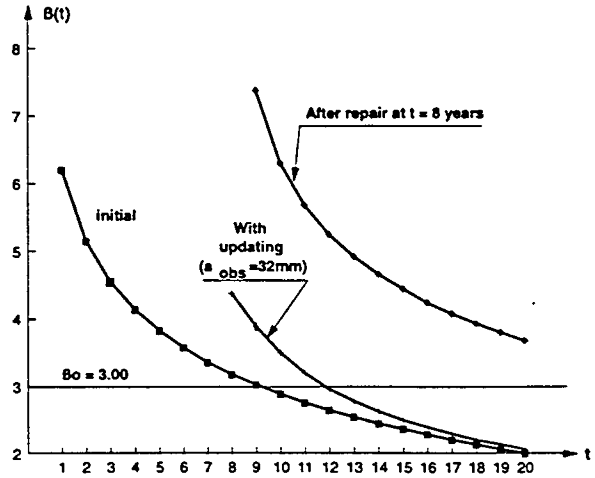


Figure 8 - Joint 46 : Reliability as function of time with repair at t = 8 years

Therefore, the manager can take a decision either to make a repair immediately or to postpone it to the time of the next inspection.

5. SENSITIVITY STUDY

As mentioned in Section 2, each IMR (Inspection-Maintenance-Repair) plan is characterized by some parameters (time to the next inspection, inspection technique, repair strategy, repair method....). One of the most important is the performance of inspection technique in terms of Probability of Detection (*POD*). In this section, we give some information about this point which cannot be too strongly emphasized insofar as it determines the significance of each possible scenario (see Figure 1).

Quality of inspection technique is defined by :

POD (*x*) = *P* ("detection of a crack with a length equal to *x*")

Here :

$$POD(x) = 1 - e^{-\lambda(x - a_{d,\min})} \text{ if } x > a_{d,\min} (= 0 \text{ otherwise})$$

In the case study presented in section 4, $a_{d,\min} = 10^{-5}$ (meters) and $\lambda = 76{,}9$ (meters).In the following, $a_{d,min}$ will be considered as fixed : Sensitivity study will turn only on λ parameter which is an indicator of inspection effort.

POD (*x*) is also equal to the distribution function of the smallest detectable crack size a_d :

$$F_{ad}(x) = P(a_d < x) = POD(x) \quad 0 < x < +\infty$$

The corresponding probability density function is :

$$f_{ad}(x) = \lambda \exp\left\{-\lambda\left(x - a_{d,\min}\right)\right\} \qquad a_{d,\min} < x < +\infty$$

$$f_{ad}(x) = 0 \qquad \text{si} \quad x < a_{d,\min}$$

Mean value and standard deviation are :

$$E(a_d) = \frac{1}{\lambda} + a_{d,\min}$$

$$\sigma(a_d) = \frac{1}{\lambda}$$

$$(\text{for } a_{d,\min} = 10^{-5}\ ,\ E(a_d) \cong \sigma(a_d) \cong \frac{1}{\lambda})$$

We consider now the following scenario :

Initial crack depth : $a_O = 4$ mm

Most recent observation : E_1 (5 years) = "the crack is equal to 10 mm"

POD distributions : $\lambda_1 = 79{,}6$, $\lambda_2 = 167$, $\lambda_3 = 500$.

Other parameters (reference period, costs, criteria for weld or grind repairs) : as those given in section 4.

Figure 9 gives expected cost values versus time t_{insp} to the next inspection for the three qualities of inspections. It can be seen that the total costs for a given value of t_{insp} are very different from one situation to another one. Clearly the decision maker should select the less costly inspection method even if it is the less effective : repair cost is always the highest component of the total cost (for example 80 or 90 %). Improvement of inspection quality increases significantly the probability of repair and consequently the expected repair and total costs but without modify the expected failure cost. This expected failure cost remains very low whatever the quality is like and we do not need a very high inspection quality.

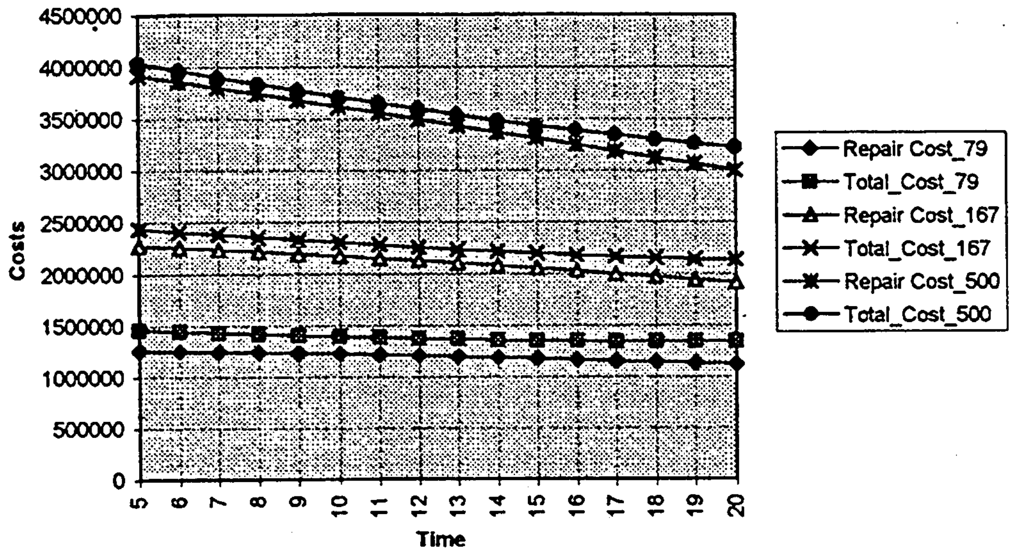


Figure 9 : Joint 46 - Expected cost values versus time t_{insp} to the next inspection : Influence of inspection quality

6. CONCLUSION

The methodology which was presented corresponds to a practical way to optimize in service life of structural parts with respect to the fatigue behaviour. It is implemented in terms of an appropriate inspection plan that will make possible an effective following-up of the damaging process of any tubular joint of a platform.

It can be included in a continuous and regular inspection plan. Based on experience feedback on the crack propagation, it permits to adapt, step by step, the cost and reliability based optimisation procedure.

It would be very useful to apply the previous approach to a significant set of joints in the same platform in order to define a practical inspection plan insofar as the times t_{insp} to the next inspection obtained for various joints have to be gather together into an optimal time t_{insp}.

This example shows that any requalification problem can be formulated as a decision problem. On this point of view, requalification procedures have to take into account time factor and IMR strategies : the decision maker has to select one among several alternatives according to available (or potentially available) information and some preference criteria (Utility function).

More generally, the process of decision would be iterative. One has to loop on the three following phases in order to better and better refine the problem description and to finally achieve consistency and clarity of action :

1- create a formal model of the problem
2- compute the logical implication of the model
3- interpret the formal recommendations in terms of reality

Communication and discussion with experts lead to a new revision of these activities in order to improve the model, and so on...(see figure 10).

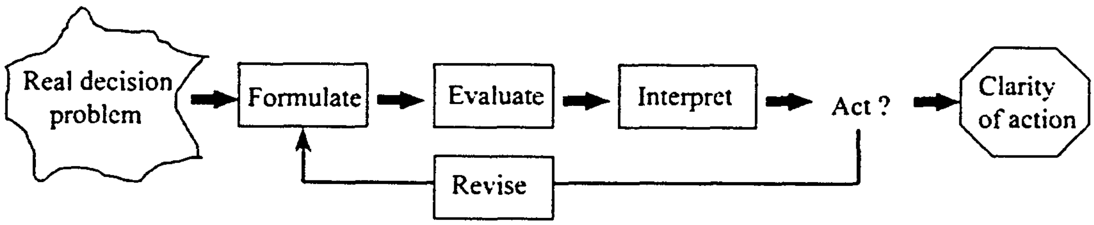


Figure 10 : The three stages closed-loop decision process

The aim is obviously to have a formal model which describes the most accurately possible the real problem in all its complexity, and that the analysis yields, in such a complex and uncertain situation, a clear and consistent recommendation.
This process in the form of a closed loop is an excellent frame for communication and discussion in dealing with a decision problem (focusing particularly on developing new possibilities for action).

References

[1] H.O. Madsen and J.D. Sorensen, "Probability-based optimization of fatigue design, inspection and maintenance", Proceedings of the International Symposium on Offshore Structures, University of Glasgow, 1990.
[2] J.D. Sorensen, M.H. Faber, R. Rackwitz and P. Thoft-Christensen, "Modelling in optimal inspection and repair", Proceedings of the 10th International Conference on Offshore Mechanics and Arctic Engineering (Volume II, Safety and Reliability), Stavanger, Norway, 1991..
[3] M. Fujita, G. Schall and R. Rackwitz, "Adaptive reliability -based inspection strategies for structures subject to fatigue", Proceedings of ICOSSAR 89, San Francisco, 1989.
[4] M.H. Faber, "IMREL : Inspection and Maintenance using RELiability methods" , Theoretical , Users and Examples Manual, 1993.
[5] H.O. Madsen, R.K. Skong, A.G. Tallin and F. Kirkemo, "Probabilistic Fatigue Crack Growth Analysis of Offshore Structures with Reliability Updating Through Inspection", Proceedings of the Marine Structural Reliability Symposium, Arlington, Virginia, 1987.
[6] "Facts for fatigue analysis of welded tubular joints", UCL NDE-Centre, 1991,

Probabilistic Reassessment of Pile Capacity - by Testing

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1 Introduction

As a part of the general evaluation of the bridges on the danish motorway system a routine reassessment of the load carrying capacity and the residual lifetime of the motorway bridge over the Gudenå river was initiated in 1994 by COWIconsult for the Danish Road Directorate.

The Gudenå bridge was originally opened in 1971 as the first part of the north-south oriented motorway of Jutland connecting as a part of the Europe road 45 system (E45), Norway in the north with the african continent in the south.

The bridge structure is a reinforced concrete pile deck structure with a total length of 400 meters and a width of 26 meters. The pile deck super structure is comprised by simply supported slab sections with spans of about 15 meters. The super structure is supported by a sub structure consisting of columns which themselves are supported on driven concrete piles. The bridge structure is illustrated in figure 1.

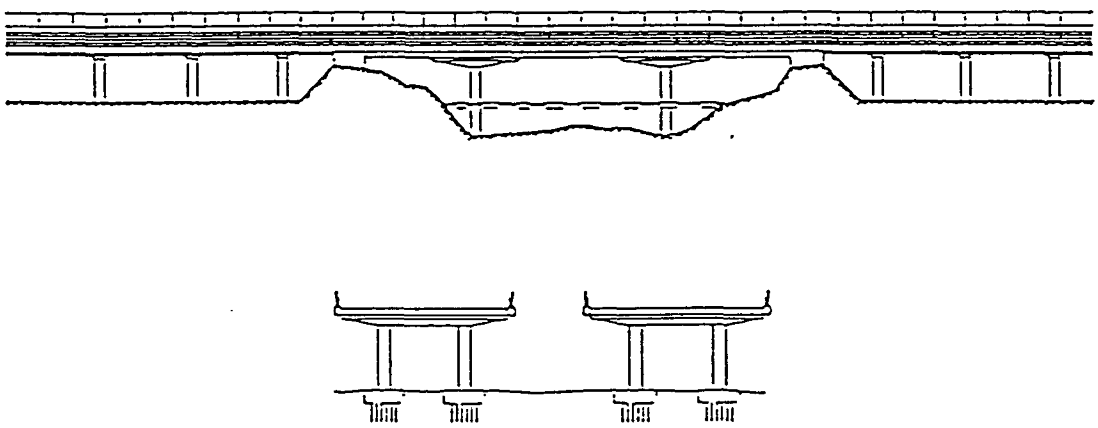

Figure 1 Illustration of the Gudenå bridge structure.

As a requirement to the load carrying capacity of the bridge the bridge shall comply with bridge classification 100 (the class roughly corresponds to the maximum allowable load - in tons - from an extraordinary vehichle simultaneously with an ordinary - 50 tons - vehichle) with reasonable maintenance costs for a residual service life exceeding 20 years.

The initial reassessment of the bridge was based on the same assumptions and the same structural data as used originally in the design of the bridge. The result of the initial reassessment indicated that the actual bridge classification of the bridge was 70 for the super structure and 40 for the sub structure, i.e. a significantly lower carrying capacity than required. As a first indication of the costs associated with the necessary strengthening of the structure in order to upgrade the overall bridge classification to class 100 an amount of MECU 5 was estimated.

Recognizing that the actual condition of the bridge appeared to be excellent and that the safety format underlying the original design basis takes into account uncertainties which may be reduced or even eliminated in a reassessment situation it was decided to investigate the possibility of performing the reassessment of the bridge based on an alternative safety format. Thereby it would be ensured that the knowledge concerning the actual condition of the bridge would be taken consistently into account, leading to a less conservative assessment of the bridge which again may lead to a reduction of the upgrading costs.

2. Reassessment of the Sub Structure

The initial reassessment of the sub structure, based on original project design information and pile driving records, indicated that the carrying capacity of the sub structure corresponds to class 40. As the governing parameter for the class of the sub structure is the compression strength of the piles the attention was focused on a refined reassessment of these.

For driven piles an increase in the load carrying capacity, as compared to the strength estimated from pile driving formulas, can normally be expected after the piles have been driven and the soil in the immediate vicinity of the piles has rehabilitated. For the usual cylindrical constant diameter piles the increase in the load carrying capacity will normally take place during the first year after the piles have been driven, but for the piles used for the present structure which are piles with footing, see figure 3, the increase in carrying capacity may be expected to take place over a substantially longer period of time i.e. over several years. Furthermore as piles with footing induce more severe disturbances in the soil around the piles during pile driving, larger increases in the carrying capacity can be expected for such piles. For this reason the reassessment of the load carrying capacity of the piles concentrated on establishing and verifying models of the pile compression capacity incorporating the increase due to soil rehabilitation. The present presentation focuses on the general methodology and approach in the considered case.

First a model was established for the present load carrying capacity of the piles.. It is assumed that the pile load capacity may be described by the sum of basically two contributions namely a contribution from the pile surface (shaft and foot) and a contribution from the pile tip see figure 2. As the piles are located in a two layer soil structure the pile load carrying capacity may be expressed by

$$Q_P = Q_1 + Q_2 + Q_3 + Q_4 \quad (1)$$

where

Q_1 is the contribution from the pile shaft in cohesion soils.
Q_2 is the contribution from the pile shaft in friction soils.
Q_3 is the contribution from the surface of the pile foot.
Q_4 is the contribution from the pile tip.

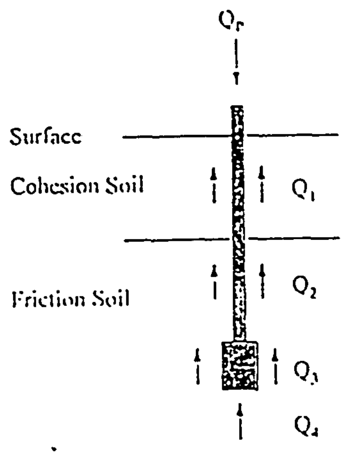

Figure 2 Principal illustration of the pile and the soil structure.

It may be assumed that the pile tip carrying capacity at the time of the pile driving can be estimated from the pile driving journals and by application of the standard pile diving formula from the danish codes (DDR). However by comparison of pile load carrying capacities determined by the DDR and static pile capacity tests performed immediately after pile driving, see e.g. [1] a discrepancy is observed.

This discrepancy may appropriately be described by a systematic term (bias) and a random term (noise).

The relation between the capacity of the piles estimated through the pile driving expressions Q_{DDR} and as obtained by compression tests Q_P can therefore be given by

$$Q_P = K\,Q_{DDR} + \Sigma \tag{2}$$

where the bias factor K and the noise term Σ are model parameters estimated by the maximum likelihood method.

One month after the piles were installed four static pile compression tests were performed and the results of these tests can be used to estimate the K and Σ for the present pile capacities. At the time of the pile compression tests it may be assumed that full friction is established on the shaft area on the pile feet. The relation between the pile load carrying capacity estimated by DDR and the static test results can hence be given by

$$Q_P = Q_3 + K\,Q_{DDR} + \Sigma \tag{3}$$

Maximum Likelihood fitting of equation (3) to the four static pile compression tests yields the mean values $E[K] = 0.762$, $E[\Sigma] = 163.3$ and the standard deviations $S[K] = 0.067$, $S[\Sigma] = 57.7$.

In the period following the static pile capacity tests it is assumed that all possible pile shaft load capacity has been established and the present pile load carrying capacity may therefore be written as

$$Q_P = Q_1 + Q_2 + Q_3 + K\,Q_{DDR} + \Sigma \tag{4}$$

where

$$Q_1 = c_u A_{cl}$$
$$Q_2 = S_u A_{fs} N_m$$
$$Q_3 = S_u A_{ff} N_m$$

where S_u is the effective stress, A_{cl} and A_{fs} are the surface areas of the pile shafts in the cohesion and the friction soil respectively and A_{ff} is the surface ares of the pile foot shaft. N_m is a factor modelling the participating friction.

Modelling N_m by a Weibull distributed random variable with mean value $E[N_m] = 0.65$ and a coefficient of variation $V[N_m] = 0.2$ and c_u as a Normal distributed random variable with mean value $E[c_u] = 20$ and a coefficient of variation $V[c_u] = 0.2$, the probability distribution function $F_{Qp}(x)$ for the piles may readily be determined through

$$F_{Qp}(q_P) = P(q_P > Q_P) \tag{5}$$

where the right hand side gives the probability that the uncertain pile capacity Q_p is lower than a certain value q_p (determined according to equation (4)).

The distribution function for one of the piles is shown in figure 3.

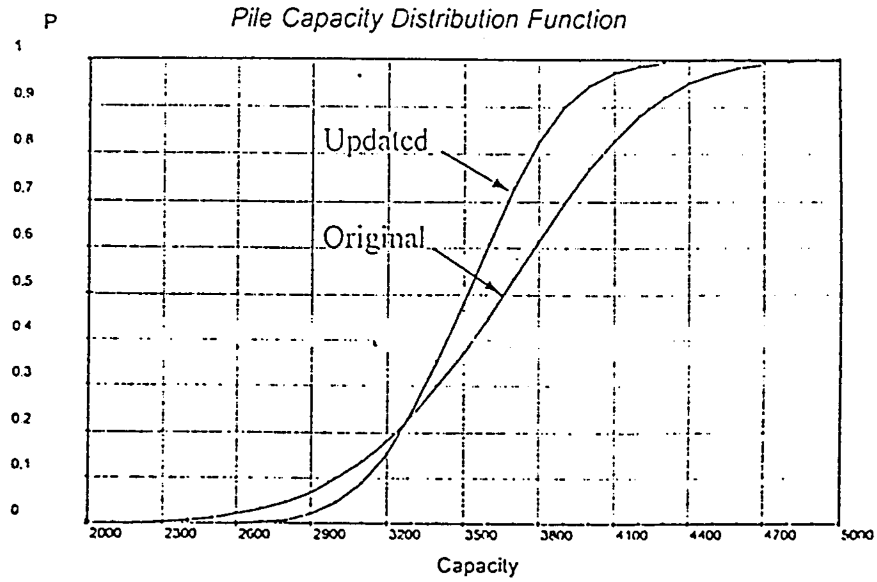

Figure 3 Distribution function for one of the piles before and after compression capacity testing.

Corresponding to normal practice the characteristic pile capacities to be used with the deterministic safety formats, in the classification of the bridge, shall be assessed as the 50 % percentile value i.e. the mean value of the pile capacity. In this way there is no benefit gained by having an estimated pile capacity with a low coefficient of variation in comparison to an estimated pile capacity with a high coefficient of variation why in general the mean value is a bad choice for a characteristic value.

In order to verify and update the probabilistic model for the pile compression strength it was decided to perform on-site compression tests of three of the four piles tested at the time of construction. Furthermore the results of the compression tests were planned to be used in order to update the probabilistic model of the pile compression tests. The updated probability distribution function for the pile compression strength, i.e. the distribution function of the pile compression strength, conditional on the outcome of the experiments X, x may be determined by

$$F_{Q_P}(q_P|x) = P(q_P > Q_P|x_1,x_2,x_3) \tag{6}$$

where the right hand side gives the probability that the uncertain pile capacity Q_P is lower than a certain value q_P conditional on the observed results from the pile compression tests x_1, x_2 and x_3.

In advance of each of the tests the probabilistic model of the pile compression strength was used in order to predict what the result of the next experiment would be by use of equation (6). It is worth noticing that all the predicted mean values of the pile strengths were within 10% of the test results.

Based on the updated probabilistic models for the pile compression strengths, updated probability distribution functions were established. The updated probability distribution function for one of the piles is shown in figure 3.

Using the updated characteristic values (50 % percentile) for the pile compression strength in the reassessment of the sub structure it was found that only 10 of the piles did not meet the requirements for upgrading the bridge to class 100. For this reason it was decided not to use the deterministic safety format and to use reliability analysis directly instead.

For the assessment of the reliability of the individual piles a probabilistic model was developed for the traffic loading on the bridge.

It should however be noted that reliability estimates at all times will be relative to the applied probabilistic model which is usually influenced by a certain amount of subjectivity. Therefore it is important to be able to compare, using the same probabilistic model, the result of a reliability analysis of a structure with an unknown reliability with the result of a reliability analysis of a structure which is known to be safe.

As the traffic loading relevant for punching shear failure of the super structure and for compression failure of the piles is the same and the punching shear strength of the super structure is governed by the concrete compression strength for which frequentistic material was available the reliability with respect to punching shear failure of the super structure was determined first. The result of this reliability analysis indicated that the failure probability was around 10^{-5} per year (which is in the order of magnitude to be expected using the deterministic safety format for the considered type of structure. Using the same probabilistic model for the traffic load and the updated probabilistic pile compression strength it was found that the failure probability of the most critical piles is in the order of 10^{-7} per year clearly indicating that the pile compression strength is sufficient for upgrading the sub structure to class 100 without strengthening.

3. References

1 Denver, H., Rikard Skov (1988), *Pile Capacities Assessed by the Impact Wave Measurement* (in danish), Geoteknisk Institut & CENTRUM PÆLE A/S.

Evaluation of the Structural State of Prestressed Girder Slabs

M. Hergenroeder

1. Introduction

A case study is given which describes a specific assessment problem on prestressed girder slabs in cattle stables. Due to the aggressive deterioration mechanism it was possible to separate the time dependency of the damage process from the assessment decisions. On the basis of material investigations the structural state of the numerous slabs in use was estimated by the statistical evaluation of a small sample of investigated objects. For more details see [1].

2. Background

The construction consists of prefabricated beams (I or ⊥ -cross section, pretensioning) hollow blocks and a reinforced cast in-situ layer. Figure 1 shows an example for a typical slab cross section. The girders were prestressed by quenched wires (A_Z = 20 / 30 mm², St 145/160 kg/mm²). Following the code specifications of DIN 1045 (1953) the minimum concrete cover of the prestressing wires was 14-15 mm, which is 10 mm plus the stirrup diameter. The girders have usually been produced using parallel installed sliding forms. At the bottom side of the beams mostly a heat insulation layer (brickwork, bituminum saturated felt, polystyrene) was inserted in the prestressing mold before casting the concrete. The investigations included the product from three manufacturers covering a total amount of about 5000 slabs in the period from 1954 to 1963.

In 1980 and 1984 prestressed girder slabs collapsed in two cattle stables due to many brittle wire fractures of the prestressing steel. As a result of material investigations the important fact was established that fractures always occur near to superficially corroded areas. Corrosion was observed at areas where the concrete cover was not sufficiently consolidated or too small

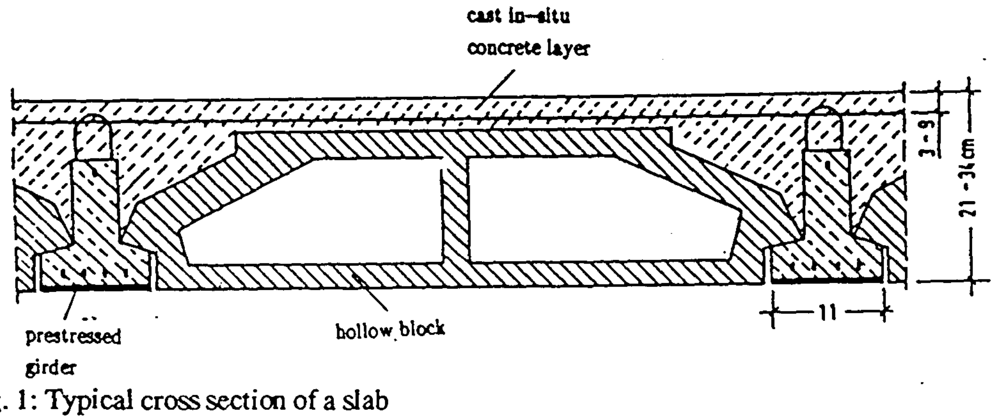

Fig. 1: Typical cross section of a slab

$$p(y) = \frac{n+1}{s+y+1} \begin{bmatrix} m \\ y \end{bmatrix} \begin{bmatrix} n \\ s \end{bmatrix} \left[\begin{bmatrix} m+n+1 \\ s+y+1 \end{bmatrix} \right]^{-1} \qquad (1)$$

with y the number of damaged elements, s the amount of damaged elements in the random sample and m = N-d-n with N the amount of elements in the population, d the amount of collapsed slabs and n the amount of examined slabs.

Input data and results of the calculation are shown in table 1.

manufacturer	N	sample			probability that more than
		n	damaged	undamaged	10% of all slabs are damaged
A	2000	5	2	3	0,97
B	475	6	2	4	0,98

Table 1: Input data and calculation results

Figure 2 gives an example of a density function. Although only five slabs from manufacturer A were investigated, finding damage in two of them, an estimation could be made with high probability (P = 0.97) that more than 10 % of all slabs (N ≃ 2000) are damaged. The expected value was 40 %. The slabs from the third manufacturer had to be assessed on a different basis.

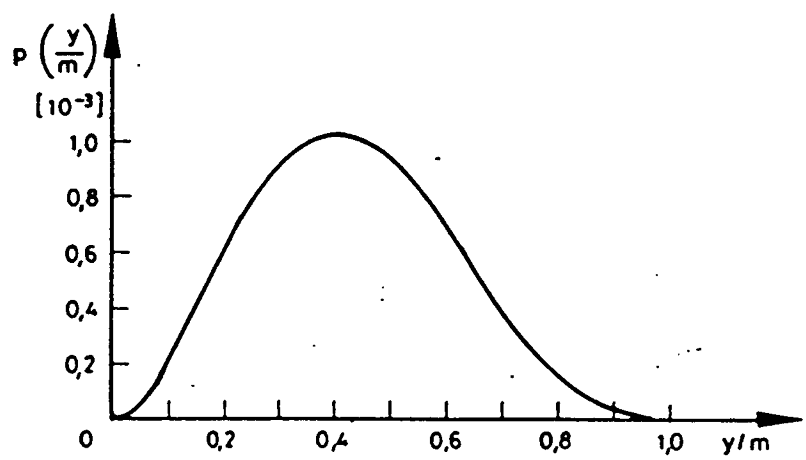

Fig. 2: Bayesian density function for the estimation of the relative frequency of damaged slabs

The Bavarian authorities set dates until which the owners have to get their slabs examined. This date is 7/31/92 for manufacturer A and 3/31/92 for the manufacturers B and C. In Germany another about 80000-100000 slabs of this type have been produced. As a part of these slabs probably is damaged in the same way the owners also are recommended to let the test be performed.

5. References

[1] Hergenroeder, M.: Long-term Behaviour of Prestessed Girder Slabs in Cattle Stables. Proc. of the Fifth International Conference on Durability of Building Materials ans Components, Brighton 1990, E &FN Spon, London, S. 325-331.

[2] Benjamin, J.R. and Cornell, C.A.: Probability, Statistics and Decision for Civil Engineers. McGraw-Hill Book Company 1980.

[3] Hillemeier, B., Flohrer, C., Schaab, A.: Die zerstoerungsfreie Ortung von Spannstahlbruechen in Spannbeton-Deckentraegern. Beton- und Stahlbetonbau 84 (1989), S. 268-270.

[4] Gerling, H. and Hergenroeder, M.: Magnetfeldmessungen und Infrarotthermographie zur Untersuchung der Standsicherheit von Spannbetondecken. In Qualitaet und Zuverlaessigkeit durch Materialpruefung im Bauwesen und Maschinenbau, VMPA-Tagung Muenchen 1990, S. 108-119.

ANHANG IV:

JCSS PROBABILISTISCHER MODEL CODE

Joint Committee
on Structural Safety
JCSS-OSTL/DIA/VROU -01-8-1998

Eight draft
April 1998

PROBABILISTIC MODEL CODE

Part 1 - BASIS OF DESIGN

Contents

1 Introduction

This part treats the general principles for a probabilistic design of load bearing structures. The more detailed aspects dealing with the probabilistic description of loads are treated in part 2. In the same way the probabilistic description of structural resistance parameters is treated in part 3.

This part doesn't give any detailed information about methods for the calculation of probabilities. It is assumed that the user of a probabilistic code is familiar with such methods.

2 Requirements

2.1 Basic requirements

Structures and structural elements shall be designed, constructed and maintained in such a way that they are suited for their use during the design working life and in an economic way.

In particular they shall, with appropriate levels of reliability, fulfil the following requirements:

- They shall remain fit for the use for which they are required (*serviceability limit state requirement*)

- They shall withstand extreme and/or frequently repeated actions occurring during their construction and anticipated use (*ultimate limit state requirement*)
- They shall not be damaged by accidental events like fire, explosions, impact or consequences of human errors, to an extent disproportionate to the triggering event (*robustness requirement*, see Annex A).

2.2 Reliability differentiation

The expression "*with appropriate levels of reliability*" used above means that the degree of reliability should be adopted to suit the use of the structure, the type of structure or structural element and the situation considered in the design, etc.

The choice of the various levels of reliability should take into account the possible consequences of failure in terms of risk to life or injury, the potential economic losses and the degree of social inconvenience, as described in chapter 8. It should also take into account the amount of expense and effort required to reduce the risk of failure. It is further noted, that the term "failure" as used in this document refers to either inadequate strength or inadequate serviceability of the structure.

The consequences of a failure generally depend on the mode of failure, specially in those cases when the risk to human life or injury exists.

In order to provide a structure corresponding to the requirements and to the assumptions made in the design, appropriate *quality measures* shall be adopted. These measures comprise definition of reliability requirements, organisational measures and controls at the stages of design, execution and use and the maintenance of the structure.

2.3 Requirements for durability

The durability of the structure in its environment shall be such that it remains fit for use during its design working life. This requirement can be considered in one of the following ways:

a) By using materials that, if well maintained, will not degenerate during the design working life.

b) By giving such dimensions that deterioration during the design working life is compensated.

c) By chosing a shorter lifetime for structural elements, which may be replaced one or more times during the design working life.

d) By inspection at fixed or condition dependent intervals and appropriate maintenance activities.

In all cases the reliability requirements for long and short term periods should be met. Analysis aspects on durability are described in Annex B.

3 Principles of limit state design

3.1 Limit states and adverse states

The structural performance of a whole structure or part of it should be described with reference to a specified set of limit states which separate desired states of the structure from adverse states.

The limit states are divided into the following two basic categories:

- the *ultimate limit states*, which concern the maximum load carrying capacity as well as the maximum deformability
- the *serviceability limit states*, which concern the normal use.

The exceedance of a limit state may be irreversible or reversible. In the irreversible case the damage or malfunction associated with the limit state being exceeded will remain until the structure has been repaired. In the reversible case the damage or malfunction will remain only as long as the cause of the limit state being exceeded is present. As soon as this cause ceases to act, a transition from the adverse state back to the desired state occurs.

It is further noted here that in cases of a considerable range between the two limit states an in-between limit state called "*reusable limit state*" can be identified. For example in case of earthquake damage of plant structures such limit state is associated to the safe shut down of the plant.

Ultimate limit states may correspond to the following adverse states:

- loss of equilibrium of the structure or of a part of the structure, considered as a rigid body (eg. overturning)

- attainment of the maximum resistance capacity of sections, members or connections by rupture or excessive deformations

- rupture of members or connections caused by fatigue or other time-dependent effects
- instability of the structure or part of it

- sudden change of the assumed structural system to a new system, (eg. snap through)

The exceedance of an ultimate limit state is almost always irreversible and the first time that this occurs causes failure.

Serviceability limit states may correspond to the following adverse states:

- local damage (including cracking) which may reduce the durability of the structure or affect the efficiency or appearance of structural or non-structural elements.
- observable damage caused by fatigue or other time dependent effects
- unacceptable deformations which affect the efficient use or appearance of structural or non-structural elements or the functioning of equipment.
- excessive vibrations which cause discomfort to people or affect non-structural elements or the functioning of equipment.

In the cases of permanent local damage or permanent unacceptable deformations the exceedance of a serviceability limit state is irreversible and the first time that this occurs causes failure.

In other cases the exceedance of a serviceability limit state may be reversible and then failure occurs:

a) the first time the serviceability limit state is exceeded, if no exceedance is considered as acceptable

b) if exceedance is acceptable but the time when the structure is in the undesired state is longer than specified

c) if exceedance is acceptable but the number of times that the serveciability limit state is exceeded is larger than specified

d) if a combination of the above criteria occur.

These cases may involve temporary local damage (eg. temporarily wide cracks), temporary large deformations and vibrations. Limit values for the serviceability limit state should be defined on the basis of *utility considerations*.

3.2 Design procedure

For each specific limit state the relevant basic variables should be identified, i.e. the variables which characterize:

- actions and environmental influences
- properties of materials and soils
- geometrical parameters

Such variables may be time dependent. Models, which describe the behaviour of a structure, should be established for each limit state. These models include mechanical models, which describe the structural behaviour, as well as other physical or chemical models, which describe the effects of environmental influences on the material properties. The parameters of such models should in principle be treated in the same way as basic variables.

Serviceability constaints (limit values according to 4.1) should in principle be regarded as random and may in many cases be treated in the same way as basic variables.

Where calculation models are available, the limit state can be described with aid of a function, g, of the basic variables $\underline{X}(t) = X_1(t), X_2(t), \ldots$ so that

$$g\,(\underline{X}(t)) = 0 \qquad (1)$$

Eq. (1) is called the limit state equation, and

$$g\,(\underline{X}(t)) < 0 \qquad (2)$$

identifies the adverse state.

In a component analysis where there is one dominating failure mode the limit state condition can normally be described by one equation according to eq. (1). In a system analysis, where more than one failure mode may be determining, there are several such equations.

3.3 Design situations

Actions, environmental influences and structural properties may vary with time. Such variations, which occur throughout the lifetime of the structure, should be considered by selected design situations, each one representing a certain time intervall with associated hazards, conditions and relevant structural limit states.

The design situations may be classified as:

Persistent situations, which refer to conditions of normal use of the structure and are generally related to the working life of the structure.

Transient situations, which refer to temporary conditions of the structure, in terms of its use or its exposure.

Accidental situations, which refer to exceptional conditions of the structure or its exposure.

4 Basic variables and uncertainty modelling

4.1 Basic variables

The calculation model for each limit state considered should contain a specified set of basic variables, i.e. physical quantities which characterize actions and environmental influences, material and soil properties and geometrical quantities. The model should also contain model parameters which characterize the model itself and which are treated as basic variables (compare 4.2). Finally there are also parameters which describe the requirements (e.g. serviceability constraints) and which may be treated as basic variables. The basic variables (in the wide sence given above) are assumed to carry the entire input information to the calculation model.

The basic variables may be random variables (indlucing the special case deterministic variables) or stochastic processes or random fields. Each basic variable is defined by a number of parameters such as mean, standard deviation, parameters determining the correlation structure etc.

4.2 Uncertainty modelling

Uncertainties from *all essential sources* must be evaluated and integrated in a basic variable model. Types of uncertainty to be taken into account are:

- intrinsic physical or mechanical uncertainty
- statistical uncertainty, when the design decisions are based on a small sample of observations or when there are other similar conditions
- model uncertainties (see 5.6).

Within given classes of structural design problems the types of probability distributions of the basic variables should be standardized. These standardizations are defined in the parts 2 and 3 of the probabilistic model code.

4.3 Definition of populations

The random quantities within a reliability analysis should always be related to a meaningfull and consistent set of populations. The description of the random quantities should correspond to this set and the resulting failure probability is only valid for the same set.

The basis for the definition of a population is in most cases the physical background of the variable. Factors which may define the population are:

- the nature and origin of a random quantity
- the spatial conditions (e.g. the geographical region considered)
- the temporal conditions (e.g. the intended time of use of the structure considered)

The choice of a population is to some extent a free choice of the designer. It may depend on the objective of the analysis, the amount and nature of the available data and the amount of work that can be afforded.

In connection with theoretical treatment of data and with the evaluation of observations it is often convenient to divide the largest population into sub-populations which in turn are further divided in smaller sub-populations etc. Then it is possible to study and distinguish variability within a population and variability between different populations.

In an analysis for a specific structure it may be efficient to define a population as small as possible as far as use, shape and location of the structure are concerned (microzonation). When the results are used for design in a national or international code, it may be necessary or convenient to put the sub-populations together to the large population again in order not to get too complicated rules (randomizing). This means that the variability within the population is increased.

4.4 Hierarchical models

This section contains a convenient and recommended mathematical description in general terms of a hierarchical model which can be used for different kinds of actions and materials. The details of this model have to be stated more precisely for each specific variable. The model is associated with a hierarchical set of subpopulations.

The hierarchical model assumes that a random quantity X can be written as a function of several variables, each one representing a specific type of variability:

$$X = f(X_1, X_2, X_3..) \quad (3)$$

The X_i represent various origins, time scales of fluctuation or spatial scales of fluctuation.

For instance, X_1 may represent the constant in time variability, X_2 a slowly fluctuating time process and X_3 a fast fluctuating time process.

In a similar way X_1 may represent the building to building variation, X_2 the floor to floor variation and X_3 the point to point variation on one floor.

5 Analysis

5.1 General

Calculation models shall describe the structure and its behaviour up to the limit state under consideration, accounting for relevant actions and environmental influences. Models should generally be regarded as simplifications which take account of decisive factors and neglect the less important ones.

One can often distinguish between:

- action models
- structural models which give action effects (internal forces, moments etc.)
- resistance models which give resistances corresponding to the action effects, and are based on.
- material models and geometry models .

However, in some cases it is not possible or convenient to make this distinction, for example, if the instability or loss of equilibrium of an entire structural system is studied or if interactions between loads and structural response are of interest.

Calculation models should be in general verified by experimental results.

5.2 Action models

A complete action model should describe several properties of the action such as its magnitude, position, direction, duration etc. In some cases there is an interaction between the different properties and also between these properties and the response of the structure. Such interactions should be taken into account.

The magnitude F of an action may often be described by two different types of variables so that

$$F = \varphi (F_0, W) \tag{4}$$

where φ is an appropriate function and

F_0 is a basic action variable, often with time and space dependent variations (random or non-random) and is generally independent of the structure

W is a random or non-random variable or a random field which may depend on the structural properties and which transformes F_0 to F.

Eq. (4) should be regarded as a symbolic expression where F_0 and W may represent several variables.

One example may be snow load where F_0 is the time dependent snow load on ground and W is the conversion factor for snow load on ground to snow load on roof which normally is assumed to to be time independent.

Further information on action models is provided in part 2. It is noted that action models may include material properties (earthquake action depends for example on material damping).

5.3 Geometrical models

A structure can generally be described by a model consisting of one-dimensional elements (beams, columns, cables, arches, etc), two-dimensional elements (slabs, walls, shells, etc) and three-dimensional elements.

The geometrical quantities which are included in the model generally refer to nominal values, i.e. the values given in drawings, descriptions etc. Normally, the geometrical quantities of a real structure differ from their nominal values, i.e. the structure has geometrical imperfections. If the structural behaviour is sensitive to such imperfections, these shall be inlcuded in the model.

In many cases the deformation of a structure causes significant deviations from nominal values of geometrical quantities. If such deformations are of importance for the structural behaviour, they have to be considered in the design in principally the same way as imperfections. The effects of such deformations are generally denoted *geometrically nonlinear* or *second order effects* and should be accounted for.

5.4 Material models

When strength or stiffness is considered the material model normally consists of relations between forces or stresses and deformations i.e *costitutive relationships*. The parameters of such relations are modulus of elasticity, yield limit, ultimate strength etc. which generally are considered as random variables, Sometimes they are time dependent or space dependent. There is often an correlation between the parameters e.g. the modulus of elasticity and the ultimate strength of concrete.

Other material properties, e.g. resistance against material deterioration may often be treated in a similar way. However the principles are strongly dependent on type of material and the property considered.

Further information related to models of several material types is given in part 3.

5.5 Mechanical models

In almost all design calculations some assumptions concerning the relation between forces or moments and deformations (or deformation rates) are necessary. These assumptions can vary and depend on the purpose and type of calculation. The most general relationship assumed in design follows elastic behaviour under low action effetcs (when the overall structural response is considered to be elastic) developing into plastic behaviour in certain parts of the structure at high action effects. In other parts of the structure intermediate stages occur. Such relationships may be used generally. However the use of any theory taking into account in-elastic or post-critical behaviour may have to take into account repetitions of variable actions that are free. Such actions may cause great variations of the action effects, repeated yielding and exhaustion of the deformation capacity.

The theory of elasticity may be regarded as a simplification of a more general theory and may generally be used provided that forces and moments are limited to those values, for which the behaviour of the structure is still considered as elastic. However, the theory of elasticity may also be used in other cases if it is applied as a conservative approximation.

Theories in which fully developed plasticity is assumed to occur in certain zones of the structure (plastic hinges in beams, yield lines in slabs, etc) may also be used, provided that the deformations which are needed to ensure plastic behaviour, occur before the ultimate limit state is reached. Thus theory of plasticity should be used with care to determine the load carrying capacity of a structure, if this capacity is limited by:

- brittle failure
- failure due to instability

5.6 Model uncertainties

A calculation model is a physically based or empirical relation between relevant variables, which are in general random variables:

$$Y = f(X_1, X_2, \ldots X_n) \qquad (5)$$

Y = model output

$f()$ = model function

X_i = basic variables

The model f (...) may be complete and exact, so that, if the values of X_i are known in a particular experiment (from measurements), the outcome Y can be predicted without error. This, however, is not normally the situation. In most cases the model will be incomplete and inexact. This may be the result of lack of knowledge, or a deliberate simplification of the model, for the convenience of the designer. The difference between the model prediction and the real outcome of the experiment can be written down as:

$$Y = f'(X_1 \ldots X_n, \theta_1 \ldots \theta_m) \qquad (6)$$

θ_i are referred to as parameters which contain the model uncertainties and are treated as random variables. Their statistical properties can in most cases be derived from experiments or observations. The mean of these parameters should be determined in such a way that, on average, the calculation model correctly predicts the test results.

6 Reliability models

6.1 Reliability measures

A standard reliability measure may be chosen to be the *generalized reliability index*. It is defined as

$$\beta = -\Phi^{-1}(P_f) \qquad (7)$$

where P_f is the probability of failure

$\Phi^{-1}(\cdot)$ is the inverse Gaussian distribution

Another equivalent reliability measure is the probability of the complement of the adverse event

$$P_s = 1 - P_f \qquad (8)$$

The probability P_f should be calculated on the basis of the standardized joint distribution type of the basic variables and the standardized distributional formalism of dealing with both model uncertainty and statistical uncertainty.

In special situations other than the standardized distribution types can be relevant for the reliability evaluation. In such cases the distributional assumptions must be tested on a suitable representative set of observation data.

Reliability analysis principles including time-dependent reliability problems are described in Annex C.

6.2 Component reliability and system reliability

Component reliability is the reliability of one single structural component which has one dominating failure mode.

System reliability is the reliability of a structural system composed of a number of components or the reliability of a single component which has several failure modes of nearly equal importance. The following type of systems can be classified:

- *redundant systems* where the components are „fail safe", i.e. local behaviour of one component does not directly result in failure of the structure;

- *non-redundant* systems where local failure of one component leads rapidly to failure of the structure.

Probabilistic structural design is primarily concerned with *component behaviour*. System behaviour is, however, of concern because it is usually the most serious consequence of structural failure. Therefore the likelihood of system failure following an initial component failure should be assessed. In particular, it is necessary to determine the system characteristics in relation to damage tolerance or robustness with respect to accidental events. The requirements for the reliability of the components of a system should depend upon the system characteristics.

A probabilistic system analysis should therefore be carried out to establish:

- the redundancy (alternate load-carrying paths)
- the state and complexity of the structure (multiple failure modes).

Furher aspects on system reliability are provided in Annex C.

6.3 Methods for reliability analysis and calculation

The numerical value of the reliability measure is obtained by a reliability analysis and calculation method (see Annex C and D). The reliability method used should be capable of producing a sensitivity analysis including importance factors for uncertain parameters. The choice of the method should be in general justified. The justification can be for example based by another relevant computation method or by reference to appropriate literature.

Due to the computational complexity a method giving an approximation to the exact result is generally applied. Two fundamental accuracy requirements are:

- Overestimation of the reliability due to use of an approximative calculation method shall be within limits generally accepted for the specific type of structure.

- The overestimation of the reliability index should not exceed 5 % with respect to the target level.

The accuracy of the reliability calculation method is linked to the sensitivity with respect to structural dimensions and material properties in the resulting design.

7. Target Reliability

7.1 General Aspects

In terms of a reliability based approach the structural risk acceptance criteria correspond to a required minimum reliability herein defined as *taget reliability*. The requirements to the safety of the structure are consequently expressed in terms of *the accepted minimum reliability index* or *the accepted maximum failure probability*.

In a rational analysis the target reliability is considered as a control parameter subject to *optimization*. The parameter assigns a particular investment to the material placed in the structure. The more material - invested in right places - the less is the expected loss. Such optimization is mainly possible when economic loss components dominate over life, injury, and culture components. When the expected loss of life or limb is important, the optimal reliability level becomes more controversional. Frequently, this leads to the problem of the economic equivalent of human life; *risk-benefit analyses* are then applied to circuvent this difficulty; the *reliability of the system is translated into the cost per life saved.* The target reliability may then be chosen such that the cost per life saved is at acceptable levels (for example comparable to other similar systems).

In a practical approach the required reliability of the structure is controlled by:

i) a set of assumptions about *quality assurance* and *quality management* measures; these measures are for example related to design and construction supervision and are intended to avoid *gross errors*.

ii) formal failure probability requirements, *conditional upon these assumptions,* defined by specified target values for the various classes of structures and structural members.

7.2 Influencing Parameters

The main parameters affecting the choice of the target reliability are described next.

Degree of failure consequences

Whole structures as well as structural components maybe classified according to the consequences of failure. Generally, a classification according to the following is sufficient:

Class 1 Minor Consequences: Risk to life, given a failure, is low and also economic and social consequences are small or negligible (e.g. agricultural structures, silos, masts).

Class 2 Moderate Consequences: Risk to life, given a failure, is medium or economic and social consequences are considerable (e.g. office buildings, industrial buildings, appartment buildings).

Class 3 Large Consequences: Risk to life, given a failure, is high, or economic or social consequences are significant (e.g. main bridges, theatres, hospitals, high rise buildings).

Class 4 Extreme Consequences: Risk to life, given a failure, is extreme as well as social and economic impact (e.g. nuclear power plants, important dam structures).

At a similar way the relative costs of safety measures can be subdivided into classes, e.g. low, moderate and high.

System behaviour

Appart from the classification of structures a classification of structural elements is needed. The failure consequences of elements in one structure may differ quite substancially. This means that one should take into account *the system behaviour* as characterized by the type of systems e.g.: *redundant systems* and *non-redundant systems* as identified in 6.2.

Failure modes

The following types of failure can be classified:

a) ductile failure with reserve strength capacity resulting from strain hardening;

b) ductile failure with no reserve capacity;

c) brittle failure.

Consequently a structural element which would be likely to collapse suddenly without warning should be designed for a higher level of reliability than one for which a collapse is preceded by some kind of warning which enables measures to be taken to avoid severe consequences.

Limit State Type

Ultimate and *serviceability limit states* are considered. For specific cases a limit state between those two can be distinguished as mentioned in 3.1.

7.3 Recommended target values

Possible schemes for the formal target safety levels to be used in design have been proposed by various national and international associations. Thereby either

- lifetime target safety levels or target safety levels for a reference period of one year

- target safety levels for the whole structure or for the structural members

have been proposed as a function of the failure consequences, of the relative costs of structural strengthening and of the type of the limit state (*serviceability or ultimate*). Examples of desired lifetimes for different structures are provided in Table 1. The tentative target values proposed in the background documentation of the Eurocode 1 are recommended here and are presented in Tables 2 and 3. These values shall be considered in reliability analyses in association with the stochastic models for the influencing variales as described in parts 2 and 3. In case of structures with extreme failure consequences the target values shall be defined based on risk-benefit studies.

Table 1: Examples of design lifetime

Type of structure	Design Lifetime in years
Temporary	<5
Renewable (silos, offshore)	<25
Ordinary buildings	40-75
Bridges	>100

Table 2: Tentative target reliabilty indices related to design life and to ultimate limit states

Relative Cost of Safety Measure	Minor consequences of failure	Moderate consequences of failure	Large consequences of failure
Low	**2.8**	**3.3**	**3.8**
Moderate	**3.3**	**3.8**	**4.3**
Large	**3.8**	**4.3**	**4.8**

Table 3: Tentative target reliabilty indices related to design life and to serviceability limit states

Relative Cost of Safety Measure	Target serviceability Index
Low	**1.0**
Moderate	**1.5**
Large	**2.0**

ANNEX A: THE ROBUSTNESS REQUIREMENT

A1. Introduction

In clause 3.1 the following robustment requirement has been formulated:

> *"A structure shall not be damaged by events like fire explosions or consequences of human errors, deterioration effects, etc. to an extend disproportionate to the severeness of the triggering event".*

This annex is intended to give some further guidance. No attention is being paid to terrorist actions and actions of war. The general idea is that, whatever the design, proper destructive actions can always be succesful.

A2. Structural and nonstructural measures

In order to attain adequate safety in relation with accidental loads one or more of the following strategies may be followed:

1. reduction of the probability that the action occurs or reduction of the action intensity (prevention)
2. reduction of the effect of the action on the structure (protection)
3. making the structure strong enough to withstand the loads
4. limiting the amount of structural damage
5. migitation of the consequences of failure

The strategies 1, 2 and 5 are so called non-structural measures. These measures are considered as being very effective for some specific accidental action.

The strategies 3 and 4 are so called structural measures. In general strategy 3 is extremely expensive in most cases. Strategy 4, on the other hand accepts some members to fail, but requires that the total damage is limited. This means that the structure should have sufficient redundancy and possibilities to mobilise so called alternative load paths.

In the ideal design procedure, the occurrence and effects of an accidental action (impact, explosion, etc.) are simulated for all possible action scenarios. The damage effect of the structural members is calculated and stability of the remaining structure assessed. Next the consequences are estimated in terms of number of casualties and economic losses. Various measures can be compared on the basis of economic criteria.

A3. Simplified design procedure

The approach sketched in A2 has two disadvantages:

(1) it is extremely complicated
(2) it does not work for unforseeable hazards

As a result other more global design strategies have been developed, like the classical requirements on sufficient ductility and tying of elements.

Another approach is that one considers the situation that a structural element (beam, column) has been damaged, by whatever event, to such an extend that its normal load bearing capacity has vanished almost completely. For the remaining part of the structure it then required that fore some relatively short period of time (repair period T) the structure can withstand the "normal" loads with some prescribed reliability:

$$P(R < S \text{ in } T \mid \text{one element removed}) < p_{target} \qquad \text{(A1)}$$

The target reliability in (A1) depends on:

- the normal safety target for the building
- the period under consideration (hours, days or months)
- the probability that the element under consideration is removed (by other causes then already considered in design).

The probability that some element is removed by some cause, not yet considered in design, depends on the sophistication of the design procedure and on the type of structure. For a conventional structure it should, at least in theory, be possible to include all relevant collapse origins in the design. Of course, it will always be possible to think of failure causes not covered by the design, but those will have a remote likelihood and may be disregarded on the basis of decision theoretical arguments. For unconventional structures this certainly will not be the case.

A4. Recommendation

For *unconventional* structures, as for instance large structures, the probability of having some unspecified failure cause is substantial. If in addition new materials or new design concepts are used, unexpected failure causes become more likely. This would indicate that for unconventional structures the simplified approach should be recommended.

For *conventional* structures there is a choice:

(1) one might argue that, as one never succeeds in dealing with all failure causes explicitly in a satisfactory way, it has no use to make refined analyses including system effect, accidental actions and so on; this leads to the use of the simplified procedure.
(2) one might also eliminate the use of an explicit robustness requirement (A1) as much as possible by taking into the design as many aspects explicitly as possible.

Stated as such it seems that the second approach is more rational, as it offers the possibility to reduce the risks in the most economical way, e.g. by sprinklers (for fire), barriers (for collision), QA (for errors), relief openings (for explosions), artificial damping (for earth quake), maintenance (for deterioration) and so on.

certain structure meets the reliability requirements. Note further that the probability of repair is given by:

$$P = P[Z_{ins} < 0]$$

Repair may be considered like some serviceability limit state. The designer should also make sure that the probability of repair is below some economic limit value.

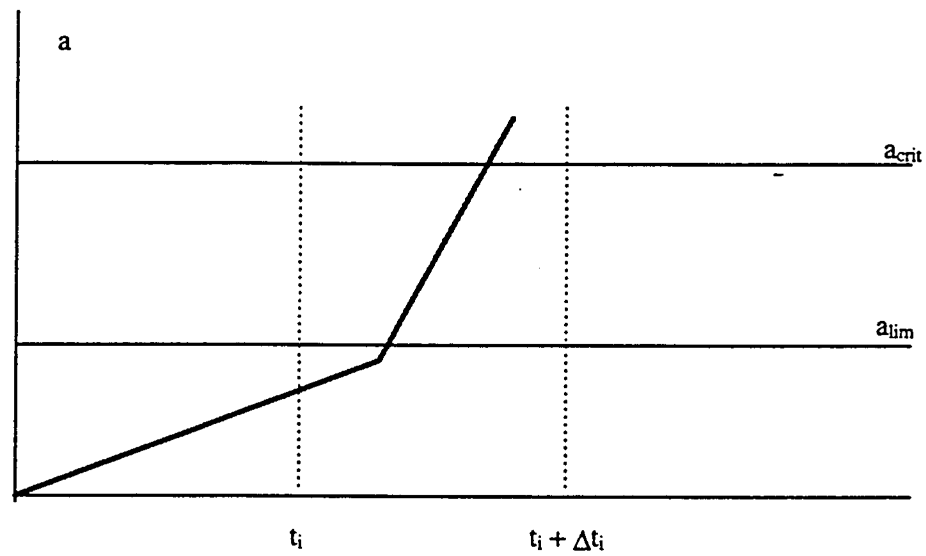

Figure B1: Fatigue failure in the interval t_i, $t_i + \Delta t_i$ with $a(\tau) < a_{lim}$ at the beginning of the interval.

JCSS-VROU-12-03-97

JCSS PROBABILISTIC MODEL CODE

PART 2: LOAD MODELS

Third draft

2.0: GENERAL PRINCIPLES

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2.0 GENERAL PRINCIPLES

2.0.1 Introduction

The environment in which structural systems function gives rise to internal forces, deformations, material deterioration and other short-term or long-term effects in these systems. The causes of these effects are termed actions. The environment from which the actions originate can be of a natural character, for example, snow, wind and earthquake. It can also be associated with human activities such as living in a domestic house, working in a factory, etc.

The following concepts of actions are used in this document.

1) An action is an assembly of concentrated or distributed forces acting on the structure. This kind of action is also denoted by "load".
2) An action is the cause of imposed displacements or thermal effects in the structure. This kind of action is often denoted by "indirect action".
3) An action is an environmental influence which may cause changes with time in the material properties or in the dimensions of a structure.

Action descriptions are in most cases based on suitably simple mathematical models, describing the temporal, spatial and directional properties of the action across the structure. The choice of the level of richness of details is guided by a balance between the quality of the available or obtainable information and a reasonably accurate modelling of the action effect. The choice of the level of realism and accuracy in predicting the relevant action effects is, in time, guided by the sensitivity of the implied design decisions to variations of this level and the economical weight of these decisions. Thus the same action phenomenon may give rise to several very different action models dependent on the effect and structure under investigation.

2.0.2 Classifications

Loads can be classified according to a number of characteristics. With respect to the type of the loads, the following subdivision can be made:

- self weight of structures
- occupancy loads in buildings, e.g. loads from persons and equipment
- actions caused by industrial activities, e.g. silo loads
- actions caused by transport: traffic, liquids in pipelines, cranes, impact, etc.
- climatic actions, e.g. snow. wind, outdoor temperature etc.
- hydraulic actions, e.g. water and ground water pressures
- actions from soil or rock, including earth quake

This classification does not cover all possible actions hut most of the common types of actions can be included in one or more classes. Some of the classes belong as a whole either to uncontrollable actions or to controllable actions. Other actions may belong to both e.g. water pressure.

With respect to the variations in time the following classification can be made:

- **permanent actions,** whose variations in time around their mean is small and slow (e.g. self weight, earth pressure) or which montonically a limiting value (C.g. prestressing, imposed deformation from construction processes, effects from temperature, shrinkage, creep or settlements)
- **variable actions,** whose variations in time are frequent and large (e.g. all actions caused by the use of the structure and by most of the external actions such as wind and snow)
- **exceptional** actions, whose magnitude can be considerable but whose probability of occurrence for a given structure is small related to the anticipated time of use. Frequently the duration is short (e.g. impact loads, explosions, earth and snow avalanches).

As far as the spatial fluctuations are concerned it is useful to distinguish between fixed and free actions. **Fixed actions** have a given spatial intensity distribution over the structure. They are completely defined if the intensity is specified in a particular point of the structure (e.g. earth or water pressure). For **free actions** the spatial intensity distribution is variable (e.g. regular occupancy loading, involved although they are variable actions.

2.0.3 Modelling of actions

There are two main aspects of the description of an action: one is the physical aspect, the other is the statistical aspect. In most cases these aspects can be clearly separated. Then the physical description gives the types of physical data which characterise the action model, for example, vertical forces distributed over a given area. The statistical description gives the statistical properties of the variables, for example, a probability distribution function. In some cases the physical and statistical aspects are so integrated that they cannot be considered separately.

A complete action model consists in general, of several constituents which describe the magnitude, the position, the direction, the duration etc. of the action. Sometimes there is an interaction between the components. There may in certain cases also be an interaction between the action and the response of the structure.

One can in many cases distinguish between two kinds of variables (constituents) Fo and W describing an action F (see also part 1, Basis of Design).

$$F = \varphi (F_O, W) \tag{2.0.3.1}$$

F_o is a basic action variable which is directly associated with the event causing the action and which should be defined so that it is, as far as possible, independent of the structure. For example, for snow load F_o is the snow load on ground, on a flat horizontal surface

W is a kind of conversion factor appearing in the transformation from the basic action to the action F which affects the particular structure. W may depend on the form and size of the structure etc. For the snow load example W is the factor which transforms the snow load on ground to the snow load on roof and which depends on the roof slope, the type of roof surface etc.

$\varphi(-)$ is a suitable function, often a simple product.

The time variability is normally included in F_o, whereas W can often be considered as time independent. A systematic part of the space variability of an action is in most cases included in W, whereas a possible random part may be included in F_o or in W. Eq. (2.0.3.1) should be regarded as a schematic equation. For one action there may be several variables F_o and several variables W.

Any action model contains a set of parameters and variables that must be evaluated before the model can be used. In **probabilistic** modelling all action variables are in principle assumed to be random variables or processes while other parameters may be time or spatial co-ordinates, directions etc. Sometimes parameters may themselves be random variables, for example when the model allows for statistical uncertainty due to small sample sizes.

An action model often includes two or more variables of different character as is described by eq. (2.0.3.1). For each variable a suitable model should be chosen so that the complete action model consists of a number of models for the individual variables.

These models may be described in terms of:

- stochastic processes or random fields
- sequences of random variables
- individual random variables
- deterministic values or functions

The definition of the models for these quantities require probability distributions (see annex 2) and a description of the correlation patterns.

2.0.4 Models for fluctuations in time

2.0.4.1 Types of models

To describe time depended loads, one needs the probability distribution for the "arbitrary point in time values" and a description of the variations in time. Some typical process models are (see figure 2.0.4.1):

a) Continuous and differentiable process
b) Random sequence
e) Point pulse process with random intervals
d) Rectangular wave process with random intervals
e) Rectangular wave process with equidistant intervals Δ

If the load intensities in subsequent time intervals of model (e) are independent, the model is referred to as a **FBC model** (Ferry Borges Castanheta model).

In many applications a combination of models is used, e.g. for wind the long term average is often modelled as an FBC model while the short term gust process is a continuous Gaussian process. Such models are referred to as **hierarchical models** (see Part 1, Basis of Design, Section 5.4). Each term in such a model describes a specific and independent part of the time variability. For a number of further definitions and notions, reference is made to Annex 1.

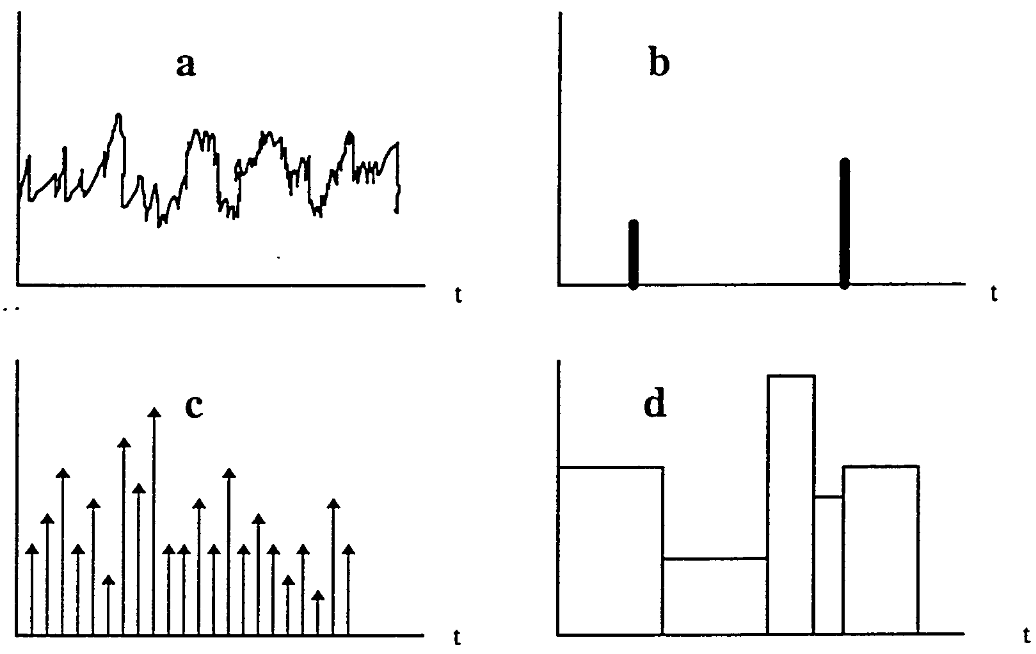

Figure 2.0.4.1: Various types of load models

2.0.4.2 Distribution of extremes for single processes

At the design the main interest is normally directed to the maximum value of the load in some reference period of time t_o. A quite general and useful upperbound formula to calculate the distribution of the maximum is given by:

$$F_{\max Q}(a) \cong \exp[-t_o \, v^+(a)] \tag{2.0.4.1}$$

The upcrossing frequency $v^+(a)$ is given by:

$$v^+(a) = P\{Q_t < a \text{ and } Q_{t+dt} > a\} / dt \tag{2.0.4.2}$$

For the FBC model $v^+(a)$ is simply given by:

$$v^+(a) = (1-F_Q(a))\, F_Q(a) / \Delta t \cong (1-F_Q(a))/\Delta t \tag{2.0.4.3}$$

And for a continuous Guassian process:

$$v^+(a) = \frac{1}{2\pi}\sqrt{-\rho''(0)} \quad \exp(-\beta^2/2) \tag{2.0.4.4}$$

where $\beta = (a-\mu(Q)) / \sigma(Q)$ and ρ = the correlation function.

2.0.4.3 Distribution of extremes for hierarchical processes

Consider the case that the load model contains slowly and rapidly varying parts, as well as random variables that are constant in time (see figure 2.0.4.2).

$$F = R + Q + S \qquad (2.0.4.5)$$

R = random variables, constant in time
Q = slow FBC process with mean renewal rate λ
S = fast varying process

In that case the following expression (see Annex 3, A.3.5) can be used:

$$F_{max\,Q}(a) = E_R\left[\exp[\lambda t_o[1 - E_Q \exp(-\Delta t\, \nu_s^+(a|RQ))]]\right] \qquad (2.0.4.6)$$

$\nu_s^+(a|RQ)$ = upcrossing rate of level "a" for process S, conditional upon R and Q
$\Delta t = 1/\lambda$ = time interval for the FBC process Q

E_R and E_Q denote the expectation operator over all variables R and Q respectively.

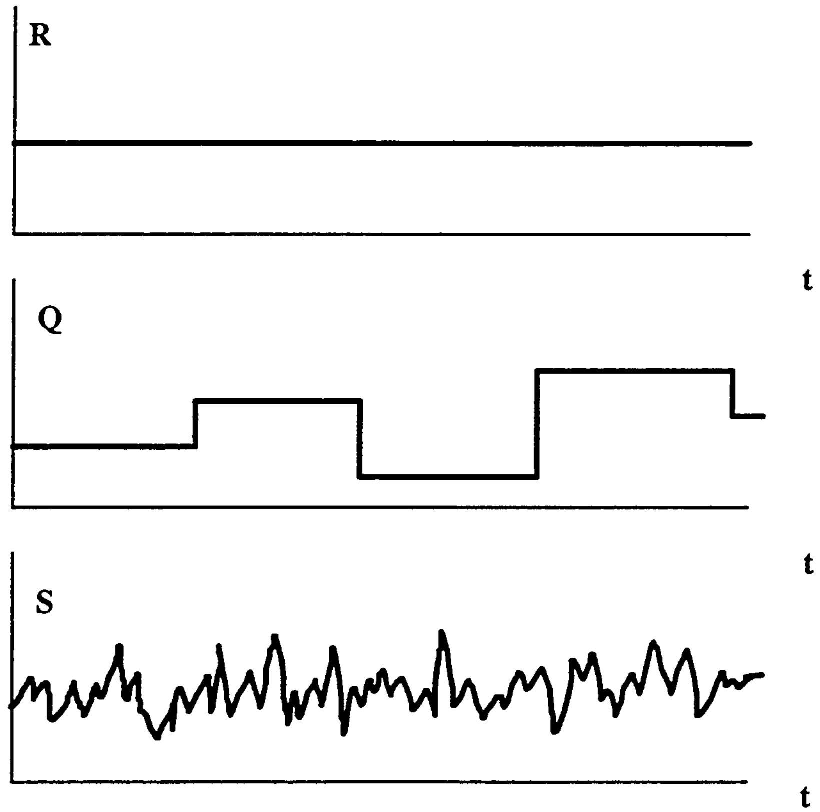

Figure 2.0.4.2: Hierarchical model for time dependent loads

2.0.5 Models for Spatial variability

2.0.5.1 Hierarchical models

As an example for the spatial modelling of actions using a hierarchical model consider the live load in an office building:

$$Q = m + \Delta Q_1 + \Delta Q_2 + \Delta Q_3(x,y) \quad (2.0.5.1)$$

where:

m is a general mean value for the whole population

ΔQ_1 is a stochastic variable which describes the variation **between** the load on different floors. The distribution function for ΔQ_1 has the mean value zero and the standard deviation σ_1

ΔQ_2 is a stochastic variable which describes the variation **between** the load in rooms on the same floor but with different floor areas. The distribution function for ΔQ_2 has the mean value zero and the standard deviation σ_2

ΔQ_3 is a random field which describes the spatial variability of the load **within** a room.

The total variability of the samples taken from the total population is described by $\Delta Q_1 + \Delta Q_2 + \Delta Q_3$. The variability **within** the subpopulation of floors is described by $\Delta Q_2 + \Delta Q_3$.

2.0.5.2 Equivalent uniformly distributed load (EUDL)

Consider a simple hierarchical distribution load model given by:

$$q(x,y) = q_o + q_{loc}(X,Y) \quad (2.5.0.2)$$

q_o = the variability between the various structures or structural elements
q_{loc} = the small scale or point to point fluctuation.

In many cases the random field q is replaced by a so called Equivalent Uniformly Distributed Load (EUDL). This load is defined as:

$$q_{EUDL}(t) = \frac{\int q(x,y,t) i(xy) dA}{\int i(xy) dA} \quad (2.5.0.3)$$

when i(x,y) is the influence function for some specific load effect (e.g. the midspan bending moment).

For given statistical properties of the load field q(x,y) the mean and standard deviation of Q_{EUDL} can be evaluated. For a homogeneous field, that is a random field where the statistical properties of q(x,y) do not depend on the location, we give here the resulting formulas:

$$\mu(qEUDL) = \mu(q_o) \quad (2.5.0.4)$$

$$\sigma+(Q_{EUDL}) = \sigma^2(q_o)+\sigma^2(q_{loc})\int\int\int\int i(x,y)(\xi,\eta)\rho(d)dxdyd\xi d\eta/[\int\int i(x,y)dxdy]^2 \quad (2.5.0.5)$$

Here $\rho(d)$ is the correlation function describing the correlation between the small scale load qlo, on the two points (x,y) and (ξ,η). This function may be of the form:

$$\rho(\Delta r) = \exp\{-\Delta r^2/d_c^2\} \quad (2.5.0.6)$$

with $\Delta r^2 = (X-\xi)^2 + (y-\eta)^2$, Δr being the distance between the two points, and d_c some scale distance. The correlation function tends to zero for distances Δr much larger than d_c.

If the field can be schematised as an FBC-field, the formula for $\sigma^2(q_{EUDL})$ can be simplified to:

$$\sigma^2(q_{EUDL}) = \sigma^2(q_i) + \sigma^2 + (q_{loc})\ \kappa A_o/A \quad (2.5.0.7)$$

Here A_o is the reference area and A stands for the total area under consideration, the so called tributary area.

The parameter κ is a factor depending on the shape of the influence line i(x,y). Values are presented in Figure 2.5.0.1. The figure $\kappa = 1$ corresponds to a constant value of i(x,y).

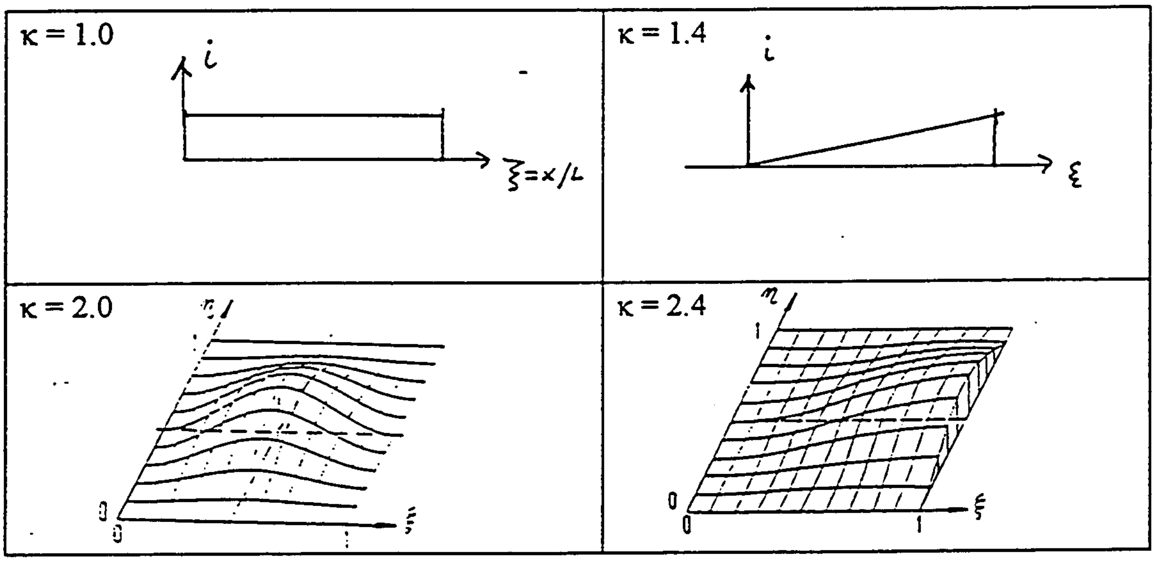

Figure 2.5.0.1: Random fields and corresponding κ-values.

2.0.6 Dependencies between different actions

Dependencies between different actions may occur for actions of different kinds, e.g. between actions due to snow and wind and different actions of the same kind e.g. between floor loads on different floors in a building.

In the first case there are generally no difficulties to distinguish between the different actions. In the second case one has to define what is **one** action. If the loads on several floors in a building are of the same character, e.g. loads in offices, the loads on these floors may be regarded as **one** entity and the mutual dependency between the loads on different parts of the floors is described by the correlation structure of the load.

In the following only dependencies between different actions are treated. The cause of dependency between actions can normally be found in the causal events behind the actions. One can distinguish two cases:

- There is a common causal event which cause two or more actions. One example can be an earthquake which, due to rupture of gas pipes may lead to fires and explosions.

- There is a sequence of causal events where one event causes the following and where each event is the cause of an action. One example is the event of a storm which causes the event of waves. The storm causes wind forces and the waves cause wave forces. Another, more complex example could be traffic and wind on a bridge: traffic leads to higher wind loading (greater loaded area), but a heavy storm may lead to a reduction of the traffic.

The mathematical description of the dependencies between various actions depend on the nature of the physical relationship and the nature of the processes themselves. For two stationairy continous gaussian processes x(t) and y(t) the correlation is normally described by the cross correlation function $R_{xy}(\tau)$ or the alternatively by the cross spectrum $S_{xy}(\omega)$. For pulse type processes we may have to distinguish between the correlation in arrival time and and the correlation in amplitude.In many cases it may be convenient to define one of the processes as the "leading one" and describe arrival times and amplitudes of the second process conditional upon the occurrence and amplitude of the first one. So one may model the probability of a fire given an earth quake of a certain intensity.

In this model code none or little guidance is presented to this matter. However, the user of this model code is always entitled to be aware of these possible correlations and interactions.

2.0.7 Combinations of actions

From a mathematical modelling point of view the load on a structure is a joint set Q(t) of of time varying random fields. This set of loads gives a scalar load effect S(t) in a given cross section or point of the structure at time t as a function of Q(t) (i.e. a random process). In the simplest case we have:

$$S(t) = c_1 Q_1(t) + c_2 Q_2(t) + .. \qquad (2.0.7.1)$$

The reliability problem related to the considered point is to evaluate the probability P_f that S(t) satisfies $S_L(t) < S(t) < S_U(t)$ for all future time where $S_L(t)$ and $S_U(t)$ are bounds defined by the strength properties at the considered point and limit state. By allowing the possibility that $S_l(t) = -\infty$ and $S_u(t) = \infty$ for some t, focus on a time interval of finite duration is included in the formulation.

The load combination problem is to formulate a reasonably simple but for the considered engineering purpose sufficiently realistic mathematical model that defines Q(t). The needed level of detailed modelling of Q(t) depends on the filtering effect of the function that maps Q(t) into the load effect S(t). This filtering effect is judged under due consideration of the sensitivity of the probability p_f to the detailing. The sensitivity question is tied to the last part of the load combination problem which is actually to compute the value of P_f. Thus, to be operational, the modelling of Q(t) should be simple enough to make practicable at least a computer simulation of the scalar process S(t) to an extend that allows an estimation of P_f.

First the relevant set of different action types is identified. This identification defines the number of elements in the set Q(t) and the subdivision of Q(t) into stochastically independent subsets. The modelling is next concentrated on each of these subsets with dependent components.

The mathematical difficuty to solve probabilities for outcrossing rates of the type (2.0.7.1) is the possible very different nature of the various contributors Q_i . Each of these processes may be of a completely different nature, including all kinds of contiuous and intermittent processes. Numerical solutions will often prove to be necessary, but also analytical solutions may prove to be very helfpul [Rackwitz..].

ANNEX 1 - DEFINITIONS

Covariance function

The covariance function r (t_1, t_2) is defined by:

$$r(t_1, t_2) = E[(Q(t=) - m_1)(Q(t_2) - M_2)]$$

$$m_1 = E[Q(t_l)] \qquad m_2 = E[Q(t_2)]$$

Stationary processes

The process is defined for $-\infty < t < \infty$. If, for all values t_1 and for all values τ, chosen such that $0 \leq t_i \leq$ to and $0 \leq t_i + \tau \leq t_o$, the stochastic variable x $(t_i + \tau)$ has the same distribution function as the stochastic variable x (t_i) the stochastic process x (t) is stationary.

If the mcan value function m (t) is constant and the covariance function r (t_1, t_2) depends solely on the difference $\tau = (t_2 - t_l)$ the process is sald to be wide-sense stationary.

Thus the covariance function for a stationary or a wide sense stationary process may be written

$$r(\tau) = E[(Q(t + \tau) - m)(Q(t) - m)]$$

The concept of stationary applied to action processes should in most cases be interpreted as wide-sense stationary.

Ergodic processes

A process is ergodic if averaging over several realisations and averaging with respect to time (or another index parameter) give the same result.

For ergodic processes a relation between the point-in-time value distribution function F and the excursion time t is determined for a chosen reference period t_o, by

$$1 - F_Q(Q) = t/t_o$$

The correlation function

The correlation function for a stationary process is:

$$\rho(\tau) = \frac{r(\tau)}{r(0)}$$

For ergodic processes $\rho(\tau = \infty) = 0$

Spectrum

A stationary stochastic process may be characterised with ald of a spectrum:

$$S(n) = \int_{-\infty}^{\infty} e^{-i2\pi n\tau} r(\tau) d\tau$$

S(n) may be regarded as a measure of how the process is built up of components with different frequences. The total variance of the process is:

$$\mathrm{Var}\, Q = 2\int_{0}^{\infty} S(n) dn$$

Gaussian processes

A stochastic process Q(t) is a Gaussian process if the multidimensional probability distribution functions for all the stochastic variables Q (t_i) are Gaussian. The stochastic properties of a Gaussian process is completely determined by the mean value and the covariance function or by the spectrum.

Scalar Nataf Processes

A special but important class of non-Gaussian, scalar and differentiable processes are built by a memoryless transformation from a normal process, i.e.

$$S(t) = h(U(t))$$

where U(t) is a standard normal process and h(u) is an arbitrary function. For S(t) any admissible (unimodal) distribution function can be chosen thus defining a certain class of functions h(u). In addition the autocorrelation function $\rho_s(t_1,t_2)$ has to be specific. However, there are some restrictions on the type of autocorrelation function. Many results for Nataf processes can be found in Grigoriu (1995).

Scalar Hermite Processes

The Hemite process is a special case of the Nataf process. All marginal distribution must be of Hermite type. For this process the solution of the integral equation occurring for the autocorrelation function of the equivalent (or better generating) standard normal process is analytic. The standard Hermite process has the representation, i.e. a special case of the function h(u)

$$S(t) = \kappa(U(t) + \tilde{h}_{3,i}(U(t)^2 - 1) + \tilde{h}(U(t)^3 - 3U(t)))$$

For the coefficients depending on the first four moments of the marginal distribution of the non-normal process. In addition, the Hermite process requires specification of the autocorrelation function of S(t). Again, there are certain restrictions on the moments of the marginal distributions as well as on the autocorrelation function.

Scalar Rectangular Wave Renewal Processes

Scalar rectangular wave renewal processes are useful models for processes changing their amplitude at random renewal points in a random fashion. A scalar rectangular wave renewal process is characterised by the jump rate λ, and the distribution function of the amplitude. The renewals occur independently of each other. No specific distribution is assigned to the interarrival times. Therefore, the renewal process characterised only by a jump rate captures only long term statistics. The mean duration of pulses is asymptotically equal to $1/\lambda_{,}$. For the special case of a Poisson rectangular wave process the interarrival times and so the durations of the pulses are exponentially distributed with parameter $1/\lambda_{.}$. In the special case of a Ferry Borges-Castanheta process the durations are constant and the repetition number $r = (t_2 - t_1)/\Delta$ with Δ the duration of pulses is equal to $\lambda(t_2 - t_1)$. Also, the sequence of amplitudes is an independent sequence.

The jump rate can be a function of time as well as the parameters of the distribution function of the amplitudes.

It is assumed that rectangular wave processes jump from a random value S(t) to a new value $S^{+}(t+\delta)$ with $\delta \rightarrow$ at a renewal without returning to zero. Rectangular wave renewal procesesses must be regular processes, i.e. the occurrence of any two or more renewals in a small time interval must be negligible (of o-order). Non-stationary rectangular wave renewal processes are processes which have either time-dependent parameters of the amplitude distributions and/or time-dependentjump rates.

Random fields

A random field may be regarded as a one-, two- or three-dimensional stochastic process. The time t is substituted by the space co-ordinates x, y, z.

For the two-dimensional case the covariance function is written (for a stationary random field)

$$r\,(d_x, d_y) = E\,[(Q\,(x + d_x, y + d_y) - m)\,(Q\,(x, y) - m)]$$

The concepts of stationary, ergodicy etc. are in principle the same as for the stochastic processes.

ANNEX 2 - DISTRIBUTIONS FUNCTIONS

Distribution type	Parameters	Moments
Rectangular $a \le x \le b$ $f_X(x) = \frac{1}{b-a}$	$1 = a$ $2 = b$	$m = \frac{a+b}{2}$ $s = \frac{b-a}{\sqrt{12}}$
Normal $\sigma > 0$ $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$	$1 = \mu$ $2 = \sigma$	$m = \mu$ $s = \sigma$
Lognormal $x > 0, \zeta > 0$ $f_X(x) = \frac{1}{x\zeta\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln x - \lambda}{\zeta}\right)^2\right)$	$1 = \lambda$ $2 = \zeta$	$m = \exp\left(\lambda + \frac{\zeta^2}{2}\right)$ $s = \exp\left(\lambda + \frac{\zeta^2}{2}\right)\sqrt{\exp(\zeta^2) - 1}$
sLognormal $x > \varepsilon, \zeta > 0$ $f_X(x) = \frac{1}{(x-\varepsilon)\zeta\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln(x-\varepsilon) - \lambda}{\zeta}\right)^2\right)$	$1 = \lambda$ $2 = \zeta$ $3 = \varepsilon$	$m = \varepsilon + \exp\left(\lambda + \frac{\zeta^2}{2}\right)$ $s = \exp\left(\lambda + \frac{\zeta^2}{2}\right)\sqrt{\exp(\zeta^2) - 1}$
sExponential $x \ge \varepsilon, \lambda > 0$ $f_X(x) = \lambda \exp(-\lambda(x-\varepsilon))$	$1 = \varepsilon$ $2 = \lambda$	$m = \varepsilon + \frac{1}{\lambda}$ $s = \frac{1}{\lambda}$
Gamma $x \ge 0, b > 0, p > 0$ $f_X(x) = \frac{b^p}{\Gamma(p)} \exp(-bx) x^{p-1}$	$1 = p$ $2 = b$	$m = \frac{p}{b}$ $s = \frac{\sqrt{p}}{b}$
Beta $a \le x \le b,\ r, t \ge 1$ $f_X(x) = \frac{(x-a)^{r-1}(b-x)^{t-1}}{(b-a)^{r+t-1} B(r,t)}$	$1 = a$ $2 = b$ $3 = r$ $4 = t$	$m = a + (b-a)\frac{r}{r+t}$ $s = \frac{b-a}{r+t}\sqrt{\frac{rt}{r+t+1}}$
Gumbel (L) $-\infty < x < +\infty, \alpha > 0$ $f_X(x) = \alpha \exp(-\alpha(x-u) - \exp(-\alpha(x-u)))$	$1 = u$ $2 = \alpha$	$m = u + \frac{0.577216}{\alpha}$ $s = \frac{\pi}{\alpha\sqrt{6}}$
Frechet (L) $\varepsilon \le x < +\infty, \quad u, k > 0$ $f_X(x) = \frac{k}{u-\varepsilon}\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{-k-1} \exp\left(-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{-k}\right)$	$1 = u$ $2 = k$ $3 = \varepsilon$	$m = \varepsilon + (u-\varepsilon)\Gamma\left(1 - \frac{1}{k}\right)$ $s = (u-\varepsilon)\sqrt{\Gamma\left(1 - \frac{2}{k}\right) - \Gamma^2\left(1 - \frac{1}{k}\right)}$
Weibull (S) $\varepsilon \le x < +\infty, \quad u, k > 0$ $f_X(x) = \frac{k}{u-\varepsilon}\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{-k-1} \exp\left(-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^{-k}\right)$	$1 = u$ $2 = k$ $3 = \varepsilon$	$m = \varepsilon + (u-\varepsilon)\Gamma\left(1 + \frac{1}{k}\right)$ $s = (u-\varepsilon)\sqrt{\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)}$

ANNEX 3 MATHEMATICAL TECHNIQUES

Ferry Borges-Castanheta model

Consider the case that two actions Q_1 (t) and Q_2 (t) are to be combined. Assume that these actions can be described as FBC-models (Figure A3.1). The following assumptions are made about the processes:

- Q_1 (t) and Q_2 (t) are stationary ergodic processes
- All intervals τ_1 are equal; all intervals τ_2 are equal
- $\tau_1 \geq \tau_2$
- Q_1 and Q_2 are constant during each interval τ_1 and τ_2 respectively
- The values of Q_1 for the different intervals are mutually independent; same for Q_2
- Q_1 and Q_2 are independent

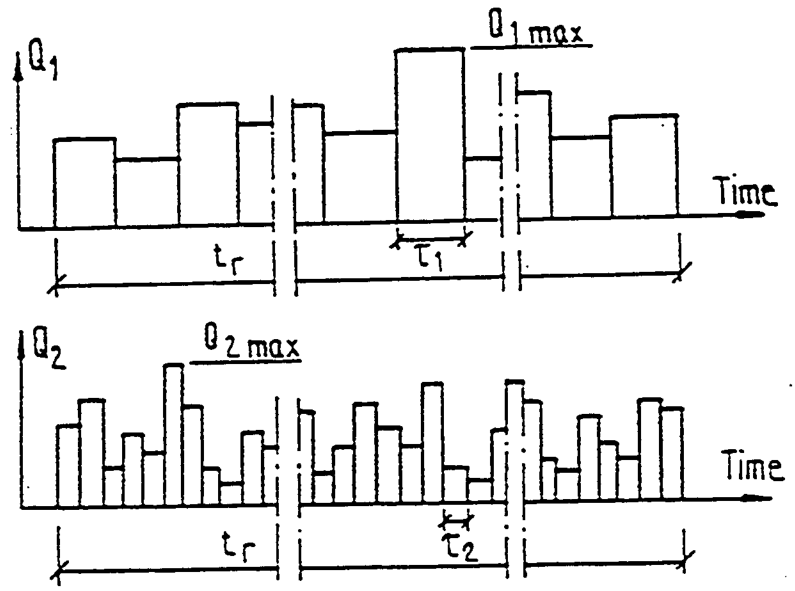

Figure A3.1: Square wave processes for Q_1 (t) and Q_2 (t)

Define Q_{2c} as the maximum value of Q_2 occurring during the interval τ_1 with the probability distribution function:

$$F_{Q2c}(Q) = \left[F_{Q^*}(Q)\right]^{\tau 2/\tau 1}$$

F_Q^* = the arbitrary point in time distribution for Q_2 (A3.1)

Assume a linear relationship between the actions effect S and the actions:

$$S = c_1\, Q_1 + c_2\, Q_2 \qquad \text{(A3.2)}$$

The maximum action effect S_{max} from Q_1 and Q_2 during the reference period t_o can then be written as:

$$S_{max} = \max \{c_1 Q_1 + c_2 Q_{2c}\} \qquad \text{(A3.3)}$$

The maximum should be taken over all intervals τ_1 within the reference period t_o.

Turkstra rule

As an approximation, the resulting action effects could be calculated as the maximum of the following two combinations (Turkstra's rule):

- $S\{Q_{1\max}, Q_{2c}\}$ if Q_1 is considered as the dominating action
- $S\{Q_{2\max}, Q_{1c}\}$ if Q_2 is considered as the dominating action

Written as a formula for the case $S = c_1 Q_1 + c_2 Q_2$

$$S_{max} = \max \{c_1 Q_{max} + c_2 Q_{2c}; c_1 Q_{1c} + c_2 Q_{2,max}\} \qquad \text{(A3.4)}$$

It should be noted that the Turkstra Rule gives a lower bound for the failure probability.

Oucrossing approach

Consider the event that a realization $z(\tau)$ of a random state vector $Z(\tau)$ representative for a given problem, enters the failure domain

$$V = \{Z(\tau) \mid g(z(\tau),\tau) < 0, 0 < \tau < t\};$$

where g(.) is the limit state function. $Z(\tau)$ may conveniently be separated into three components as:

$$Z(\tau)^T = (R^T, Q(\tau)^T, S(\tau)^T)$$

where R is a vector of random variables independent of time t, $Q(\tau)$ is a slowly varying random vector sequence and $S(\tau)$ is a vector of not necessarily stationary but sufficiently mixing random process variables having fast flunctuations as compared to Q(t).

In the general case where all the different types of random variables R, $Q(\tau)$ and $S(\tau)$ are present, the failure probability $P_f(t)$ not only must be integrated over the time invariant variables R, but an expectation operation must also be performed over the slowly varying variables $Q(\tau)$:

$$P_f(t_{min}, t_{max}) \approx 1 - E_R[\exp[nE_Q[1 - \exp(-E[N^+(\Delta t, R, Q))]]]] \qquad \text{(A3.5a)}$$

Dt is the characteristic fluctuation time of Q and $n = (t_{max} - t_{min}) / \Delta t$

Or, one step further simplified:

$$P_f(t_{min}, t_{max}) \approx 1 - E_R[\exp[-E_Q[E[N^+(t_{min}, t_{max}; R, Q)]]]] \quad \text{(A3.5b)}$$

It should be observed that the expectation operation with respect to Q is performed inside the exponent, whereas the expectation operation with respect to R is performed outside the exponent operator. If the point process of exits is a regular process which can be assumed in most cases, the conditional expectation of the number of exits in the time interval [t_{min}, t_{max}] can be determined from (see [..]):

$$E[N^+(t_{min}, t_{max}; r, q)] = \int_{t_{min}}^{t_{max}} \nu^+(\tau; r, q) d\tau \quad \text{(A3.6)}$$

where $\nu^+(\tau;p,r.q)$ is the outcrossing rate defined by:

$$\nu^+(\tau; r, q) = \lim_{\Delta \to 0} \frac{1}{\Delta} P(N^+(\{S(\tau) \in \bar{V}\} \cap \{S(\tau + \Delta) \in V\} | r, q) \quad \text{(A3.7)}$$

If the vector $\underline{S}$ consists out of n components (S_1, ... S_n), all of ractangular wave type, the following formula can be used:

$$\nu^+ = \sum_{i=1}^{n} \nu_i P\{(S_1, S_2, \ldots S_i, \ldots S_n) \in \bar{V}) \cap (S_1, S_2, \ldots S_i^+, \ldots S_n) \in V\} \quad \text{(A3.8)}$$

where S_i^- and S_i^+ are two realisations of S_i, one before and one after some particular jump and ν_i is the jump rate of S_i.

Example:

TO BE COMPLETED

Intermittent processes

Intermittent processes are a practically important generalisation for all types of random processes. Although more general forms are known only the simplest type of intermittancies is discussed below. The renewals of times where the process is "on" follow a Poisson renewal process with rate κ (or mean interarrival time l/κ). At a renewal the process activates an "on"-state (state "1"). The "off"-states are denoted by "0". The initial durations of "on"-states will have exponential distribution with mean l/μ independent of the arrival times. However, we will assume that a "on"-time is also finished if a next renewal occurs so that the durations have a truncated distribution. By assuming random initial conditions the probabilities of the "on/off-states are then determined by

$$p_{off}(t) = \frac{\mu}{\kappa + \mu} + \frac{\kappa - \mu}{\kappa + \mu} \exp[-(\kappa + \mu)t] \quad \text{(A.3.9)}$$

$$p_{on}(t) = \frac{\mu}{\kappa + \mu} + \frac{\kappa - \mu}{\kappa + \mu} \exp[-(\kappa + \mu)t] \quad \text{(A.3.10)}$$

In general it is assumed that the "on/off"-process is already in its stationary state where the last terms in these equations vanishes. In contrast to rectangular wave renewal processes where the duration of the rectangular pulse is exactly until the next renewal and the duration of the rectangular pulse is exponentially distributed with mean $1/\lambda$ for a Poissonian renewal process the "on"-times are now truncated at the next renewal. It is easily shown that the effective duration of the "on"-times then are also exponential but with mean $1/(\kappa+\mu)$. The so-called interarrival-duration intensity is defined by $\rho = \kappa/\mu$. For $\rho = \kappa/\mu \rightarrow \infty$ the processes are almost always active. For $\kappa/\mu \rightarrow 0$ one obtains spike-like processes.

Intermittancies can also be defined for differentiable processes. If this is a dependent vector process the entire vector process must have a common ρ, that is all components of the vector must have the same κ and μ. Independent differentiable vector processes, however, can have different ρ's.

In the case of a *single* intermittent process with $\kappa t_0 > 1$ and $\mu t_o << 1$ the periods where the intermittent load are present can conveniently be put together. The failure probability is then approximately given by:

$$P_f(t_{min}, t_{max}) = \nu_{failure\ lintermittent\ load\ off}\ (T / t_o) + \nu_{failure\ lintermittent\ load\ on}\ (to - T) / t_o \qquad (A.3.11)$$

where $T = \kappa\, t_0 / \mu$ the total expected time that the intermittent load is active on and $t_o = t_{max} - t_{min}$

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PROBABILISTIC MODEL CODE
PART 2 - LOAD MODELS

2.1 SELF WEIGHT

Table of contents:

List of symbols:

d	correlation length
V	volume described by the boundary of the structural part
γ	weight density of the material.
$\bar{\gamma}$	average weight density for a structural part
ρ_o	correlation between two far away points in one member
Δr	distance between two points within a member

2.1.1 Introduction

The self weight concerns the weight of structural and non-structural components. The main characteristics of the self weight can be described as follows:

- The probability of occurrence at an arbitrary point-in-time is close to one
- The variability with time is normally negligible
- The uncertainties of the magnitude is normally small in comparison with other kinds of loads.

Concerning the uncertainties one can distinguish between (hierarchical model):

- variability **within** a structural part
- variability **between** different structural parts of the same structure
- variability **between** various structures

The variability within a structural part is normally small and can often be neglected. However, for some types of problem (e.g. static equilibrium) it may be important.

2.1.2 Basic model

The self weight, G, of a structural part is determined by the relation

$$G = \int_{Vol} \gamma \, dV \qquad (1)$$

where:

V is the volume described by the boundary of the structural part. The volume of V is Vol.
γ is the weight density of the material.

For a part where the material can be assumed to be reasonably homogeneous eq. (1) can be written

$$G = \bar{\gamma} \, V \qquad (2)$$

where

$\bar{\gamma}$ is an average weight density for the part (see further section 2.1.4).

2.1.3 Probability density distribution functions

The weight density and the dimensions of a structural part are assumed to have Gaussian distributions. To simplify the calculations the self weight, G, may as an approximation be assumed to have a Gaussian distribution.

2.1.4 Weight density

Total variability

Mean values, μ_γ, and coefficients of variation, V_γ, for the total variability of the weight density of some common building materials are given in table 2.1.1.

Table 2.1.1. Mean value and coefficient of variation for weight density [1)]

Material	Mean value [kN/m³]	Coefficient of variation
Steel	77	< 0.01
Concrete		
Ordinary concrete [2)] f_c = 20 MPa	23.5	0.04
40 MPa	24.5	0.03
Lightweight aggregate concrete	[4)]	0.04-0.08
Cellular concrete	[4)]	0.05-0.10
Heavy concrete for special purposes	[4)]	0.01-0.02
Masonry	-	≈ 0.05
Timber [3)]		
Spruce, fir (Picea)	4.4	0.10
Pine (Pinus)	5.1	0.10
Larch (Larix)	6.6	0.10
Beech (Fagus)	6.8	0.10
Oak (Quercus)	6.5	0.10

[1)] The values refer to large populations. They are based on data from various sources.

[2)] The values are valid for concrete without reinforcement and with stable moisture content. In case of continuous drying under elevated temperature the stable volume weight after 50 days is 1.0-1.5 kN/m³ lower.

[3)] Moisture content 12%. An increase in moisture content from 12% to 22% causes a 10% rise in weight density.

[4)] Depends on mix, composition and treatment

Correlations

Between densities of two points within one member, the following correlation can considered to be present:

$$\rho(\Delta r) = \rho_o + (1-\rho_o) \exp\{-(\Delta r/d)^2\} \tag{3}$$

where

d is a so called correlation length which characterises the correlation structures
Δr is the distance between two points within a member
ρ_o correlation between two far away points in one member

Only correlation in the length dimensions of a structural part are of importance. For beams the weight density over the cross section and for plates over the height may be considered as fully correlated.

Between points in two different members, but within one building, a constant correlation ρ_m is assumed to be present.

In the absence of more detailed information the following values can be used:

d	5 m
ρ_o	0.85
ρ_m	0.70

Note: For large members the variability of the weight density may be taken as V ρ_o; for a whole structure consisting out of many members the variability may be taken as V ρ_m , where V is the total variability according to table 2.1.1.

2.1.5 Volume

In most cases it may be assumed that the mean values of the dimensions are equal to the nominal values i.e. the dimensions given on drawings, in descriptions etc. The mean value of the volume, V, of the structural parts is calculated directly from the mean values of the dimensions.

The standard deviation of the volume, V, is calculated directly from the values of the standard deviation for the dimensions. Standard deviations for cross section dimensions are given in table 2.1.2 for some common building materials and types of structural elements.

Table 2.1.2. Mean values and standard deviations for deviations of cross-section dimensions from their nominal values.

Structure or structural member	Mean value	Standard deviation
Rolled steel		
steel profiles, area A	$0.01\ A_{nom}$	$0.04\ A_{nom}$
steel plates, thickness t	$0.01\ t_{nom}$	$0.02\ t_{nom}$
Concrete members [2]		
$a_{nom} \leq 1000$ m	$0.003\ a_{nom}$	$4 + 0.006\ a_{nom}$
$a_{nom} \geq 1000$ m	3 mm	10 mm
Masonry members		
unplastered	$0.02\ a_{nom}$	$0.04\ a_{nom}$
plastered	$0.02\ a_{nom}$	$0.02\ a_{nom}$
Structural timber		
sawn beam or strut	$0.05\ a_{nom}$	2 mm
laminated beam, planed	≈ 0	1 mm

1) The values refer to large populations. They are based on data from various sources and they concern members with currency acceptation dimension accuracy.

2) The values are valid for concrete members cast in situ. For concrete members produced in a factory the deviations may be considerably smaller.

The variability within a component (e.g. the variability of the cross section area along a beam) may be treated according to the same principles that is presented for the weight density in section 2.1.4.

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2.6 LOADS IN CAR PARKS

Table of contents:

List of symbols:

i = influence coefficient
L = weight of car in kN
S = load effect
T = reference time in years
t_y = busy days per year
t_d = busy time per day
λ_d = renewal rate in [1/d]
τ = mean dwell time in hours

2.6.1. Basic Model

In car parks the loads on parking areas and drive ways may be distinguished. In general, the loads for regulated parking are dominating the loads for spatially free parking. Further, the entries and parking places are such that only certain categories of vehicles can use the facility. It is sufficient to distinguish between facilities for light vehicles like normal passenger cars, station wagons and vans and for heavy vehicles like trucks and busses. For each parking facility it can conservatively be assumed that the vehicles form an independent sequence each vehicle with random weight remaining the same at arrival and when leaving the place. At the beginning of the busy period of a day the parking facility will be filled up with cars and emptied at the end of the busy period. During the busy periods it can conservatively be assumed that parking places left by a car will immediately be occupied by another car. Thus, the loading process due to vehicles is a rectangular wave renewal process.

2.6.2 Stochastic Model

With respect to the temporal fluctuations one can distinguish the following usage categories for light vehicles:

- car parks belonging to residential areas
- car parks belonging to factories, offices etc.
- car parks belonging to commercial areas
- car parks belonging to assembly halls, sport facilities etc.
- car parks connected with railway stations airports etc.

The temporal fluctuations are summarized in table 1.

For parking facilities for heavy vehicles similar distinctions can be made.

The mean weight of light vehicles can be assumed to be about $E[L] \approx 15$ kN with coefficient of variation of 15 to 30 % depending on the usage of the parking facility and the traffic mixture. The parking place covers an area of about $2.4 \cdot 5.0$ m^2. A normal distribution can be assumed. In general light vehicles can be modeled by point loads located in the middle of the parking places.

Location of car park	Busy days per year t_y[d]	Busy time per day t_d[h]	Mean dwell time τ[h]	Number of cars per day λ_d[1/d]
Commercial areas	312	8 4	2.4	3.2
Railway stations airports	360	14-18	10-14	1.3
Assembly halls	50-150	2.5	2.5	1.0
Offices, factories	260	8-12	8-12	1.0
Residential areas	360	17	8	2.1

Table 1: Temporal fluctuation in car parks

Calculation of load effects has to take proper account of influence functions according to

$$S(t) = \sum_{j=i}^{n} i_j L_j \tag{1}$$

If the negative parts of the influence functions can be neglected [ggg] the distribution of extreme load effects can be computed from

$$F_{\max\{S\}}(x) \approx \exp\left[-\lambda_d \; t_y t_d T P\left(\sum_{j=1}^{n} i_j L_j \geq\rangle x\right)\right] \tag{2}$$

with

$$P\left(\sum_{j=1}^{n} i_j L_j \rangle x\right) \approx \Phi\left(-\frac{x - \left(\sum_{j=1}^{n} i_j E\left[L_j\right]\right)}{\left(\sum_{j=1}^{n} i_j Var\left[L_j\right]\right)^{1/2}}\right) \tag{3}$$

T is the reference time in years. On drive ways where only one vehicle determines the load effect one has

$$F_{\max\{S\}}(x) \approx \exp\left[-\lambda_d \; t_y T \; N\left\{1 - \Phi\left(\frac{x - E[L]}{Var[L]}\right)\right\}\right] \tag{4}$$

where N is the number of parking places associated with the drive way.

References

CIB W81, Actions on Structures: Live Load in Buildings, Rep. N0. 116, Rotterdam, 1989

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PART 2: LOAD MODELS

2.12 SNOW LOAD

Table of contents:

List of symbols:

C_e = exposure coefficient
C_r = redistribution (due to wind) coefficient
C_t = deterministic thermal coefficient
d = snow depth
h = altitute of the building site
h_r = reference altitude
k = coefficient for altitude conversion
r = conversion factor of snow load on ground to snow load on roofs
S_r = snow load on the roof
S_g = snow load on ground at the weather station
$\gamma(d)$ = average weight density of the snow for depth d
η_a = shape coefficient

2.12 SNOW LOAD

2.12.1. Basic Model for Snow Load on roofs

The snow load on roofs, S_r, is determined by the relation

$$S_r = S_g \; r \; k^{h/h_r} \tag{1}$$

where

S_g is the snow load on ground at the weather station
r is a conversion factor of snow load on ground to snow load on roofs (see 2.12.3).
h is the altitute of the building site
h_r is a reference altitude (= 300 m)
k is a coefficient: k = 1.25 for coastal regions, k = 1.5 for inland mountainous regions

The snow load S_r acts vertically and refers to a horizontal projection of the area of the roof. S_g is time dependent but not space dependent within a specified region with similar climatic conditions and with approximately the same altitude.

The characteristics of the ground snow load S_g should be determined on the basis of observations from weather stations. The results of such observations are either water-equivalents of snow or depths of snow. In the first case the values can be used directly to determine the ground snow load. In the second case the data on snow depth must be converted to snow load by the relation

$$S_g = d \; \gamma(d) \tag{2}$$

where

d is the snow depth
$\gamma(d)$ is the average weight density of the snow

The density $\gamma(d)$ follows from:

$$\gamma(d) = \frac{\lambda\gamma(\infty)}{d} \ln\left\{1 + \frac{\gamma(0)}{\gamma(\infty)}[\exp(\frac{d}{\lambda}) - 1]\right\} \tag{3}$$

where

$\gamma(\infty) = 5$ kN/m^3, $\gamma(0) = 1.70$ kN/m^3 and x = 0.85 m

2.12.2. Probabilistic model for S_g

A probability model of the ground snow load S_g is presented by:

- a probability distribution function for the total duration T of the load
- a probability distribution function for the maximum load S_{gmax} within one year.

The distribution function $F_{sg\,max}$, its mean μ and its coefficient of variation V are denoted as:

for maritime climate : F_{s1} , μ_1 , V_1
for continental climate : F_{s2} , μ_2, V_2

The probability distribution functions in these two cases are gamma distributions. The parameters should be based on local observations. As prior distribution a vague prior should be used. In some cases data from "similar stations" can be used as prior with n' = 3 and v' = 2.

In those cases when the climate is a mixture of maritime and continental climate, a part p of the observations are associated with a continental climate and a part 1-p with a maritime climate. The combined probability distribution function F for the mixed climates can then be written as $F_s = (1-p)F_{s1} + p F_{s2}$.

2.12.3. Conversion ground to roof snow load

2.12.3.1 General

The conversion factor r is subdivided into a number of factors and terms according to the expression

$$r = \eta_a C_e C_t + C_r \qquad (6)$$

where

η_a is a shape coefficient, a random variable according to 2.12.3.2
C_e is a deterministic exposure coefficient according to 2.12.3.2
C_t is a deterministic thermal coefficient according to 2.12.3.3
C_r is a redistribution (due to wind) coefficient, a random variable according to 2.12.3.4. If redistribution is not taken into account $C_r = 0$

2.12.3.2 The exposure coefficient C_e and shape factor η_a

The exposure coefficient, C_e and the shape factor η_a are a reduction coefficients taking account of the exposure to wind of a building and the slope of the roof α:

$\alpha = 0°$	$C_e\eta_a = 0.4 + 0.6 \exp(-0{,}1\ u(H))$
$\alpha = 25°$	$C_e\eta_a = 0.7 + 0.3 \exp(-0{,}1\ u(H))$
$\alpha = 60°$	$C_e\eta_a = 0$

u(H) is the wind speed, averaged over a period of one week, at roof level H.
For intermediate values of α linear interpolation should be used.

2.12.3.3 The thermal coefficient C_t

The thermal coefficient, C_t, accounts for the reduction of snow load on roofs with high thermal transmittance, in particular glass covered roofs. C_t is equal to 1.0 for buildings which are not heated and for buildings where the roofs are highly insulated. A value of 0,8 shall be used for most other cases.

2.1.3.4 The redistribution coefficient C_r

The redistribution coefficient, C_r, takes account of the redistribution of the snow on the roof caused by wind but in some cases also by other causes.

For monopitch roofs the redistribution of snow load may be neglected.

For symmetrical duopitch roofs the coefficient C_r is assumed to be constant and equal to $\pm C_{ro}$ for each half of the roof according to FIG 1. C_{ro} has a β-distribution with $\mu(C_{ro})$ according to FIG 2; the coefficient of variation of C_r is equal to 1.0. For other types of roofs the numerical values given in ENV 1991-2-3 and ISO 4355 shall be used. These values can assumed to correspond to the mean value plus one standard deviation.

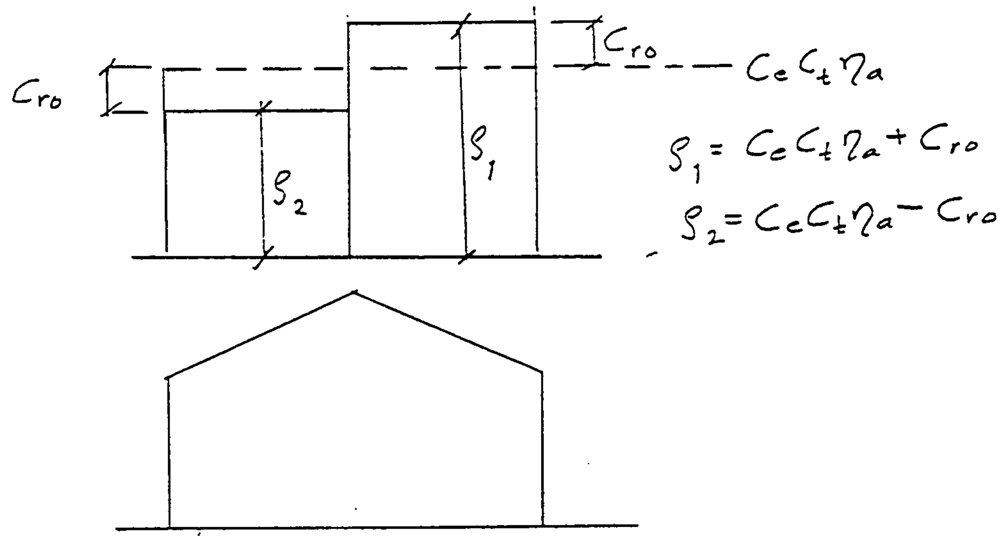

Figure 1: The redistributed snow load on a duopitch roof

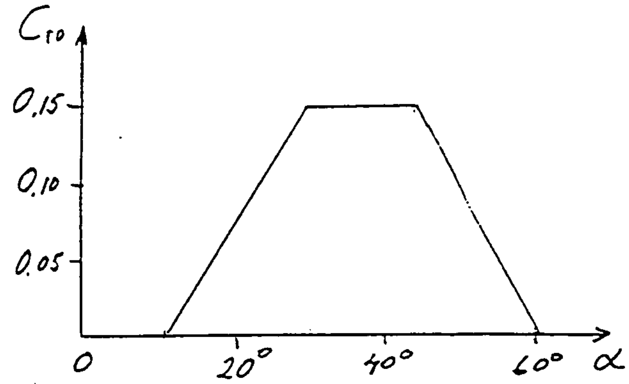

Figure 2: C_{ro} as function of the roof angle

Summary of snow load variables

X	designation	distribution	mean	scatter
S_g d_g	snow depth on the grund snowload on the ground	gamma	observation[1)]	observation[1)]
ρ	climate type parameter	det	observation	observation
k	parameter	det	1.5/1.25 m	-
h_r	reference height	det	300 m	-
$\gamma(0)$	unit weight at t = 0	det	1,7 kN/m^3	-
$\gamma(\infty)$	unit weight at t = ∞	det	5.0 kN/m^3	-
λ	parameter	det	0.85 m	-
$C_e\eta_a$	shape coefficient	beta	2.13.3.2	V = 0.15
C_t	insulation parameter	det	0.8-1.0	-
C_{ro}	redistribution coefficient	beta	Fig. 2	V = 1.0

[1)] Data from similar stations can be used as prior with n' = 3 and ν' = 2.

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2.18 IMPACT LOAD

Table of contents:

List of symbols:

a	=	deceleration
A_b	=	the area of the building including the shadow area
d	=	distance from the structural element to the road
$f_s(y)$	=	distribution of initial object position in y direction
$F_c(x)$	=	static compression strength at a distance x from the nose
k	=	stiffness
m	=	mass
$m'(x)$	=	mass per unit length
n	=	number of vehicles, ships or planes per time unit
$n(t)$	=	number of moving objects per time unit (traffic intensity)
P_a	=	the probability that a collision is avoided by human intervention.
$P_{fq}(xy)$	=	the probability of structural failure given a mechanical or human failure on the ship, vehicle, etc. at point (x,y).
r	=	$d/\sin\alpha$ = the distance from "leaving point" to "impact point"
R	=	radius of airport influence circle
T	=	period of time under consideration

v_c	=	the object velocity at impact
$v_c(t)$	=	velocity of the crashed part
$v_c(xy)$	=	object velocity at impact, given initial failure at point (x,y)
v_o	=	velocity of the vehicle when leaving the track
x,y	=	coordinate system;
α	=	angle between collision course and track direction
$\Lambda(r)$	=	collision rate for crash at distance r from the airport with $r < R$
$\lambda(x,t)$	=	failure intensity as a function of the coordinate x and the time t.

2.18 IMPACT LOAD

2.18.1 Basic Model for Impact Loading

2.18.1.1 Introduction

The basic model for impact loading constitutes of (see figure 2.18.1):

- potentially colliding objects (vehicles, ships, airplanes) that have an intended course, which may be the centre line of a traffic lane, a shipping lane or an air corridor; the moving object will normally have some distance to this centre line;
- the occurrence of a human or mechanical failure that may lead to a deviation of the intended course; these occurences are described by a homogeneous Poison process;
- the course of the object after the initial failure, which depend on both object properties and environment;
- the mechanical impact between object and structure, where the kinetic energy of the colliding object is partly transferred into elastic-plastic deformation or fracture of the structural elements in both the building structure and the colliding object.

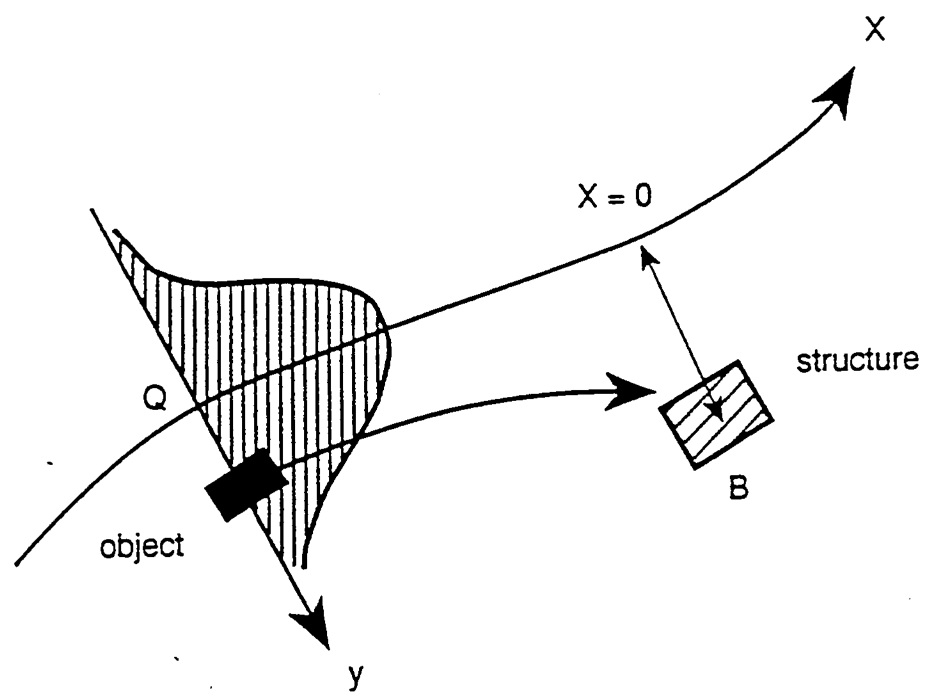


Figure 2.18.1: Probabilistic collision model

2.18.1.2 Failure probability

The probability of structural failure is presented as:

$$P_f(T) = 1 - \exp\{-\int\int\int n(t)\ \lambda(x,t)\ P_{fq}(xy)\ f_s(y)\ dx\ dy\ dt\} \qquad (2.18.1)$$

or for small probability and constant n and l:

$$P_f(T) = nT\lambda\int\int P_{fq}(x,y)\, f_s(y)\, dydx \qquad (2.18.\ 2)$$

where:

T	=	period under consideration
n(t)	=	number of moving objects per time unit (traffic intensity)
$f_s(y)$	=	distribution of initial object position in y direction
$P_{fq}(xy)$	=	the probability of structural failure given a mechanical or human failure on the ship, vehicle, etc. at point (x,y).
x,y	=	coordinate system; the x coordinate follows the centre line of the traffic lane, while the y coordinate represents the (horizontal) distance of the object to the centre; the structure that potentially could be hit, is located at the point with coordinates x=0 and y=d.
$\lambda(x,t)$	=	failure intensity as a function of the coordinate x and the time t. The length dependency expresses the variability in circumstances along the centre line (for instance curved versus straight trajectories). The time dependency indicates differences in summer and winter, day and night, etc. Note that although $\lambda(x,t)$ is a function of x and t, its dimension is [1/Length].

2.18.1.3 Distribution function for the impact load

In principle, impact is an interaction phenomenon between the object and the structure. It is not possible to formulate a separate action and a separate resistance function. However, an upper bound for the impact load can be found using the "rigid structure" assumption. If the colliding object is modelled as an elastic single degree of freedom system, with stiffness k and mass m, the maximum possible resulting interaction force equals:

$$F_c = v_c \sqrt{(km)} \qquad (2.18.3)$$

v_c = the object velocity at impact

Note that (2.18.3) gives the maximum for the external load; dynamic effects within the structure still need to be considered.

Based on formulation (2.18.3) the distribution function for the load F_c can be found:

$$P\{F_c < X\} = 1-\exp\{-\int\int\int n\lambda P[v_c(xy)\sqrt{km} > X]\ f_s(y)\ dxdydt\} \qquad (2.18.4)$$

$v_c(xy)$ = object velocity at impact, given initial failure at point (x,y)

For small probabilities:

$$P\{F_c > X\} = P_f(T) = nT\lambda\int\int P[v_c\sqrt{km} > X]\ f_s(y)\,dydx \qquad (2.18.5)$$

For the designation of the variables, see clause 2.18.1.2.

2.18.2 Impact from vehicles

2.18.2.1 Distribution of impact force

Consider a structural element in the vicinity of a road or track. Impact will occur if some vehicle, travelling over the track, leaves its intended course at some critical place with sufficient speed (see Figure 2.18.2).

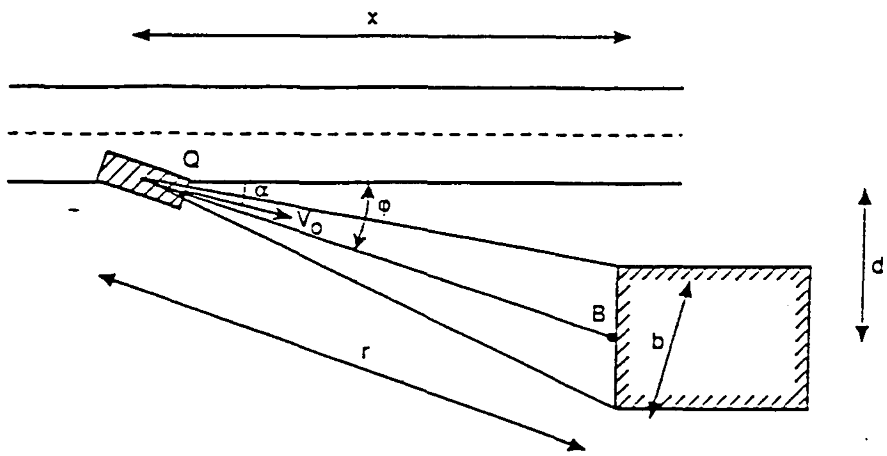


Figure 2.18.2: A vehicle leaves the intended course at point Q with velocity v_0 and angle a. A structural element at distance r is hit with velocity v_r.

The collision force probability distribution based on (2.18.5), neglecting the variability in y-direction is given by:

$$P(F_c > X) = n\,T\,\lambda\,\Delta x\,P\,[\,\sqrt{\{\,m\,k\,(v_o^2 - 2ar)\,\}} > X] \qquad (2.18.6)$$

n = number of vehicles per time unit
T = period of time under consideration
λ = probability of a vehicle leaving the road per unit length of track
Δx = part of the road from where collisions may be expected
v_o = velocity of the vehicle when leaving the track
a = deceleration
r = d/sin α = the distance from "leaving point" to "impact point"
d = distance from the structural element to the road
α = angle between collision course and track direction

λ Δx is the probability that a passing vehicle leaves the road at the interval Δx, which is approximated by:

$$\Delta x = b / \sin \mu(\alpha) \tag{2.18.7}$$

The value of b depends on the structural dimensions. However, for small objects such as columns a minimum value of b follows from the width of the vehicle, so b > 2.5 m.

Numerical values and probabilistic models can be found in Table 2.18.1.

variable	designation	type	mean	stand dev
λ	accident rate	deterministic	10^{-10} m^{-1}	-
α	angle of collision course	rayleigh	10 °	10°
v	vehicle velocity - motorway - urban area - court yard - parking garage	 lognormal lognormal lognormal lognormal	 80 km/hr 40 15 10	 10 km/hr 7 6 5
a	deceleration	lognormal	4 m^2/s	1.3 m/s^2
m	vehicle mass - truck - car	 normal normal	 20.000 kg 1500 kg	 12.000 kg -
k	vehicle stiffness	lognormal	300 kN/m	60 kN/m

Table 2.18.1: Numerical values for vehicle impact

2.18.2.2 Specifications of impact force

The collission force is a horizontal force; only the force component perpendicular to the structural surface needs to be considered.

The collision force for passenger cars affects the structure at 0.5 m above the level of the driving surface; for trucks the collision force affects it at 1.25 m above the level of the driving surface. The force application area is 0.25 m (height) times 1.50 m (width).

For impact loads on horizontal structural elements above traffic lanes the following rules hold (see Figure 2.18.3):

a) on vertical surfaces the impact actions follow from 2.18.2.1 and the height reduction as specified at c)
b) on horizontal lower side surfaces upward inclination of 10% should be considered. The force application area is 0.25 m (heigh) times 0.25 m (width).
c) for free heights h larger than 6.0 m the forces are equal to zero; for free heights between 4.0 m and 6.0 m a linear interpolation should be used

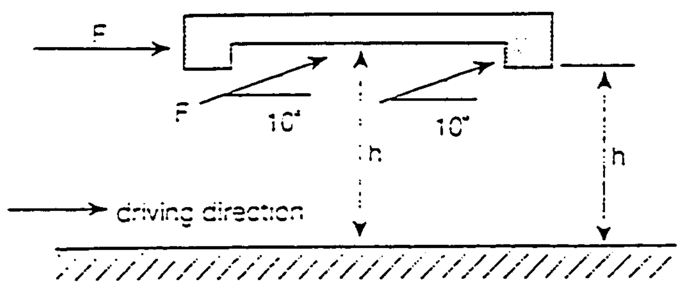


Figure 2.18.3: Impact loads on horizontal structural elements above traffic lanes

2.18.3 Impact from ships

2.18.3.1 Distribution of impact force

A co-ordinate system (x,y) is introduced as indicated in Figure 2.18.4. The x coordinate follows the centre line of the traffic lane, while the y co-ordinate represents the (horizontal) distance of the ship to the centre. The structure that potentially could be hit is located at the point with co-ordinates x=0 and y=d.

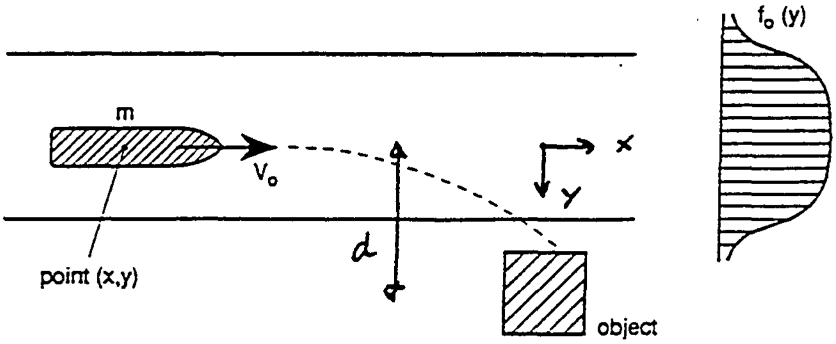


Figure 2.18.4: Ingredients for a ship collision model

Ship impact may be the result of:
- either a ship being on collision course, while no avoidance action is taken
- or the result of a mechanical or human failure leading to a change of course.

Both origins are present in the following model:

$$P(F > X) = n\,T\,(1-P_a) \int\int_{\Delta y} P[v_c(x,y)\sqrt{km} > X]\; f_s(y)\,dx\,dy + n\,T\,\lambda \int\int_{-\infty}^{+\infty} P[v_c(x,y)\sqrt{km} > X]\; f_s(y)\,dx\,dy \tag{2.18.8}$$

T	=	period of time under consideration
n	=	number of ships per time unit (traffic intensity)
λ	=	probability of a failure per unit travelling distance
v(x,y)	=	impact velocity of ship, given error at point (x,y)
k	=	stiffness of the ship
m	=	mass of the ship
$f_s(y)$	=	distribution of initial ship position in y direction
P_a	=	the probability that a collision is avoided by human intervention.
Δy	=	values of y coinciding with a collision course

For the evaluation in practical cases, it may be necessary to evaluate (2.18.8) for various ship types and traffic lanes, and add the results in a proper way at the end of the analysis.

Table 2.18.2 gives a number of standard ship characteristics and velocities that could be chosen by the designer.

variable	designation	type	mean	standard dev
P_a	avoidance probability - small - medium - large - very large	-	 0.995 0.997 0.998 0.999	-
λ	failure rate	-	$10^{-6}\,km^{-1}$	-
v	velocity - harbour - canal - sea	 lognormal lognormal lognormal	 1.5 m/s 3 6	 0.5 m/s 1.0 1.5
m	mass - small - medium - large - very large	 lognormal lognormal lognormal lognormal	 1000 ton 4000 20000 200000	 2000 ton 8000 40000 200000
k	equivalent stiffness	lognormal	15 MN/m	3 MN/m

Table 2.18.2: Numerical values for ship impact

2.18.3.2 Specifications of impact force

Bow, stern and broad side impact shall be considered where relevant; for side and stern impact the design impact velocities may be reduced.

Bow impact shall be considered for the main sail direction with a maximum deviation of 30°.

If a wall structure is hit under an angle a, the following forces should be considered:
- perpendicular to the wall: $F_y = F \sin\alpha$
- in wall direction: $F_x = f F \sin\alpha$

where F is the collision force at $\alpha = 90°$ and $f = 0.3$ is the friction coefficient.

The hydrodynamic mass for a ship drifting sideways is 80 percent of the mass of the ship.

Impact is to be considered as a free horizontal force; the point of impact depends on the geomertry of the structure and the size of the vessel. As a guideline one could take the most unfavourable point ranging from 0.1 L below to 0.1 L above the design water level. The impact area is 0.05 L * 0.1 L unless the stuctural element is smaller.

L is the typical ship length (L = 15, 40, 100 and 300 m for respectively small, medium, large and very large ship size).

The forces on the superstructure of the bridge depend on the height of the bridge and the type of ships te be expected. In general the force on the superstructure of the bridge will be limited by the yield strenght of the ships superstructure. A maximum of 10 000 kN for large and very large ships and 3000 kN for small and medium ships can be taken as a guideline averages.

2.18.4 Impact from airplanes

2.18.4.1 Distribution of impact force

The probability of a structure being hit by an airplane is very small. Only for exceptional structures like nuclear power plants, where the consequences of failure may be very large, is it mandatory to account for aircraft impact during design.

For air corridors, using (2.18.3) and for small probabilities:

$$P(F_c > X) = n\,T\,\lambda\,A_b\,(1-P_a)\,P(F_c > X|\text{impact})\,f_s(y) \qquad (2.18.9)$$

n	=	number of planes passing per time unit through an air corridor (traffic intensity)
T	=	time period of interest (for instance reference period)
λ	=	probability of a crash per unit distance of flying
$f_s(y)$	=	distribution of ground impact perpendicular to the corridor direction, given a crash
A_b	=	the area of the building including the shadow area
P_a	=	probability of avoiding a collision, given an airplane on collision course

The area A_b is the area of the building itself, enlarged by a so called shadow area (see figure 2.18.5). The strike angle is 10%

For the vicinity of an airport (at a distance r) the impact force distribution is based on:

$$P(F_c > X) = n\,T\,(1-P_a)\,\Lambda(r)A_b\,P\{F_c > X|\text{impact}\} \qquad (2.18.10)$$

$$\Lambda(r) = \frac{\overline{\Lambda}\,R}{2r} \qquad (2.18.11)$$

$\bar{\Lambda}$ = average air plane collision rate for a circular area with radius R = 8 km
$\Lambda(r)$ = collision rate for crash at distance r from the airport with $r < R$
n = number of planes approaching the airport per windtunnel
R = radius of airport influence circle
r = distance to the airport

Numerical values are presented in Table 2.18.3

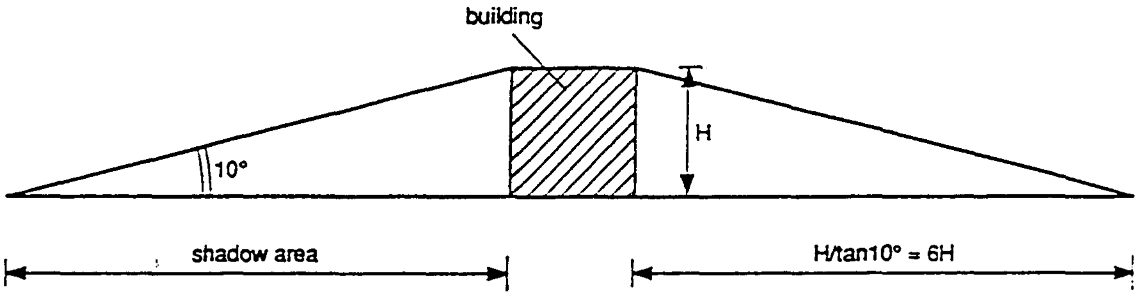


Figure 2.18.5: Strike area A_b for an airplane crash.

For airplanes the impact model (2.18.3) is not sufficient. A better model is given by:

$$F_c(t) = F_c(\xi) + m'(\xi)\ v_c^2(t) \qquad (2.18.12)$$

$$\xi = \int_0^t v_c(\tau)\ d\tau \qquad (2.18.13)$$

$F_c(x)$ = static compression strength at a distance x from the nose
$m'(x)$ = mass per unit length at a distance x from the nose
$v_c(t)$ = velocity of the crashed part of the plane at time t

Sometimes $v_c(t)$ is taken as constant and equal to v_r for further simplification. Results from calculations based on this model can be found in table (2.18.4).

It is recommended to make the analysis for each type of aircraft (small, large, civil, military) separately and add the results afterwards.

λ	Crash rate - military plane - civil plane	 10^{-8} km^{-1} 10^{-9} km^{-1}
$\overline{\Lambda}$	Average collission rate for airport area - small planes (< 6 ton) - large planes (> 6 ton)	 10^{-4} yr^{-1} km^{-2} 4 10^{-5} yr^{-1} km^{-2}
R	Radius of aiport influence circle	8 km

Table 2.18.3: Numerical values for the air plane impact model

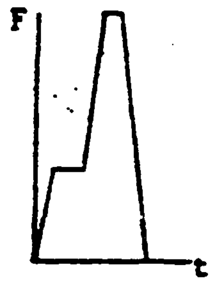
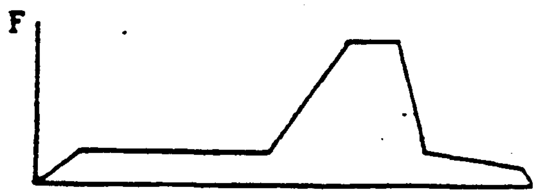
type	t [ms]	F [MN]	
<u>Cessna 210A</u> m = 1.7 ton v = 100 m/s A = 7 m^2 engine: m = 0.2 ton A = 0.5 m^2	0 3 6 18 18	0 7 7 4 0	
<u>Lear Jet 23A</u> m = 5.7 ton v = 100 m/s A = 12 m^2	0 20 35 50 70 80 100	0 2 6 6 12 10 0	
<u>MRCA (Multi Role Combat Aircraft)</u> m = 25 ton v = 215 m/s A = 4 m^2 engine: m = 1.2 ton A = 0.5 m^2	0 10 30 40 50 701	0 55 55 154 154 0	
<u>Boeing 707-320</u> m = 90 ton v = 100 m/s A = 36 m^2	0 30 150 200 230 250 320 330	0 20 20 90 90 20 10 0	

Table 2.18.4: Impact characteristics for various aircrafts (perpendicular on immovable walls

A = cross sectional area of the plane or engine
m = mass
v_r = velocity at impact

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PART 2: LOAD MODELS

2.20 FIRE

Table of contents:

List of symbols:

A = considered floor area
A_f = floor area
A_t = total internal surface area
f = ventilation opening
H_i = specific combustible energy for material i
q_o = fire load density per unit floor area
t = time
t_{eq} = equivalent time of fire duration

α = parameter
β_f = coefficient (model uncertainty)
μ_i = derating factor between 0 and 1, describing the degree of combustion
Δm_i = combustible mass present at ΔA for material i
θ = temperature in the compartment
θ_o = temperature at the start of the fire
θ_A = parameter

2.20. FIRE

2.20.1 Fire ignition model

The probability of a fire starting in a given building or area is modelled as a Poisson process with constant occurrence rate:

$$P\{\text{ignition in } (t,t+dt) \text{ in a compartment}\} = \nu_{fire}\, dt \qquad (2.20.1)$$

The occurrence rate ν_{fire} can be written as a summation of local values over the floor area:

$$\nu_{fire} = \iint_{A_f} \lambda(x,y)\, dxdy \qquad (2.20.2)$$

where $\lambda(x,y)$ corresponds to the probability of fire ignition per year per m^2 for a given occupancy type; A_f is the floor area of the fire compartment. As in most applications $\lambda(x,y)$ is a constant, this can be simplified to:

$$\nu_{fire} = A_f\, \lambda \qquad (2.20.3)$$

Values for l are presented in Table 2.20.1.

Type of building	λ [m^{-2} $year^{-1}$]
dwelling/school	0.5 to 4 * 10^{-6}
shop/office	1 to * 10^{-6}
industrial building	2 to 10 * 10^{-6}

Table 2.20.1: Example values of annual fire probabilities λ per unit floor area for several types of occupancy.

2.20.2 Flashover occurrence

After ignition there are various ways in which a fire can develop. The fire might extinguish itself after a certain period of time because no other combustible material is present. The fire may be detected very early and be extinguished by hand. An automatic sprinkler system may operate or the fire brigade may arrive in time to prevent flash over. Only in a minority of cases does a fire develop fully into a complete room or compartment fire; sometimes the fire may break through a barrier and start a fire in another compartment. From the structural point of view only these fully developed or post flashover fires (see Figure 2.20.1) may lead to failure. For fire compartments having a very large volume, e.g. industrial buildings and sports halls, a local fire of high intensity also may lead to structural damage.

The occurrence rate of flashover is given by:

$$\nu_{flash\ over} = P\{\text{flash over} \mid \text{ignition}\}\ \nu_{fire} \qquad (2.20.4)$$

The probability of a flashover once a fire has taken place, can obviously be influenced by the presence of sprinklers and fire brigades. Numerical values for the analysis are presented in Table 2.20.2.

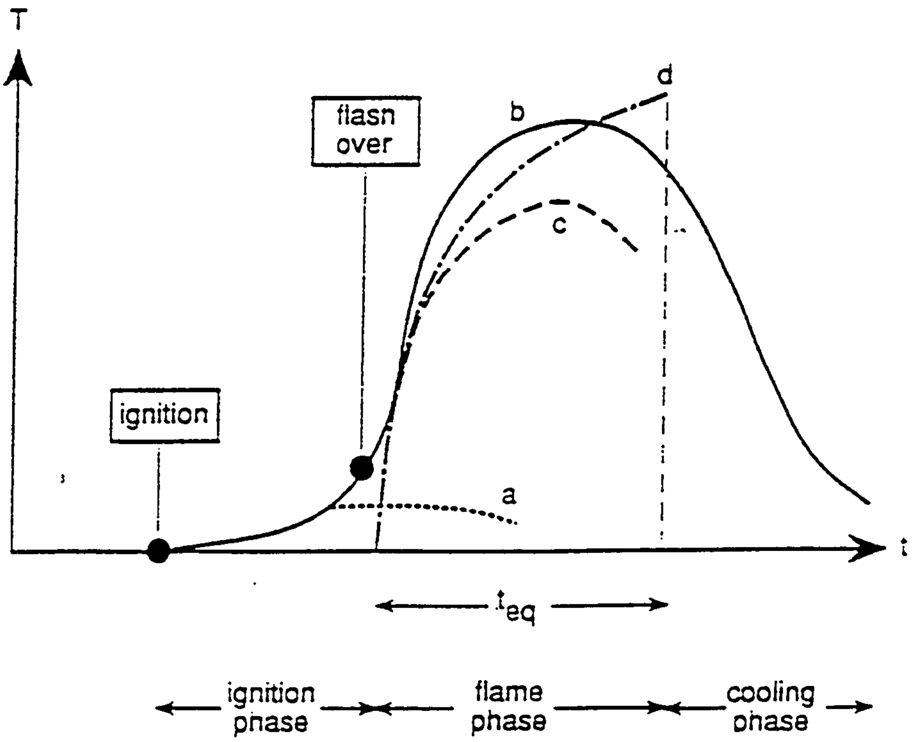

Figure 2.1: Schematic presentation of a temperature-time curve

* Curve (a) represents the temperature-time curve when a sprinkler system or a timely fire brigade action is successful.
* Curve (b) presents the temperature-time relation for a fully developed fire.
* Curve (c) indicates the limited influence of a fire brigade arriving after flashover has taken place.
* Curve (d) indicates the ISO-standard temperature curve (see section 2.20.4.2).

Protection method	P{flashover|ignition}
Public fire brigade	10^{-1}
Sprinkler	$2 * 10^{-2}$
High standard fire brigade on site, combined with alarm system (industries only)	10^{-3} to 10^{-2}
Both sprinkler and high standard residential fire brigade	10^{-4}

Table 2.20.2: Probability of flashover for given ignition, depending on the type of active protection measures

2.20.3 Combustible material modelling

The available combustible material can be considered as a random field, which in general might be nonhomogeneous as well as nonstationary. The intensity of the field q at some point in space and time is defined as:

$$q = \frac{\Sigma \mu_i \Delta m_i H_i}{A} \qquad (2.20.5)$$

μ_i = derating factor between 0 and 1, describing the degree of combustion
Δm_i = combustible mass present at ΔA for material i
H_i = specific combustible energy for material i
A = considered floor area

In some cases the intensity q may also depend on a vertical ordinate.

The non-dimensional factor μ_i is a function of the fuel type, the geometrical properties of the fuel, and the position of the fuel in the fire compartment, among other things. For some types of fire load components, μ_i depends on the time of fire duration and on the gas temperature-time characteristics of the compartment fire. Probabilistic models for q are presented in tabel 2.20.3.

Type of fire compartment	$\mu(q_0)$ [MJm^{-2}]	Coefficient of variation $v(q_0)$
1 : Dwellings	500	0.20
2 : Offices	600	0.30
3 : Schools	350	0.20
4: Hospitals	450	0.30
5: Hotels	300	0.25

Table 2.20.3: Recommended values for the average fire load intensity q_o

2.20.4 Temperature-time relationship

2.20.4.1 Scientific models

For known characteristics of both the combustible material and the compartment, the post flash over period of the temperature time curve can be calculated from energy and mass balance equations.

Many variables can be introduced as random in the model, for instance:
- the amount and spatial distributions of combustible material;
- the effective energy value;
- the rate of combustion;
- the ventilation characteristics;
- air use and gas production parameters;
- thermal conductivity properties;
- model uncertainties.

In addition, the development of the fire may depend on events like collapse of windows or containments, which may change the ventilation conditions or the available amount of combustible material respectively.

As a simplification the following assumptions may be used.

1. the combustible material is wood;
2. the wood is spread uniformly over the floor area;
3. the fire compartment is of a standard building material (brick, concrete);
4. the fire is controlled by ventilation and not by the amount of fuel load (this is conservative);
5. the initial temperature is 20 ° C .

In this case the temperature time curve depends on two parameters:

- the floor averaged fire load density q_o ;
- the opening factor f.

The opening factor f is defined as:

$$f = \frac{A}{A_t}\sqrt{h}; \quad \text{with } h = \frac{\Sigma A_i h_i}{A}; \; A = \Sigma A_i \qquad (2.20.7)$$

where:

A_t = total internal surface area of the fire compartment, i.e. the area of the walls, floor and ceiling, including the openings [m^2]
A_i = area of the vertical opening i in the fire compartment [m^2]
h_i = value of the height of opening i [m]

For a fire compartment which also contains horizontal openings, the opening factor can be calculated from a similar expression. In calculating the opening factor, it is assumed that ordinary window glass is immediately destroyed when fire breaks out.

In many cases it will be possible to indicate a physical maximum f_{max}. The actual value of f in a fire should be modelled as a random quantity according to:

$$f = f_{max}(1 - \zeta) \qquad (2.20.8)$$

ζ = random parameter (see Table 2.20.4)

To avoid negative values of f, this lognormal distribution should be cut off at $\zeta = 1$. In addition one should multiply the resulting temperatures by an overall model uncertainty factor θ_{model}.

2.20.4.2 Engineering models

In many engineering applications, use is made of equivalent standard temperature-time-relationship according to ISO 824:

$$\theta = \theta_o + \theta_A \log_{10} \{\alpha t + 1\} \text{ for } 0 < t < t_{eq} \qquad (2.20.9)$$

with:

$$t_{eq} = \frac{\beta_f \; q_o \; A_f}{A_t \; \sqrt{f}} \qquad (2.20.10)$$

θ = temperature in the compartment
θ_o = temperature at the start of the fire
θ_A = parameter
α = parameter
t = time
t_{eq} = equivalent time of fire duration
β_f = coefficient (model uncertainty)
q_o = fire load density per unit floor area
A_f = floor area
A_t = total internal surface area
f = ventilation opening (see 2.20.7, 2.20.8)

Numerical values and probabilistic models are given in Table 2.20.4.

Variable	Distribution	Mean	Standard deviation
ζ	truncated lognormal [1)]	0.2	0.2
β_f	lognormal	4.0 $sm^{2.25}/MJ$	1.0
θ_o	deterministic	20°C	-
θ_A	deterministic	235 K	-
α	deterministic	0.125 s^{-1}	-

[1)] values of $\zeta > 1$ should be supressed

Table 2.20.4: Numerical values for random variables

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Part 2 : Loads

Section 2.13 : Wind

SYMBOLS

f_c	Coriolis parameter (= $2\Omega \sin \phi$)
f_0	mean frequency of zero up crossing, in Hz
g	peak factor (no dimension)
$G_u(n)$, $G_v(n)$, $G_w(n)$	half-sided power spectral density for longitudinal, transversal and vertical components of velocity fluctuations
$I_u(z)$	turbulence intensity of longitudinal velocity fluctuations (dimensionless)
k	von Karman`s constant (= 0.4)
$L_u(z)$	integral length scale for longitudinal velocity fluctuations, in m
$L_v(z)$	integral length scale for transversal velocity fluctuations, in m
$L_w(z)$	integral length scale for vertical velocity fluctuations, in m
N	number of reference time, in years
n	frequency, in Hertz
n_u, n_v, n_w	dimensionless frequency of fluctuations in longitudinal, transversal and vertical direction
$\overline{Q}_{ref}$	reference wind velocity pressure
$\overline{Q}(z)$	mean velocity pressure at height z (=(1/2) ρ $\overline{U}^2(z)$)
$S_{ij}(n)$	cross spectral power density
T	reference time
$\overline{T}(u_p)$	mean recurrence interval of maximum annual mean velocity, in years
$\overline{U}_{ref}$	reference wind velocity, in m/s
$\overline{U}(z)$	mean longitudinal velocity of the wind at height z
u_1	mode of the maximum annual mean wind speed in Gumbel distribution
u(x,z,t)=u	longitudinal component of the wind velocity fluctuations, in m/s
v(y,z,t)=v	transversal component of wind velocity fluctuations, in m/s
w(z,t)=w	vertical component of wind velocity fluctuations, in m/s
z	height above ground, in m
z_0	roughness length, in m
z_r	a reference height above ground, in m
z_{ref}	the reference height above ground (10 - 30 m)
α_1	dispersion parameter for the maximum annual mean wind speed in Gumbel distribution
δ	height of the atmospheric boundary layer
κ	surface drag coefficient (dimensionless) (=$[k/\ln(z_{ref}/z_0)]^2$)
λ_k	k-th moment of spectral density
$\nu(x)$	mean upcrossing rate for level x
ϕ	geographical latitude
ρ	air density (= 1.25 kg/m^3)
$\sigma_u, \sigma_v, \sigma_w$	standard deviation of velocity fluctuations in x-, y- and z-direction, in m/s

2.13.1 Introduction

Wind effects on buildings and structures depend on the general wind climate, the exposure of buildings, structures and their elements to the natural wind, the dynamic properties, the shape and dimensions of the building (structure). The section presents basic data and procedures for the estimation of wind loads on buildings and structures. Tropical cyclones, tornados, thunderstorms and orographic wind phenomena require separate treatment.

The field of wind velocities over horizontal terrain is decomposed into a mean wind (average over 10 minutes) in the direction of general air flow (x-direction) averaged over a specified time interval and a fluctuating, turbulent part with zero mean and components in the longitudinal (x-) direction, the transversal (y-) direction and the vertical (z-) direction

2.13.2 Wind forces

The wind force acting per unit area of structure is determined with the relations:

(i) For rigid structures of smaller dimensions:

$$w = c_a c_g c_r \overline{Q}_{ref} = c_a c_e \overline{Q}_{ref} \qquad (1)$$

(ii) For structures sensitive to dynamic effects (natural frequency < 1Hz) and for large rigid structures:

$$w = c_d c_a c_e \overline{Q}_{ref} \qquad (2)$$

where:

$\overline{Q}_{ref}$ is the reference (mean) velocity pressure
c_r - roughness factor
c_g - gust factor
c_a - aerodynamic shape factor
c_d -dynamic factor.

2.13.3 Mean wind velocity

The reference wind velocity, $\overline{U}_{ref}$ is the mean velocity of the wind averaged over a time interval of 10 min, determined at an elevation of 10 m above ground, in horizontal open terrain exposure (z_0 = 0.03 m).[1]

The distribution of the mean wind velocities (for any terrain category, height above ground and averaging time interval) is the Weibull distribution for minima:

$$F_{\overline{U}}(x) = 1 - \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma}\right)^k\right] \qquad (3)$$

with k close to 2.

The same distribution is valid for direction dependent mean wind flows. Generally, it can not be assumed that the mean wind direction is uniformly distributed over the circle.

---

[1] For other than 10 min averaging intervals, in open terrain exposure, the following relationships may be used: $1.05\overline{U}^{1h} = 1.0\overline{U}^{10min} = 0.84\overline{U}^{1min(fastest\ mile)} = 0.67\overline{U}^{3sec}$.

Mean wind velocities vary over the year. If no data are available it can be assumed in the northern hemisphere that $\sigma(t) \approx \sigma[1+ a \cos(2\pi(t-t_0)/365]$ with the constant a between 1/3 and 1/2 and $t_0 \approx 15$ to 45, with t in days.

The mean wind velocities are highly autocorrelated. Mean wind velocities with separation of about 4 to 12 (8 on average) hours can be considered as independent.

If seasonal variations are neglected, the mean time the mean wind velocities are between levels x_1 and x_2 $(x_2 \geq x_1)$ is asymptotically

$$E\left[T_{x_1,x_2}\right] = T[F_{\bar{U}}(x_2) - F_{\bar{U}}(x_1)] \tag{4}$$

with T the reference time. For higher levels of x_2 the distribution of individual times above x is approximately $[1 - F_{\bar{U}}(x)] / \nu(x)$ with $\nu(x)$ the mean upcrossing rate for level x.

The distribution of maximum mean wind speeds follows a Gumbel distribution for maxima. Generally, it is not possible to infer the maxima over more years from observations covering only a few years. If the annual maxima are used, provided that the maximum annual data are homogenous as exposure and averaging time, the distribution function is :

$$F_{\max \bar{U}}(x) = \exp\{-\exp[-\alpha_1(x - u_1)]\} \tag{5}$$

The mode u and the parameter α_1 of the distribution are determined from the mean m_1 and the standard deviation σ_1 of the set of maximum annual velocities: $u_1 = m_1 - 0.577 / \alpha_1$, $\alpha_1 = 1.282 / \sigma_1$. The coefficient of variation of maximum annual wind speed, $V_1 = \sigma_1 / m_1$ depends on the climate and is normally between 0.10 and 0.35. For reliable results, the number of the years of available records must be of the same order of magnitude like the required mean recurrence interval.

The lifetime (N years) maxima of wind velocity is also Gumbel distributed and the mean and the standard deviation of lifetime maxima are functions of the mean and of the standard deviation of annual maxima: $m_N = m_1 + \frac{\ln N}{1.282}\sigma_1$, $\sigma_N = \sigma_1$. The reference wind velocity having the probability of non-exceedance p = 0.98 is so called "characteristic" velocity, $U_{0.98}$. The mean recurrence interval of the characteristic velocity is $T(U_{0.98}) = 50$ yr.

Under special climatic conditions, the distribution of mean wind speeds is a mixed distribution reflecting different meteorological phenomena.

2.13.4 Terrain roughness (category)

The roughness of the ground surface is aerodynamically described by the roughness length z_0 (in meters), which is a measure of the size and spacing of obstacles on the ground surface. Alternatively, the terrain roughness can be described by the surface drag coefficient, κ corresponding to the roughness length z_0:

$$\kappa^2 = \frac{k}{\ln \frac{z_{ref}}{z_0}} \tag{6}$$

where $k \cong 0.4$ is von Karman's constant and z_{ref} is the reference height (Table 2, Table 3). Various terrain categories are classified in Table 1 according to their approximate roughness lengths. The distribution of the surface roughness with wind direction must be considered.

Table 1. Roughness length z_0, in meters, for various terrain categories [1) 2)]

Terrain category	Terrain description	Range of z_0, in m	Recom-mended value
A. Open sea. Smooth flat country	Areas exposed to the wind coming from large bodies of water; snow surface; Smooth flat terrain with cut grass and rare obstacles.	0.001 ⋮ 0.005	0.003
B. Open country	High grass (60 cm) hedges, and farmland with isolated trees; Terrain with occasional obstructions having heights less than 10 m (some trees and some buildings)	0.01 ⋮ 0.1	0.03
C. Sparsely built-up urban areas. Wooded areas	Sparsely built-up areas, suburbs, fairly wooded areas (many trees)	0.1 ⋮ 0.7	0.3
D. Densely built-up urban areas. Forests	Dense forests in which the mean height of trees is about 15m; Densely built-up urban areas; towns in which at least 15% of the surface is covered with buildings having heights over 15m	0.7 ⋮ 1.2	1.0
E. Centers of very large cities	Numerous large high closely spaced obstructions: more than 50% of the buildings have a height over 20m	1.0 ≥ 2.0	2.0

1) Smaller values of z_0 provoke higher mean velocities of the wind

2) For the full development of the roughness category, the terrains of types A to D must prevail in the up wind direction for a distance of at least of 1000m, respectively. For category E this distance is more than 5 km.

2.13.5 Variation of the mean wind with height

The variation of the mean wind velocity with height over horizontal terrain of homogenous roughness can be described by the logarithmic law. The logarithmic profile is valid for moderate and strong winds (mean hourly velocity > 10 m/s) in neutral atmosphere (where the vertical thermal convection of the air may be neglected).

$$\overline{U}(z) = \frac{1}{k} u_*(z_0)\left(\ln\frac{z}{z_0} + 5.75\frac{z}{\delta} - 1.87\left(\frac{z}{\delta}\right)^2 - 1.33\left(\frac{z}{\delta}\right)^3 + 0.25\left(\frac{z}{\delta}\right)^4 \right) \quad (z > d_0 >> z_0) \quad (7)$$

where:

$\overline{U}(z)$ mean velocity of the wind at height z above ground in m/s

z = Height above ground in m

z_0 = roughness length in m

$u_*(z_0) = \dfrac{\overline{U}(z)}{2.5\ln\frac{z}{z_0}}$ = friction velocity in m/s

k - von Karman's constant ($k \cong 0.4$

d_0 - the lowest height of validity of Eq.(7) in m

$\delta = \dfrac{u_*(z_0)}{6 f_c}$ = Depth of boundary layer in m

$f_c = 2\Omega\sin(\varphi)$ = Coriolis parameter in 1/s

$\Omega = 0.726\ 10^{-4}$ = angular rotation velocity in rad/s
ϕ = latitude of location in degree

For lowest 0.1 δ or 200m of the boundary layer only the first term needs to be taken into account (Harris and Deaves, 1981). The lowest height of validity for the Eq.(7), d_0 is close to the average height of dominant roughness elements : i.e. from less than 1 m, for smooth flat country to more than 15 m, for centers of cities. For $z_0 \le z \le d_0$ a linear interpolation is recommended. In engineering practice, Eq.(7) is conservatively used with $d_0 = 0$.

With respect to the reference (open terrain) exposure, the relation between wind velocities in two different roughness categories at two different heights can be written approximately as (Bietry, 1976, Simiu, 1986):

$$\frac{\overline{U}(z)}{\overline{U}_{ref}} = \frac{\ln\dfrac{z}{z_0}}{\ln\dfrac{z_{ref}}{z_{0,ref}}}\left(\frac{z_0}{z_{0,ref}}\right)^{0.07} = p. \tag{8}$$

At the reference height z_{ref}, the ratio of the mean wind velocity in various terrain categories to the mean wind velocity in open terrain is given by the factor p in Table 2. The corresponding ratio for the mean velocity pressure is p^2 .

Table 2. Scale factors for the mean velocity (and the mean velocity pressure) at reference height in various terrain exposure

Terrain category	A. Open sea. Smooth flat country	B. Open country	C. Sparsely built-up urban areas. Wooded areas	D. Densely built-up urban areas. Forests	E. Centers of large cities
z_{ref}, m	10	10	10	15	30
p	1.19	1.00	0.71	0.56	0.39
p^2	1.40	1.00	0.50	0.30	0.15

2.13.6 Intensity of turbulence

The turbulent fluctuations of the wind velocity can be assumed to be normally distributed with mean zero. The root mean squared value of the velocity fluctuations in the airflow, deviating from the longitudinal mean velocity, may be normalised to the friction velocity as follows:

$$\frac{\sigma_u}{u_*} = \beta_u\left(1 - \frac{z}{\delta}\right) \quad \text{Longitudinal} \tag{9a}$$

$$\frac{\sigma_v}{u_*} = \beta_v\left(1 - \frac{z}{\delta}\right) \quad \text{Transversal} \tag{9b}$$

$$\frac{\sigma_w}{u_*} = \beta_w\left(1 - \frac{z}{\delta}\right) \quad \text{Vertical} \tag{9c}$$

The approximate linear variation with height (Hanna, 1982) can be used only in moderate and strong winds. For neutral atmosphere, the ratios σ_v/σ_u and σ_w/σ_u near the ground are constant irrespective the roughness of the terrain (ESDU 1993):

$$\frac{\sigma_v}{\sigma_u} = 1 - 0.25\cos^4\left(\frac{\pi}{2}\frac{z}{\delta}\right) \tag{10a}$$

$$\frac{\sigma_w}{\sigma_u} = 1 - 0.55\cos^4\left(\frac{\pi}{2}\frac{z}{\delta}\right) \quad (10b)$$

For z<<δ the variance of the velocity fluctuations can be assumed independent of height above ground :

$$\sigma_u = \beta_u u_* \quad (11a)$$

$$\sigma_v = \beta_v u_* \quad (11b)$$

$$\sigma_w = \beta_w u_* \quad (11c)$$

and, for $z < 0.1\ \delta$:

$$\frac{\sigma_v}{\sigma_u} \cong 0.75 \quad (12a)$$

$$\frac{\sigma_w}{\sigma_u} \cong 0.50 \quad (12b)$$

The variance of the longitudinal velocity fluctuations can also be expressed from non-linear regression of measurement data, as function of terrain roughness (Solari, 1987):

$$4.5 \leq \beta_u^2 = 4.5 - 0.856 \ln z_0 \leq 7.5 \quad (13)$$

The longitudinal intensity of turbulence is the ratio of the root mean squared value of the longitudinal velocity fluctuations to the mean wind velocity at height z (i.e. the coefficient of variation of the velocity fluctuations at height z :

$$I_u(z) = \frac{\overline{u(z,t)^2}^{1/2}}{\overline{U}(z)} = \frac{\sigma_u(z)}{\overline{U}(z)} \quad (14)$$

The turbulence intensity at height z can be approximated by:

$$I(z) = \frac{\beta_u}{2.5 \ln \frac{z}{z_0}} \approx \frac{1}{\ln \frac{z}{z_0}} \quad (15)$$

The transversal and vertical intensities of turbulence can be determined by multiplication of the longitudinal intensity $I_u(z)$ by the ratios σ_v/σ_u and σ_w/σ_u. Representative values for intensity of turbulence at the reference height are given in Table 3.

Table 3: Wind parameters depending on terrain category

Terrain category	A. Open sea. Smooth flat country	B.Open country	C. Sparsely built-up urban areas. Wooded areas	D. Densely built-up urban areas. Forests	E. Centers of large cities
z_0, m	0.01	0.05	0.3	1.0	2.0
d_0 m	0	2	8	15	30
κ	0.0024	0.0047	0.013	0.022	0.022
β_u	3.1	2.7	2.3	2.1	2.0
β_v	2.3	2.1	1.8	1.6	1.5
β_w	1.55	1.35	1.15	1.05	1.0
z_{ref}, m	10	10	10	15	30
$I(z_{ref})$	0.15	0.19	0.26	0.31	0.39

2.13.7 Power spectral density and autocorrelation functions of gustiness

The normalised half-sided von Karman power spectral densities and autocorrelation functions of gust velocity are given in Table 4.

Table 4. The von Karman model of isotropic turbulence

Component of gust velocity	Normalised spectral density $\frac{nG_i(n)}{\sigma_i^2}$	Normalised autocorrelation function $\rho_i(\tau_i)$
Longitudinal I = u	$\frac{4n_u}{\left(1+70.8\,n_u^2\right)^{5/6}}$	$\frac{2^{2/3}}{\Gamma(1/3)}\bar{\tau}_u^{1/3}\,K_{1/3}\left(\bar{\tau}_u\right)$
Transversal I = v Vertical i = w	$\frac{2n_i\left(1+188.6\,n_i^2\right)}{\left(1+70.8\,n_i^2\right)^{11/6}}$	$\frac{2^{2/3}}{\Gamma(1/3)}\bar{\tau}_i^{1/3}\left[K_{1/3}(\bar{\tau}_i)-\frac{1}{2}\bar{\tau}_i K_{2/3}(\bar{\tau}_i)\right]$

The notations in Table 4 are as follows:

σ_i^2 = variance of velocity fluctuations in direction i, in m^2/s^2; i = u, v or w

$n_i = n_i(z) = \frac{n\,L_i(z)}{\bar{U}(z)}$ = is a non-dimensional height dependent frequency

n = frequency, in Hertz

$\bar{U}(z)$ = longitudinal mean velocity at height z, in m/s

$L_i(z)$ = length of integral scale of turbulence in direction i, in m/s.

$\bar{\tau}_i = \frac{\tau\bar{U}(z)}{aL_i(z)}$ = non-dimensional time (a = 1.339)

$K_\mu(\;)$ = modified Bessel function of second kind of order μ and argument

τ = time lag, in s

The integral length scale of turbulence in direction i at the height z is:

$$L_i(z) = U(z)\int_0^\infty \rho_i(\bar{\tau}_i)\,d\tau_i \qquad (16)$$

where the autocorrelation $\rho_i(\tau_i)$ is the Fourrier transform of spectral density.

An estimation of the length of the integral scale of longitudinal turbulence, for heights up to 300 m is given by ESDU, 1993, as:

$$L_u(z) = \frac{A^{3/2}(\sigma_u/u_*)^3 z}{2.5K_z^{3/2}(1-z/h)^2(1+5.75z/h)} \qquad (17)$$

where

$$A = 0.115\left[1+0.315\left(1-\frac{z}{\delta}\right)^6\right]^{2/3}$$

$$K_z = 0.188[1-(1-z/z_c)^2]^{1/2}$$

$$z_c/\delta = 0.39\left[\frac{u_*}{f_c z_0}\right]^{-1/8}$$

For the lateral and vertical direction (ESDU, 1993):

$$L_v(z) = 0.5\,(\sigma_v/\sigma_u)^3\,L_u(z) \qquad (18a)$$

$L_w(z) = 0.5\ (\sigma_w/\sigma_u)^3\ L_u(z)$ (18b)

$L_v(z) \cong 0.24\ L_u(z)$ (18c)

$L_w(z) \cong 0.08\ L_u(z)$ (18d)

2.13.8 Coherence functions

The cross-spectral density for two separated points P_1 and P_2 with distance r perpendicular to direction i are given by:

$$S_{ij}(n, P_1, P_2) \approx S_{ii}^{1/2}(n, P_1, P_2) S_{jj}^{1/2}(n, P_1, P_2) \cdot Coh_{ij}^{1/2}(n, P_1, P_2) \qquad (19)$$

with:

Longitudinal
$$Coh_{uu}^{1/2}(r, \bar{k}) = \frac{\left(\frac{\psi_u}{2}\right)^{5/6}}{\Gamma(5/6)} \left[2K_{5/6}(\psi_u) - \psi_u K_{1/6}(\psi_u)\right] \qquad (20a)$$

Transversal
$$Coh_{vv}^{1/2}(r, \bar{k}) = \frac{\left(\frac{\psi_v}{2}\right)^{5/6}}{\Gamma(5/6)} \left[2K_{5/6}(\psi_v) + \frac{6(r\bar{k})^2}{3\psi_v^2 + 5(r\bar{k})^2} \psi_v K_{1/6}(\psi_v)\right] \qquad (20b)$$

Vertical
$$Coh_{ww}^{1/2}(r, \bar{k}) = \frac{\left(\frac{\psi_w}{2}\right)^{5/6}}{\Gamma(5/6)} \left[2K_{5/6}(\psi_w) - \frac{6(rL)^2}{3\psi_w^2 + 5(r\bar{k})^2} \psi_w K_{1/6}(\psi_w)\right] \qquad (20c)$$

where $\bar{k} = \frac{2\pi n}{\overline{U}_m}$ and $\psi_i^2 = \left(r^2\bar{k}^2 + r^2 / L_i^2\right)$.

The longitudinal coherence can also be approximated by (Kareem, 1987):

$$Coh_{uu}^{1/2}(n, r) \approx \exp\left[-\left\{\left(\frac{r}{L_u}\right)^2 + \left(\frac{nr}{\overline{U}_m}\right)^2 \left(12 + \frac{11r}{z_m}\right)^2\right\}^{1/2}\right] \qquad (21)$$

implying a coherence coefficient of $C = 12 + 11r / z_m$ and where

$$z_m = \sqrt{z_1 z_2}$$

$$\overline{U}_m = \sqrt{\overline{U}_1(z_1)\overline{U}_2(z_2)}\,.$$

For structures of small dimension, i.e. r much smaller than L_u, r can be taken as zero.

2.13.9 Peak velocities

Spectral moments, λ_i of higher than the i = 0 order do not exist for turbulence spectra (including von Karman and other spectra) fulfilling the Kolmogorov asymptote (asymptotic $f^{-5/3}$ behaviour). Truncation of these spectra at frequencies of 10÷20 Hz makes them finite.

Then, the distribution of extreme gust velocities, u_{max} is asymptotically a Gumbel distribution with mean:

$$E\left[u_{max} \middle| \lambda_0, \lambda_2, t\right] = \left(\sqrt{2 \ln \nu_0 t} + \gamma / \sqrt{2 \ln \nu_0 t}\right)\sqrt{\lambda_0} \qquad (22)$$

and variance:

$$Var\left[u_{max} \middle| \lambda_0, \lambda_2, t\right] = [(\pi^2 / 6) / 2 \ln \nu_0 t]\lambda_0 \qquad (23)$$

γ=0.5772 is Euler's constant, t = 600 s and ν_0 is the mean frequency of zero upcrossing, in Hz:

$$\nu_0 = \sqrt{\lambda_2 / \lambda_0}\,. \tag{24}$$

The mean and standard deviation of the random peak factor for gust velocities, g are defined as:

$$g = \sqrt{2\ln \nu_0 t} + 0.577 / \sqrt{2\ln \nu_0 t} \tag{25}$$

$$\sigma_g = \frac{\pi}{6} \frac{1}{\sqrt{2\ln \nu_0 t}} \tag{26}$$

The calculation of g from turbulence spectra is sensitive to the choice of cut-off frequency (5-20 Hz). Empirically and theoretically one can assume that the mean of g is about 3.2 and its standard deviation about 0.4. Since the fluctuating velocity pressure is a linear function of fluctuating velocity of gusts, the above values of g and σ_g also apply to the peak pressure.

2.13.10 Mean velocity pressure and exposure factor

The mean wind velocity pressure [21)] at height z is defined by:

$$\overline{Q}(z) = \frac{1}{2}\rho \overline{U}^2(z) \tag{27}$$

where ρ is the air density (ρ=1.25 kg/m^3 for standard air).

The coefficient of variation of the maximum annual velocity pressure is approximately the double of the coefficient of variation of the maximum annual velocity, V_1 : $V_Q \cong 2\ V_1$.

The roughness factor describes the variation of the mean velocity pressure with height above ground and terrain roughness as function of the reference velocity pressure. From Eq.(13) one gets:

$$c_r(z) = \frac{\overline{Q}(z)}{\overline{Q}_{ref}} = \frac{\overline{U}(z)^2}{\overline{U}^2_{ref}} = \left[\frac{\left(\dfrac{z}{z_{0,ref}}\right)^{0.07}}{\ln \dfrac{z_{ref}}{z_{0,ref}}} \right]^2 \left(\ln \frac{z}{z_0}\right)^2 \tag{28}$$

and $\overline{Q}(z)^2 = c_r(z)\ \overline{Q}_{ref}$ (29)

2.13.11 Gust factors for velocity pressure

The gust factor for velocity pressure is the ratio of the peak velocity pressure to the mean velocity pressure of the wind:

$$c_g(z) = \frac{q_{peak}(z)}{\overline{Q}(z)} = \frac{\overline{Q}(z) + g \cdot \sigma_q}{\overline{Q}(z)} = 1 + g \cdot V_q = 1 + g[2 \cdot I_u(z)] \tag{29}$$

where:

$\overline{Q}(z)$ is the mean velocity pressure of the wind

$\sigma_q = \overline{q(z,t)^2}^{1/2}$ - the root mean squared value of the longitudinal velocity pressure fluctuations from the mean

V_Q - coefficient of variation of the velocity pressure fluctuations (approximately equal to the double of the coefficient of variation of the velocity fluctuations):

---

[2] Conversion of the open country velocity pressure for different averaging time intervals can be guided by the following values obtained from Section 2.13.2:

$$1.1\overline{Q}^{1h} = \overline{Q}^{10min} = 0.7\overline{Q}^{1min(fastest\ mile)} = 0.44\overline{Q}^{3s}$$

$V_Q \cong 2\ I(z)$

g - the peak factor for velocity pressure.

Approximately, the longitudinal velocity pressure fluctuation, q(z,t) is a linear function of the velocity fluctuation. Since:

$$\frac{1}{2}\rho\left[\overline{U}(z)^2+u(z,t)\right]^2=\frac{1}{2}\rho\overline{U}(z)^2+\rho\overline{U}(z)u(z,t)+\frac{1}{2}\rho u(z,t)^2\cong\frac{1}{2}\rho\overline{U}(z)^2+\rho\overline{U}(z)u(z,t)$$

it is:

$$\overline{Q}(z)=\frac{1}{2}\rho\overline{U}(z)^2$$

$$q(z,t)\cong\rho\overline{U}(z)u(z,t)$$

and consequently, the mean value and the standard deviation of the peak factor for velocity pressure are the same like that for the gust velocity $\overline{g} \cong 3.2$ and $\sigma_g \cong 0.4$. The values of the peak factor depend essentially on the averaging time interval of the reference velocity.[3]

2.13.12 Exposure factor for peak velocity pressure

The peak velocity pressure at the height z above ground is the product of the gust factor: the roughness factor and the reference velocity pressure;

$$Q_g(z) = c_g(z)\ c_r(z)\ Q_{ref} \qquad (30)$$

The exposure factor is defined as the product of the gust and roughness factors:

$$c_e(z) = c_g(z)\ c_r(z). \qquad (31)$$

2.13.13. Aerodynamic shape factors

The aerodynamic shape factor, c_a is the ratio of the aerodynamic pressure exerted by the wind on the surface of a structure and its components to the velocity pressure. The aerodynamic pressure is acting normal to the surface. By convention c_a is assumed positive for pressures and negative for suctions.

As the pressure exerted on a surface is not uniformly distributed over the whole area of the surface or on the different faces of a building, the aerodynamic coefficients should be assessed separately for the different parts and faces of a building.

The aerodynamic shape factors refer either to the mean pressure or to the peak pressure of the wind.

The shape factors are dependent on the geometry and the dimensions of building, the angle of attack of the wind i.e. the relative position of the body in the airflow, terrain category, Reynolds number, etc.

In certain cases the aerodynamic factors for external pressure must be combined with those for internal pressure.

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[3] Since: $q_{peak} = c_g^{1\,min}\left(c_r Q_{ref}^{1min}\right) = c_g^{10\,min}\left(c_r Q_{ref}^{1min}\right) = c_g^{1\,h}\left(c_r Q_{ref}^{1h}\right)$ from Section 2.13.8, the following approximate relations hold: $c_g^{1\,min} = 0.7 c_g^{10\,min} = c_g^{1\,h} = 1.1 c_g^{10\,min}$

There are two different approaches to the practical assessment of wind effects on rigid structures: using pressure coefficients and using force coefficients.

- In the former case the wind force is the result of the summation of the aerodynamic forces normal to a certain surface. It is intended for parts of the structure.
- In the later case, the wind force is the product of the velocity pressure multiplied by the overall force coefficient times the frontal area of the building. This approach is used within the procedures for calculating the structural response.

Typical values of the aerodynamic shape factors can be selected from appropriate national and international documents or from wind tunnel tests. The aerodynamic shape factors should be determined in wind tunnels capable of modelling the atmospheric boundary layer.

(this section is to be improved)

2.13.14 Uncertainties consideration

The factors involved in the assessment of the wind forces on structures contain uncertainties.

The mean and the coefficient of variation of the wind forces expressed through the product of uncorrelated variables in Eq.(1) or Eq.(2) may be written as follows:

$$E(w) = E(c_g)\, E(c_a)\, E(c_r)\, E(\overline{Q}_{ref}) \tag{32}$$

$$V_w^2 = V_{c_g}^2 + V_{c_a}^2 + V_{c_r}^2 + V_{\overline{Q}_{ref}}^2 \tag{33}$$

and

$$E(w) = E(c_d)\, E(c_a)\, E(c_r)\, E(\overline{Q}_{ref}) \tag{34}$$

$$V_w^2 = V_{c_d}^2 + V_{c_a}^2 + V_{c_r}^2 + V_{\overline{Q}_{ref}}^2 \,. \tag{35}$$

Statistics of the above factors are suggested in Table 5.

Table 5 Statistics of random variables involved in the assessment of the wind loading

Variable	Ratio $\frac{\text{Expected}}{\text{Computed}}$	Coefficient of variation, V	Reference
$\overline{Q}_{ref}$	0.8	0.2 - 0.3	Davenport, 1987
c_r	0.8	0.1 - 0.2	
c_a - pressure coefficients	1.0	0.1 - 0.3	
- force coefficients	1.0	0.1 - 0.15	
c_g	1.0	0.1 - 0.15	
c_d	1.0	0.1 - 0.2	
Structure period			Vanmarcke, 1992
- small amplitudes	0.85	0.3 - 0.35	
- large amplitudes	1.15	0.3 - 0.35	
Structure damping			
- small amplitudes	0.8	0.4 - 0.6	
- large amplitudes	1.2	0.4 - 0.6	

Generally, but not necessarily, the lognormal distribution is the recommended probability distribution function for each of the partial factors involved in Eq. (32) and Eq. (34).

Relevant Literature and References

Arya S.P., 1993. Atmospheric boundary layer and its parametrization. Proceedings of the NATO Advanced Study Institute on Wind Climate in Cities, Waldbronn, Germany, July 5-16, Kluwer Academic Publishers, Dordrecht/Boston/London, p.41-66

ASCE 7-93, 1993 and Draft of ASCE7-95, 1995. Minimum design loads for buildings and other structures. American Society of Civil Engineers, New York

Bierty

CIB W81 Commission, 1994. Actions on structures. Wind loads, 6th draft, May

Davenport N.G., 1995. The response of slender structures to wind. In the wind climate and cities. Kluwer Academic Publishers, p.209-239

Davenport A.G., 1987. Proposed new international (ISO) wind load standard. High winds and building codes. Proceedings of the WERC/NSF Wind engineering symposium. Kansas City, Missouri, Nov., p.373-388

Davenport A.G., 1967. Gust loading factors. Journal of the Structural Division, ASCE, Vol.93, No.3, p.1295-1313

Davenport A.G., 1964. Note on the distribution of the largest value of a random function with application to gust loading. Proceedings. Institution of Civil Engineering, London, England, Vol. 28 June, p.187-195

Davenport A.G., 1961. The application of statistical concepts to the wind loading of structures. Proceedings, Institution of Civil Engineering, London, England, Vol.19, Aug., p.449-472

ESDU 85020, Characteristics of atmospheric turbulence near the ground. Part II: single point data for strong winds (neutral atmosphere), April 1993, 36 p. London, U.K.

ESDU 86010, Characteristics of atmospheric turbulence near the ground. Part III: variation in space and time for strong winds (neutral atmosphere), Sept. 1991, 33 p., London, U.K.

European Prestandard ENV 1991-2-4, 1994. EUROCODE 1: Basis of design and actions on structures, Part 2.4 : Wind actions, CEN

Gerstoft P., 1986. An assessment of wind loading on tower shaped structures. Technical University of Denmark, Lingby, Serie R, No.213

Ghiocel D., Lungu D., 1975. Wind, snow and temperature effects on structures, based on probability. Abacus Press, Tunbridge Wells, Kent, U.K.

Hanna???

Harris R.I., Deaves D.M., 1980. The structure of strong winds. The wind engineering in the eighties. Proceedings of CIRIA Conference 12/13 Nov., Construction Industry, Research and Information Association, London, p.4.1-4.93

ISO / TC 98 / SC3 Draft International Standard 4354, 1990. Wind actions on structures. International Organisation for Standardisation

Joint Committee on Structural Safety CEB-CECM-CIB-FIP-IABSE, 1974. Basic data on loads. Second draft. Lisbon

Kareem, A., Wind Effects on Structures, Prob. Eng. Mech., 2, 4, 1987, pp. 166-200

Karman v., T., 1948. Progress in statistical theory of turbulence. Proceedings, National Academy of Science. Washington D.C., p.530-539

Lumley J.L., Panofsky H.A., 1964. The structure of atmospheric turbulence. J.Wiley & Sons

Lungu D., Gelder P., Trandafir R., 1995. Comparative study of Eurocode 1, ISO and ASCE procedures for calculating wind loads. IABSE Colloquium, Basis of design and actions on structures, Background and application of Eurocode 1, Delft, The Netherlands, 1996

NBC of Canada, 1990. Code National du Bâtiment du Canada, 1990 and Supplement du Code, Comité Associé du Code National du Bâtiment, Conseil National de Recherche, Canada

Plate E.J., 1993. Urban climates and urban climate modelling: An introduction. Proceedings of the NATO Advanced Study Institute on Wind Climate in Cities, Waldbronn, Germany, July 5-16, Kluwer Academic Publishers, Dordrecht/Boston/London, p.23-40

Plate E.J., Davenport A.G., 1993. The risk of wind effects in cities. Proceedings of the NATO Advanced Study Institute on Wind Climate in Cities, Waldbronn, Germany, July 5-16, Kluwer Academic Publishers, Dordrecht/Boston/London, p.1-20

Ruscheweyh H., 1995. Wind loads on structures from Eurocode 1, ENV 1991-2-3. In Wind climate in cities. Kluwer Academic Publishers, p.241-258

Schroers H., Lösslein H., Zilch K., 1990. Untersuchung der Windstructur bei starkwind und Sturm. Meteorol. Rdsch., 42, Oct., Gebruder Borntraeger, p.202-212

Simiu E., Scanlan R.H., 1986. Wind effects on structures. Second edition. J. Wiley & Sons

Simiu E., 1980. Revised procedure for estimating along-wind response. Journal of the Structural Division, ASCE, Vol.106, No.1, p.1-10

Simiu E., 1974. Wind spectra and dynamic along-wind response. Journal of the Structural Division, ASCE, Vol.100, No.9, p.1897-1910

Solari G., 1993. Gust buffeting. I Peak wind velocity and equivalent pressure. Journal of Structural Engineering, ASCE, Vol.119, No.2, p.365-382

Solari G., 1993. Gust buffeting. II Dynamic along-wind response. Journal of Structural Engineering, Vol.119, No.2, p.383-398

Solari G., 1988. Equivalent wind spectrum technique: theory and applications. Journal of Structural Engineering ASCE, Vol.114, No.6, p.1303-1323

Solari G., 1987. Turbulence modelling for gust loading. Journal of Structural Engineering, ASCE, Vol.113, No.7, p.1150-1569

Theurer W., Bachlin W., Plate E.J., 1992. Model study of the development of boundary layer above urban areas. Journal of Wind Engineering and Industrial Aerodynamics, Vol. 41-44, p.437-448, Elsevier

Uniform Building Code, 1991 Edition. International Conference of Building Officials, Whittier, California

Vellozi J., Cohen E., 1968. Gust response factors. Journal of the Structural Division, ASCE, Vol.97, No.6, p.1295-1313

Vickery B.J., 1994. Across - wind loading on reinforced concrete chimneys of circular cross-section. ACI Structural Journal, May-June, p.355-356

Vickery B.J., Basu R., 1983. Simplified approaches to the evaluation of the across-wind response of chimneys. Journal of Wind Engineering and Industrial Aerodynamics, Vol.14, p. 153-166.

Vickery B.J., 1970. On the reliability of gust loading factors. Proceedings, Technical meeting concerning wind loads on buildings and structures, Building Science Series 30, National Bureau of Standards, Washington D.C., p.93-104

Vickery B.J., 1969. Gust response factors. Discussion. Journal of the Structural Division, ASCE, ST3, March, p.494-501

Wieringa J., 1993. Representative roughness parameters for homogenous terrain. Boundary Layer Meteorology, Vol.63, No.4, p.323-364

Wind loading and wind-induced structural response, 1987. State of the art report prepared by the Committee on Wind effects of the Structural Division of ASCE. ASCE, N.Y.

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JCSS PROBABILISTIC MODEL CODE
PART 3: RESISTANCE MODELS

3.1 CONCRETE PROPERTIES

Table of contents:

List of symbols:

f_{co}	=	basic concrete compression strength
M_j	=	the logarithmic mean at strength job j
Σ_j	=	the logarithmic strength standard deviation at job j
$Y_{1,j}$	=	a log-normal variable representing additional variations due to the special placing, curing and hardening conditions of in situ concrete at job j
U_{ij}	=	a standard normal variable
λ	=	lognormal variable with mean 0.96 and coefficient of variation 0.005; generally it suffices to take λ deterministically
$\alpha(t,\tau)$	=	is a deterministic function which takes into account the concrete age at the loading time t and the duration of loading τ
$\varphi(t,\tau)$	=	is the creep coefficient.
β_d	=	total load and depends from the type of the structure
E_c	=	modulus of elasticity
f_c	=	in situ strength
ε_e	=	strain at yielding
ε_u	=	ultimate strain

3.1.1 Basic Properties

The reference property of concrete is the compressive strength f_{co} of standard test specimens (cylinder of 300 mm height and 150 mm diameter) tested according to standard conditions and at a standard age of 28 days (see ISO/DIS 2736 and ISO 3893). Other concrete properties are related to the reference strength of concrete according to:

In situ compressive strength: $f_c = \alpha(t,\tau)\ f_{co}^{\lambda}$ [MPa] (1)

Tensile strength: $f_{ct} = 0.3\ f_c^{2/3}$ [MPa] (2)

Modulus of elasticity: $E_c = 10.5\ f_c^{1/3}\ (\frac{1}{1+\beta_d\,\varphi(t,\tau)})$ [GPa] (3)

Ultimate compression strain: $\varepsilon_u = 6.10^3\ f_c^{-1/6}(1+\beta_d\,\varphi(t,\tau))$ [m/m] (4)

λ is a factor taking into account of systematic variation of in situ compressive strength and strength of standard tests (see 3.1.3)

$\alpha(t,\tau)$ is a deterministic function which takes into account the concrete age at the loading time t [days] and the duration of loading τ [days]. The function is given by:

$\alpha(t,\tau) = \alpha_1(\tau)\ \alpha_2(t)$

$\alpha_1(\tau) = \alpha_3(\infty) + [1-\alpha_3(\infty)]\exp[-a_\tau\,\tau]$ with $\alpha_3(\infty) \approx 0.8$ and $a_\tau = 0.04$.

$\alpha_2(t) = a + b\ \ln(t)$

In most applications $\alpha_1(\tau) = 0.8$ can be used. The coefficients a and b in $\alpha_2(t)$ depend on the type of cement and the climatical environment; under normal conditions a = 0.6 and b = 0.12.

$\varphi(t,\tau)$ is the creep coefficient.

β_d is the ratio of the permanent load to the total load and depends from the type of the structure; generally β_d is between 0.6 and 0.8.

The decrease of compressive strength due to cyclic loading after N load cycles can be considered by a deterministic reduction factor.

$$\gamma(N) = \frac{f_c(N)}{f_c(0)} = 1 - (0.125)\log(N) \qquad (5)$$

Tensile strength of concrete can be assumed to possess the same strength reduction.

3.1.2 Stress-strain-relationship

For concrete under compression the following simplified stress-strain relationship holds:

$$\sigma = E_c\, \varepsilon \qquad \text{for } \varepsilon < \varepsilon_e \tag{6}$$

$$\sigma = f_c \qquad \text{for } \varepsilon_e < \varepsilon < \varepsilon_u \tag{7}$$

$$\varepsilon_e = f_c/E_c \tag{8}$$

For calculations where the form of the stress-strain relationships is important the following relationship should be used:

$$\sigma = f_c\left[1-\left[1-\frac{\varepsilon}{\varepsilon_s}\right]^k\right] \tag{9}$$

$$\varepsilon_s = 0.0011\, f_c^{\ 1/6} \tag{10}$$

$$k = \frac{E_c \varepsilon_s}{f_c} \tag{11}$$

The relationship holds for $0 < \varepsilon < \varepsilon_s$.

3.1.3 The probabilistic model

The strength of concrete at a particular point i in a given structure j as a function of standard strength f_{c0} is given as:

$$f_{c,ij} = \alpha(t,\tau)\,(f_{co,ij})^{\lambda}\, Y_{1,j} \tag{12}$$

$$f_{co,ij} = \exp((U_{ij}\, \Sigma_j + M_j)) \tag{13}$$

in which

$f_{c0,ij}$ = log-normal variable, independent of $Y_{1,j}$, with distribution parameters M_j and Σ_j
M_j = the logarithmic mean at job j
Σ_j = the logarithmic standard deviation at job j
$Y_{1,j}$ = a log-normal variable representing additional variations due to the special placing, curing and hardening conditions of in situ concrete at job j
U_{ij} = a standard normal variable
λ = lognormal variable with mean 0.96 and coefficient of variation 0.005; generally it suffices to take λ deterministically

The variable $Y_{1,j}$ can also be taken as a spatially varying random field whose mean value function takes account of systematic influences in space.

Correspondingly, for the other three basic properties:

$$f_{ct,ij} = 0.3\ f_{c,ij}^{2/3}\ Y_{2,j} \qquad (14)$$

$$E_{c,ij} = 10.5\ f_{c,ij}^{1/3}\ Y_{3,j}(1+\varphi(t,\tau))^{-1} \qquad (15)$$

$$\varepsilon_{u,ij} = 6\ 10^{-3}\ f_{c,ij}^{-1/6}\ Y_{4,j}(1+\varphi(t,\tau)) \qquad (16)$$

where the variables $Y_{2,j}$ to $Y_{4,j}$ mainly reflect variations due to factors not well accounted for by concrete compressive strength (e.g., gravel type and size, chemical composition of cement and other ingredients, climatical conditions).

The variables U_{ij} and U_{kj} are correlated by:

$$\rho(U_{ij}, U_{kj}) = \exp\left\{\frac{(r_{ij} - r_{kj})^2}{d_c^{\,2}}\right\} \qquad (17)$$

where $d_c = 5$ m. For different jobs U_{ij} are uncorrelated.

3.1.4 Distributions of Y_{kj}

Unless direct measurements are available, the parameters of the variables $Y_{k,j}$ can be taken from Table 2. The variables are distributed according to the log-normal distibution. The variability of the variables $Y_{k,j}$ can further be split into a part depending only on the job under consideration and a part representing spatial variability.

If direct measurements are available, the parameters in Table 3.1.1 are taken as parameters of an equivalent prior sample with size n' = 10 (see Part 1 for the details of updating).

Variable	Distribution type	Mean	Coefficient of variation	Related to
$Y_{1,j}$	LN	1.0	0.06	compression
$Y_{2,j}$	LN	1.0	0.30	tension
$Y_{3,j}$	LN	1.0	0.15	E-modulus
$Y_{4,j}$	LN	1.0	0.15	ultimate strain

Table 3.1.1: Data for parameters Y_i

3.1.5 Distribution for f_{co}

The distribution of $x_{ij} = \ln(f_{co,ij})$ is normal provided that its parameters M and Σ obtained from an ideal infinite sample. In general it must be assumed that concrete production varies from production unit, site, construction period, etc. and that sample sizes are limited. Therefore, the parameters M and Σ must also be treated as random variables. As a result x_{ij} has a student distribution according to:

$$F_x(x) = F_{tv''}\left[\frac{x - m''}{s''}(1 + \frac{1}{n''})^{-0.5}\right]$$

when $F_{t_{v''}}$ is the Student distribution for ν″ degrees of freedom.

The values of m", n", s" and ν" depend on the amount of specific information. Table 3.1.2 gives the values if no specific information is available (prior information).

Table 3.1.2: Prior parameters for concrete strength distribution (f_{co} in MPa)

Concrete type	Concrete grade	Parameters			
		m'	n'	s'	v'
Site mixed	C15	3.40	1.0	0.15	3
	C25	3.65	2.0	0.12	4
	C35	3.85	3.0	0.09	4
	C45	-	-	-	-
	C55	-	-	-	-
Ready mixed	C15	3.40	1.5	0.14	6
	C25	3.65	1.5	0.12	8
	C35	3.85	1.5	0.09	10
	C45	3.98	1.5	0.07	12
	C55	-	-	-	-
Pre-cast elements	C15	-	-	-	-
	C25	3.80	2	0.09	4.5
	C35	3.95	2.5	0.08	4.5
	C45	4.08	3	0.07	5.0
	C55	4.15	3.5	0.05	5.5

If trials are made **before** design and construction yielding the statistics (m, n, s, ν = n-1) the posterior distribution of the in situ quality should be updated following the general procedure indicated in Part 1. Such trial tests must be planned to reflect the natural variation in future concrete production. If trial tests do not include the random variations in concrete production but are made from systematically varied admixtures, the mean $m(f_c)$ of the samples taken from the selected admixture must be considered as the only direct information, that is:

$m(x) = \ln m(f_c) - 0.5\ \delta^2$ with $\delta^2 = \ln[1 + (s(f_c)/m(f_c))^2]$

Where

$n(x) = 1$
$s(x) = 7\ \text{MPa}/m(f_c)$
$v(x) = 0$

The same updating procedure can be used if the concrete producer is known in the design stage and has sufficiently long records of past, stationary production.

References

[1] Hellesland, J.; Green, R.: A stress and time dependant strength law for concrete, Cement and Concrete Research, Vol. 2, pp. 261-275, 1972.

[2] Scerbakov, E.N.; Zaitsev, J.V.: Voraussage der Verformungs- und Festigkeitseigenschaften von Beton unter Dauerlast, Cement and Concrete Research, Vol. 6, pp. 512-528, 1976..

[3] Weber, J.W.: Empirische Formeln zur Beschreibung der Festigkeitsentwicklung und der Entwicklung des E-Moduls von Beton, Betonwerke + Fertigteiltechnik, Heft 12, 1979.

[4] Yamada, M.; Kawamura, H.: Ultimate Aseismic Safety of Reinforced Concrete Structures, Proc. 7 WCEE, Istanbul, 1980, pp. 351-358.

[5] Weigler, H.: Beton bei häufig wiederholter Beanspruchung, Beton 5/91, S. 189-194.

[6] König, G.; Hosser, D.: The Simplified Level II Method and its Application on the Deviation of Safety Elements for Level I, CEB-Bulletin d'Information, No. 147, 1982.

[7] Rackwitz, R.: Predictive Distribution of Strength under Control, Materials & Structures, 16, 94, 1983, pp. 259-267.

- WIGGLY LINE INDICATES UPDATES!

PROBABILISTIC MODEL CODE, PART 3, RESISTANCE MODELS

3.* Static Properties of Structural Steel (Rolled Sections)

Properties Considered

The following properties of structural steel are dealt with herein:

X_1 = yield strength (or 0.2% proof strength) [MPa]
X_2 = ultimate tensile strength [MPa]
X_3 = modulus of elasticity [MPa]
X_4 = Poisson's ratio
X_5 = ultimate strain (or percentage elongation at fracture)

A probabilistic model is proposed for the random vector $\mathbf{X} = (X_1, \ldots X_5)$ to be used for any particular steel grade, which may be defined in terms of nominal values verified by standard mill tests (e.g. following the procedures of EN 10025 for sampling and selection of test pieces and the requirements of EN 10002-1 for testing) or in terms of minimum (hereinafter referred to as code specified) values given in material specifications (e.g. EN 10025: 1990).

Only distinct points or parts of the full stress-strain curve are considered, thus the proposed model can be used in applications where this type of information is compatible with the parameters of the mechanical model used for strength analysis.

In certain cases, where an absence of a yield phenomenon may be noted, the values given for the yield strength may be used instead for the 0.2% proof strength. However, it should be noted that most of the data examined refers to steels exhibiting a yield phenomenon.

In applications where strain-hardening (and in particular the extent of the yield plateau and the initial strain-hardening) are important (e.g. inelastic local buckling) a more detailed model, which describes the full stress-strain behaviour, may be warranted. Several deterministic models exist in the literature which would allow a probabilistic model to be developed. The parameters of the model chosen to describe the full stress-strain curve should be selected in a way that does not invalidate the statistics given in Table A for the key points of the stress-strain diagram.

Probabilistic Models and Range of Applicability

Mean values and coefficients of variation for the above vector are given in Table A whereas the correlation matrix is given in Table B. A multi-variate log-normal distribution is recommended. The values given are valid for static loading.

The COV values refer to total steel production and are based primarily on European studies from 1970 onwards. In US and Canada higher COVs have been used (on average, about 50% higher). The main references on which these estimates are based are given below.

The values in Table A may be used for steel grades and qualities given in EN 10025: 1990, which have code specified yield strength of up to 360 MPa. In view of ductility and yield ratio considerations, these estimates should not be used for ultra high strength steels (e.g. 690 MPa) without verification.

Within-batch COVs can be taken as one fourth of the values given in Table A but within-batch variability for variables X_3 and X_4 may be neglected. Variations along the length of a rolled section are normally small and can be neglected.

If direct measurements are available, the numbers in Table A should be used as prior statistics with a relatively large equivalent sample size (e.g. $n' \approx 50$).

In some of the references listed below, it is noted that the yield strength distribution of mild steel (grade S275 in EN10025: 1993) is described by a bi-modal shape, possibly indicating re-classification of nominally higher grade material. This practice would also affect mean and COV estimates quoted in Table A, which are not indicative of this possibility.

Table A: Mean and COV values

Property	Mean Value, $E[X_i]$	COV, v_i
X_1	$X_{1sp} \cdot \alpha \cdot exp(-u \cdot v_1) - C$	0.07
X_2	$B. E[X_1]$	0.04
X_3	X_{3sp}	0.03
X_4	X_{4sp}	0.03
X_5	X_{5sp}	0.06

Table B: Correlation Matrix

	X_1	X_2	X_3	X_4	X_5
X_1	1	0.75	0	0	-0.45
X_2		1	0	0	-0.60
X_3			1	0	0
X_4				1	0
X_5					1

Definitions and Remarks

- X_{isp} is the code specified or nominal value for variable X_i
- α is spatial position factor (α=1.05 for webs of hot rolled sections and α=1 otherwise)
- u is a factor related to the fractile of the distribution used in describing the distance between the code specified or nominal value and the mean value; u is found to be in the range of -1.5 to -2.0 for steel produced in accordance with the relevant EN standards; if nominal values are used for X_{isp} the value of u needs to be appropriately selected.

- *C* is a constant reducing the yield strength as obtained from usual mill tests to the static yield strength; a value of 20 MPa is recommended but attention should be given to the rate of loading used in the tensile tests.
- *B* = 1.5 for structural carbon steel
  = 1.4 for low alloy steel
  = 1.1 for quenched and tempered steel

References

[1] Baker M J, 'Variability in the Strength of Structural Steels - A Study in Material Variability; Part 1: Material Variability', CIRIA Technical Note 44, 1972.

[2] Edlund L, 'Coefficients of Variation for the Yield Strength of Steel', 2nd Colloquium on Stability of Steel Structures, Final Report, Liege, 1977.

[3] Galambos T V and Ravindra M K, 'Properties of Steel for Use in LRFD', J. Str. Div., ASCE, Vol. 104, ST9, 1978.

[4] Kennedy D J L and Baker K A, 'Resistance Factors for Steel Highway Bridges', Can. J. Civ. Eng., Vol. 11, 1984, 324-34.

[5] Yamanouchi H, Kato B and Aoki H, 'Statistical Features of Mechanical Properties of Current Japanese Steels', Document for ECCS TC13: Seismic Design, 1990.

[6] Manzocchi G M E, Chryssanthopoulos M K and Elnashai A S, 'Statistical Analysis of Steel Tensile Test Data and Implications on Seismic Design Criteria', ESEE Report 92-7, Imperial College, 1992.

[7] Agostoni N, Ballio G and Poggi C, 'Statistical Analysis of the Mechanical Properties of Structural Steel', Costruzioni Metalliche, No.2, 1994, pp. 31-39.